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Letter to the Editor

Comments on the paper "Nonlinear Convective Flow of Maxwell Fluid over a Slendering Stretching Sheet with Heat Source/Sink, Mocherla Gayatri, Konda Jayaramireddy, Macherla Jayachandra Babu, J. Appl. Comput. Mech., 8(1), 2022, 60-70"

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Abstract. Exact solutions for non-Newtonian fluids are rare, particularly for Maxwell fluids [1], such solutions do not exist. Generally, in non-Newtonian fluids, the relation which connects shear stress and shear rate is non-linear and the constitutive relation forms equations of non-Newtonian fluids which are higher order and complex as compared to Navier-Stokes equation governing the flow of viscous fluid. Due to this high nonlinearity, closed form solutions for non-Newtonian fluid flows are not possible for the problems with practical interest. More exactly, when such fluids problems are tackled via Laplace transform technique, often the inverse Laplace transforms of the transformed functions do not exist. Due to this difficulty, the researchers are usually using numerical procedures for finding the inverse Laplace transform. However, those solutions are not purely regarded as exact solutions. Owing the great diversity in the physical structure of non-Newtonian fluids, researchers have proposed a variety of mathematical models to understand the dynamics of such fluids. Mostly, these models fall in the subcategory of differential type fluids or rate types fluids. However, a keen interest of the researchers is seen in studying rate types fluids due to the fact that they incorporate both the elastic and memory effects together. The present comments concern some doubtful results included in the above paper [2].

Keywords: Convective flow, Maxwell fluid, slandering stretching sheet.

1. Introduction

Many engineering fluid mechanical problems have been solved using this boundary-layer theory rendering results which compare well with experimental observations—at least as far as Newtonian fluids are concerned. In spite of the success of this theory for Newtonian fluids, an extension of the theory to non-Newtonian fluids has turned out to be a rather formidable task. The main difficulty in reaching to a general boundary-layer theory for non-Newtonian fluids lies obviously in the diversity of these fluids in their constitutive behavior, simultaneous viscous, and elastic properties such that differentiating between those effects which arise as a result of a fluid's shear-dependent viscosity from those which are attributable to the fluid's elasticity becomes virtually impossible. But some mathematical models have been proposed to fit well with the experimental observations. The simplest model for the rheological effects of viscoelastic fluids is the Maxwell model where the dimensionless relaxation time is small. However, in some more concentrated polymeric fluids the Maxwell model is also used for large dimensionless relaxation time. The first and the simplest viscoelastic rate type model which is still used widely to account for fluid rheological effects is called Maxwell model. This model can be generalized to produce a plethora of models. Initially, the Maxwell fluid model was developed to describe the elastic and viscous response of air. However, after that, it was frequently used to model the response of various viscoelastic fluids ranging from polymers to the earth's mantle. After the pioneering work of Friedrich [3], on fractional derivatives of Maxwell fluid, several other investigations were carried out in this direction. Among them, Haitao and Mingyu [4] studied fractional Maxwell model in channel, Fetecau and Fetecau [5] determined exact solutions by means of the Fourier sine transforms for an incompressible fluid of Maxwellian type subjected to a linear flow on an infinite flat plate and within an infinite edge, etc.

2. Comments

a) In the above paper, Fig. 1 is wrong. It should be as:

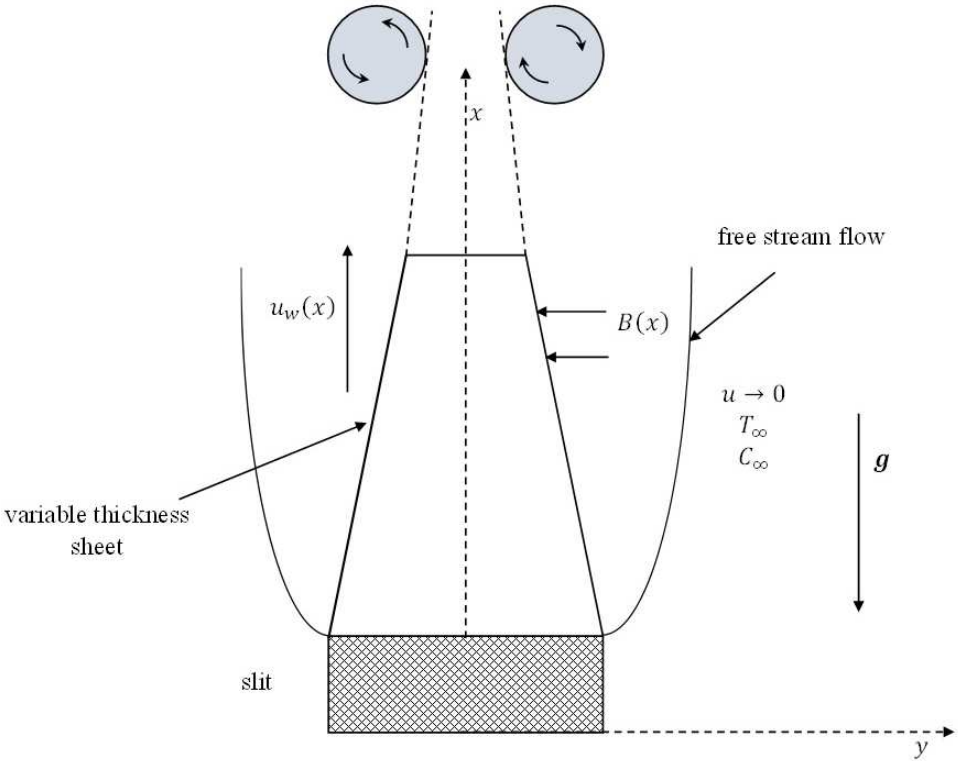

Fig. 1. Schematic diagram of Maxwell fluid flow over a slandering stretching sheet.

b) The similarity variables should have the following form:

$$\left.\begin{array}{c} \zeta = y\sqrt{\frac{r+1}{2}\frac{E}{\nu}}\,(x+q)^{(r-1)/2} \\ u = E\,(x+q)^r\,f'(\eta), \quad v = -\sqrt{\frac{r+1}{2}E\,\nu}\,(x+q)^{(r-1)/2}\left[f(\zeta)+\frac{r-1}{r+1}\,\zeta\,f'(\zeta)\right], \\ \theta(\zeta) = \frac{T-T_m}{T_\infty - T_m}, \quad \phi(\zeta) = \frac{C-C_m}{C_\infty - C_m} \end{array}\right\} \tag{6}$$

where E > 0, q and r are constants.

c) The variable temperature and concentration at the surface of the slender sheet should be defined as:

$$T_\infty = T_m(x) + [\beta_T(x+q)^{2m-1} + \beta_{1T}(x+q)^{(2m-1)/2}] \text{ and } C_\infty = C_m(x) + [\beta_C(x+q)^{2m-1} + \beta_{1C}(x+q)^{(2m-1)/2}].$$

d) In order that Eqs. (1) to (4) with the boundary conditions (5) have similarity solutions, we assume that $\lambda_0(x) = (\Lambda_0/a)\,(x+b)^{1-r}$, where Λ_0 is a constant, $\Upsilon = \lambda_0 E\,(x+q)^{1-r}\,K$, $B(x) = B_0\,(x+q)^{1-r}$, $Q_0(x) = \Gamma\,(x+q)^{1-r}$ and $Kr = \Delta\,(x+q)^{1-r}$.

e) Substituting (6) into Eqs. (2) to (4), we obtain the following ordinary (similarity) differential equations:

$$\begin{array}{c}\left(1-K\,\frac{r+1}{2}\,f^2\right)f''' + f\,f'' - \frac{2\,r}{r+1}f'^2 - K\left[\frac{2\,r\,(r-1)}{r+1}\,f'^3 - (r-1)\,\eta\,f''\,f'^2 - (3\,r-1)f\,f'f''\right] + \\ \frac{2}{r+1}(Gr\,\theta + Gr_1\theta^2 + Gr_C\phi + Gr_{C1}\,\phi^2 - M\,f') = 0\end{array} \tag{7}$$

$$\frac{1}{Pr}\,\theta'' + f\,\theta' - (2m-1)f'\theta + S\,\theta = 0 \tag{8}$$

$$\frac{1}{Sc}\,\phi'' + f\,\phi' - (2m-1)f'\phi - \frac{2}{r+1}Kr\,\phi = 0 \tag{9}$$

along with the boundary conditions:

$$\begin{array}{c} f'(0) = 1, \quad Me\,\theta'(0) + \mathrm{Pr}\left[f(0) + \frac{r-1}{r+1}\right], \qquad \theta(0) = 0, \ \ \phi(0) = 0 \\ f'(\eta) \to 1, \ \ \theta(\eta) \to 0, \ \ \phi(\eta) \to 0 \ \text{ as } \ \zeta \to \infty \end{array} \tag{10}$$

f) In Ref. [1], Eqs. (7) to (9) are wrong.

Author Contributions

T. Groşan suggested the scheme and prepared the paper; I. Pop, developed the mathematical modeling and examined the theory validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

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Data Availability Statements

Not applicable.

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