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Research Paper

Application of Newmark Average Acceleration and Ritz Methods on Dynamical Analysis of Composite Beams under a Moving Load

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Abstract. In this paper, dynamic results of carbon nanotube (CNT)-reinforced composite beams under a moving load are presented. The constitutive equations in motion are obtained by the Lagrange procedure according to Timoshenko beam theory and then solved by using the Ritz method. In the solution of the moving load problem, the Newmark average acceleration method is used in the time history. In the numerical results, the effects of CNTs' volume fraction, patterns of CNTs, and the velocity of moving load on the dynamic responses of CNT-reinforced composite beam are investigated in detail. It is observed that the reinforcement patterns and volume fraction of CNTs are very effective on the behavior of the moving load. Also, it is found that X-Beam and O-Beam have the biggest and lowest rigidities in all models, respectively.

Keywords: Reinforced composite beam, Dynamical behavior, Lagrange procedure, Newmark Average Acceleration method, Ritz method.

1. Introduction

In many engineering applications like civil, automotive, marine, aviation, and space, the most desirable properties of structural components can be specified as safe, ergonomic, aesthetic, and economical. Because the structural stiffness and efficiency can be increased while the overall cost and weight can be reduced, it may be beneficial to use variable cross-sections, composite, and reinforced members to meet the above requirements.

Tapered and stepped beams/columns with variable cross-sections are frequently used in a variety of engineering applications such as bridges, roofs, flagpoles, energy harvesters, and airplanes to save both weight and cost.

A composite material can be defined as a combination of at least two materials with different mechanical, physical, thermal, and chemical properties. The desired material can be formed, for instance, to become stronger, lighter or resistant to electricity. The main advantages of composite materials may be defined as having very high strength-to-weight and stiffness-to-weight ratios. In addition, they can have great resistance to corrosion. Composite materials can generally be classified as follows: structural, particulate and fiber-reinforced composites.

Laminated composites consist of some separate layers with various material properties. There are various types of laminated composites as symmetric, balanced, cross-ply, and angle-ply. These materials offer decisive advantages over more conventional materials due to their high specific modulus, high specific strengths, and the ability to adapt to a specific application. However, a disadvantage of laminated composites is that high stress concentrations can occur at the interfaces of adjacent layers and unfortunately this situation can cause an undesired result as delamination of the layers. Some studies on the mechanical characteristics of laminated composite structures can be given [1-7].

Functionally graded materials (FGMs) can be defined as new kind of nonhomogeneous composites composed of at least two different materials. The material properties vary gradually and smoothly throughout length, thickness, and both of them (bi-directional). FGMs have a wide range of application areas like aerospace, dental, orthopedic, military, energy, and optoelectronics. For instance, one of the most important properties of functionally graded material is its ability to inhibit crack propagation. This feature makes it useful in a defense application as a puncture-resistant material used for armor plates and bulletproof vests. A number of studies were performed on the mechanical and thermal analysis of FG structures [8-25].

Fiber-reinforced composites are other types of composite materials. They gain their strength from fibers as reinforcement set within their resin matrix. These strong and stiff fibers carry the load while the resin matrix spreads the load imposed on the composite. They have a wide range of application area as in automotive, civil, and aerospace applications, transportation industry, and high-performance sporting goods. The most used fibers are glass. Fibers can also be produced from carbon, boron, and aramid.

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While these materials offer higher tensile strength and are harder than glass, they cost much more. For this reason, carbon, boron, aramid, and graphene nanoplatelets are typically reserved for high-tech applications that require exceptional fiber properties [26-35].

Carbon nanotubes (CNTs) [36] are considered as ideal reinforcement components for new generation composite materials and have recently been used in place of the aforementioned traditional fibers in composite structures due to the exceptional properties of carbon nanotubes (CNTs) and providing good interfacial bonds. Due to the exceptional properties of them, many studies have been performed on the static and dynamic responses of CNTs [37-42]. CNT-reinforced structures have received great attention from researchers due to these unique features. Also, there are various applications of CNT-reinforced composites as tennis rackets, hockey sticks, and wind turbine blades. There are many studies on the mechanical responses carbon nanotube-reinforced composite (CNTRC) beams in the literature.

Large amplitude vibration analysis of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) beams was performed based on Timoshenko beam theory with von Kármán assumption by Ke et al. [43]. The Ritz method with a direct iterative method was employed to solve the dynamic problem for different boundary conditions. The free vibration and buckling responses of nanocomposite beams surrounded by an elastic medium were investigated by Yas and Samadi [44]. The generalized differential quadrature method (GDQM) was applied for the solution. The vibrational behaviors of FG-CNTRC beams based on Euler- Bernoulli and Timoshenko beam theories via finite element method were surveyed by Yas and Heshmati [45] and Heshmati and Yas [46]. Ke et al. [47] studied the dynamic stability analysis of FG-CNTRC beams under an axial force based on first-order shear deformation beam theory via DQM. The bending, free vibration, and buckling analyses of embedded CNTRC beams resting on Pasternak elastic foundation were made by Wattanasakulpong and Ungbhakorn [48]. Navier's method was utilized to solve these problems analytically. An investigation on the nonlinear thermo-mechanical bending behavior of simply supported FG-CNTRC plates under uniform and sinusoidal load was performed by Shen [49]. Vibrational responses of CNTRC beams were examined based on shear deformation beam theories [50-54]. The free and forced vibration analyses of CNTRC beams with constant and varying cross-sections were comprehensively addressed via finite element method. Mohseni and Shakouri [55] perused the free vibration and buckling responses of tapered FG-CNTRC beams embedded in an elastic medium on the basis of Timoshenko beam theory. The dynamic analysis in pad concrete foundation containing Silica nanoparticles was performed by applying differential quadrature and Newmark methods by Taherifar et al. [56]. Thermal buckling of FG Euler-Bernoulli beam with temperature-independent and temperature-dependent properties was examined by Chen et al. [57]. Beni et al. [58] presented the nonlinear forced vibration of visco-piezo-electro nanobeam with couple stress effect. Additionally, dynamics and stability of FG flexoelectric smart beam were given with size-dependency by Esmaeili and Beni [59]. Buckling and vibration responses of CNTRC beams resting on elastic foundation were perused with shear deformation and stretching effects by Khelifa et al. [60] and Hadji et al. [61]. The concrete structure was modeled as a thick plate and the governing equations were obtained on the basis of Mindlin plate theory. On the other hand, there have recently been many studies on the mechanical behaviors of beam type structures [62-78] and plates [79-100].

After a literature survey, it can be seen that the studies on the dynamic displacements of CNTRC beams under a moving load are limited. The purpose of this study is to peruse the vibrational behavior of a simply supported carbon nanotube-reinforced composite (CNTRC) beam subjected to a moving load. The composite beam is considered as made of polymeric matrix and reinforced by single-walled carbon nanotubes with various distribution patterns. Timoshenko beam theory is used for mathematical modeling of CNTRC beam. The governing equations are obtained by the Lagrange procedure and solved by utilizing Ritz method. In the solution of the moving load problem, the Newmark average acceleration method is employed in the time history. The influences of the velocity of moving load, volume fraction and distribution patterns of CNT on the dynamic responses of CNTRC beam are examined in detail.

2. Theory and Formulation

In Fig. 1, a schematic configuration of a simply supported CNTRC beam subjected to a moving load is displayed with three different patterns of CNTs [60, 61]. The length, height, and width of beam are represented by L, h, and b, respectively. On the other hand, Q and v_f denote the magnitude and the constant velocity of load, respectively.

The effective material properties for CNTRC beams are given below [48, 49]:

$$E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_p E^p \tag{1}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_p}{E^p} \tag{2}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_p}{G^p} \tag{3}$$

$$V_{CNT} + V_p = 1 \tag{4}$$

$$v = V_{CNT} v^{CNT} + V_p v^p \tag{5}$$

$$\rho = V_{CNT} \rho^{CNT} + V_p \rho^p \tag{6}$$

where E, G, v, and ρ denote the modulus of elasticity, shear modulus, Poisson's ratio, and density. The superscripts CNT and p respectively symbolize the related material properties of carbon nanotube and polymer matrix. η_1, η_2, η_3 can be indicated the efficiency parameters of CNT. Also, V_{CNT} and V_p are the volume fractions for CNT and polymer matrix, respectively.

Table 1. Volume fractions of CNTs as a function of thickness direction for different distributions of CNTs [48, 49].

Patterns of CNTs	V_{CNT}
UD	V_{CNT}^*
FG-O	$2V_{CNT}^*\left(1-2\frac{\lvert z\rvert}{h}\right)$
FG-X	$4V_{CNT}^*\frac{\lvert z\rvert}{h}$

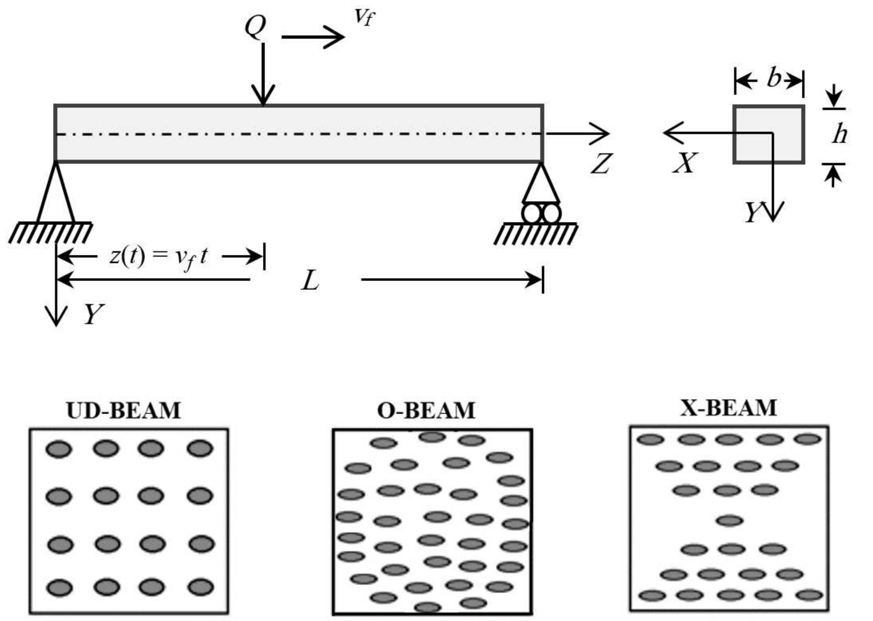

Fig. 1. A schematic configuration of a simply supported CNTRC beam under a moving load and three different patterns of CNTs.

Volume fractions of CNTs as a function of thickness direction for different patterns of CNTs [48] are presented in Table 1. In this table, V_{CNT}^{*} is the given volume fraction of CNTs. In this study, the efficiency parameters of CNTs for three different values of V_{CNT}^{*} are considered as the same with [44].

Based on Timoshenko beam theory, the displacement field of an initially straight beam can be given as:

$$u_x(z,y,t) = 0,\ u_y(z,y,t) = v_0(z,t),\ u_z(z,y,t) = u_0(z,t) - Y\emptyset(z,t) \tag{7}$$

where u_0 and v_0 are the axial and the transverse displacement of any point on the neutral axis, $\emptyset$ is the total bending rotation of the cross-sections at any point on the neutral axis.

After some mathematical manipulations, the strain energy (U_i), the kinetic energy (K), the dissipation function and potential energy of the external loads (U_e) are presented as follows:

$$U_i = \frac{1}{2}\int_0^L \left[A_0\left(\frac{\partial u_0}{\partial z}\right)^2 - 2A_1\frac{\partial u_0}{\partial z}\frac{\partial \emptyset}{\partial z} + A_2\left(\frac{\partial \emptyset}{\partial z}\right)^2\right]dZ + \frac{1}{2}\int_0^L K_s B_0\left[\left(\frac{\partial v_0}{\partial z}\right)^2 - 2\frac{\partial v_0}{\partial z}\emptyset + \emptyset^2\right]dZ \tag{8}$$

$$K = \frac{1}{2}\int_0^L\left(I_0\left(\frac{\partial u_0}{\partial t}\right)^2 - 2I_1\left(\frac{\partial u_0}{\partial t}\right)\left(\frac{\partial \emptyset}{\partial t}\right) + I_2\left(\frac{\partial \emptyset}{\partial t}\right)^2 + I_0\left(\frac{\partial v_0}{\partial t}\right)^2\right)dZ \tag{9}$$

$$U_e = -Q(t)\,v(z_p,t) \tag{10}$$

where K_s is the shear correction factor.

$$(A_0,A_1,A_2) = \int_A E(Y)(1,Y,Y^2)dA\,,\ B_0 = \int_A G(Y)dA,,\ (I_0,I_1,I_2) = \int_A \rho(Y)(1,Y,Y^2)dA \tag{11}$$

in which E and G are the Young's modulus and shear modulus.

The Lagrangian functional of the problem is presented as:

$$I = K - (U_i + U_e) \tag{12}$$

In the solution of the problem in Ritz method, approximate solution is given as a series of i terms of the following form:

$$u_0(z,t) = \sum_{i=1}^{\infty} \mathrm{a}_i(t)\alpha_i(z) \tag{13a}$$

$$v_0(z,t) = \sum_{i=1}^{\infty} \mathrm{b}_i(t)\beta_i(z) \tag{13b}$$

$$\emptyset(z,t) = \sum_{i=1}^{\infty} \mathrm{c}_i(t)\gamma_i(z) \tag{13c}$$

where a_i, b_i, and c_i are the unknown coefficients, $\alpha_i(z,t)$, $\beta_i(z,t)$, $\gamma_i(z,t)$ are the coordinate functions depending on the end conditions over the interval [0, L]. The coordinate functions for the simply supported beam are given as algebraic polynomials:

$$\alpha_i(z) = z^i \tag{14a}$$

$$\beta_i(z) = (L-z)z^{\mathrm{i}} \tag{14b}$$

$$\gamma_i(z) = z^{(i-1)} \tag{14c}$$

where i indicates the number of polynomials involved in the admissible functions. After substituting Eqs. (8)-(10) into Eq. (12), and then using the Lagrange's equation, gives the following equation:

$$\frac{\partial I}{\partial q_i} - \frac{\partial}{\partial t}\frac{\partial I}{\partial \dot{q}_i} = 0 \tag{15}$$

where q_i is the unknown coefficients which are a_i, b_i, and c_i. After implementing the Lagrange procedure, the motion equation of the problem is obtained as follows:

$$[\mathrm{K}]\{\mathrm{q(t)}\} + [\mathrm{M}]\{\ddot{\mathrm{q}}(\mathrm{t})\} = \{\mathrm{F(t)}\} \tag{16}$$

where $[K]$, $[M]$, and $\{\mathrm{F(t)}\}$ are the stiffness matrix, mass matrix, and load vector, respectively. The details of these expressions are given as follows:

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \tag{17}$$

where

$$K_{ij}^{11} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_0 \frac{\partial \alpha_i}{\partial z}\frac{\partial \alpha_j}{\partial z} dz \tag{18a}$$

$$K_{ij}^{12} = 0 \tag{18b}$$

$$K_{ij}^{13} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_1 \frac{\partial \alpha_i}{\partial z}\frac{\partial \gamma_j}{\partial z} dz \tag{18c}$$

$$K_{ij}^{21} = 0 \tag{18d}$$

$$K_{ij}^{22} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \frac{\partial \beta_i}{\partial z}\frac{\partial \beta_j}{\partial z} dz \tag{18e}$$

$$K_{ij}^{23} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \frac{\partial \beta_i}{\partial z}\gamma_j \, dz \tag{18f}$$

$$K_{ij}^{31} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_1 \frac{\partial \gamma_i}{\partial z}\frac{\partial \alpha_j}{\partial z} dz \tag{18g}$$

$$K_{ij}^{32} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \gamma_i \frac{\partial \beta_j}{\partial z} dz \tag{18h}$$

$$K_{ij}^{33} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_2 \frac{\partial \gamma_i}{\partial z}\frac{\partial \gamma_j}{\partial z} + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \gamma_i \gamma_j \, dz \tag{18i}$$

$$[M] = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \tag{19}$$

where

$$M_{11} = \sum_{\mathrm{i}=1}^{\mathrm{n}}\sum_{\mathrm{j}=1}^{\mathrm{n}}\int_0^{\mathrm{L}} \mathrm{I}_0 \, \alpha_i \alpha_j \mathrm{dz} \tag{20a}$$

$$M_{12} = 0 \tag{20b}$$

$$M_{13} = -\sum_{\mathrm{i}=1}^{\mathrm{n}}\sum_{\mathrm{j}=1}^{\mathrm{n}}\int_0^{\mathrm{L}} \mathrm{I}_1 \, \alpha_i \gamma_j \mathrm{dz} \tag{20c}$$

$$M_{21} = 0 \tag{20d}$$

$$M_{22} = \sum_{\mathrm{i}=1}^{\mathrm{n}}\sum_{\mathrm{j}=1}^{\mathrm{n}}\int_0^{\mathrm{L}} \mathrm{I}_0 \, \beta_i \beta_j \mathrm{dz} \tag{20e}$$

$$M_{23} = M_{32} = 0 \tag{20f}$$

$$M_{31} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{L} I_1\,\gamma_i\alpha_j dz \tag{20g}$$

$$M_{33} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{L} I_2\,\gamma_i\gamma_j dz \tag{20h}$$

$$\{F(t)\} = F\beta_j(v_p t) \qquad 0 \le t \le {}^{L}/_{v_f} \tag{21}$$

The constitutive equation of motions is solved by implementing implicit Newmark average acceleration method with $\alpha = 0.5$, $\beta = 0.5$ in the time domain. By this procedure, the dynamic problem is transferred to system of static problem in each step as following:

$$[\bar{K}(t,X)]\{d_n\}_{j+1} = \{\bar{F}(t)\} \tag{22}$$

in which

$$[\bar{K}(t,X)] = [K] + \frac{[M]}{\beta\Delta t^2} + \frac{[C]\alpha}{\beta\Delta t} \tag{23a}$$

$$\{\bar{F}(t)\} = \{F(t)\}_{j+1} + B_1\{d_n\}_j + B_2\{\dot{d}_n\}_j + B_3\{\ddot{d}_n\}_j \tag{23b}$$

where

$$B_1 = \frac{[M]}{\beta\Delta t^2}, \quad B_2 = \frac{[M]}{\beta\Delta t}, \quad B_3 = [M]\left(\frac{1}{2\beta} - 1\right) \tag{24}$$

After evaluating $\{d_n\}_{j+1}$ at a time $t_{j+1} = t_j + \Delta t$, the acceleration and velocity vectors can be expressed as:

$$\{\ddot{d}_n\}_{j+1} = \frac{1}{\beta\Delta t^2}\left(\{d_n\}_{j+1} - \{d_n\}_j\right) - \frac{[M]}{\beta\Delta t}\{\dot{d}_n\}_j - \left(\frac{\alpha}{2\beta} - 1\right)\{\ddot{d}_n\}_j \tag{25a}$$

$$\{\dot{d}_n\}_{j+1} = \{\dot{d}_n\}_j + \Delta t\,(1-\alpha)\{\ddot{d}_n\}_j + \Delta t\,\alpha\,\{\ddot{d}_n\}_{j+1} \tag{25b}$$

The dimensionless quantities are presented as follows:

$$\bar{v} = \frac{v_f}{L}, \quad t^* = \frac{t}{L}v_f \tag{26}$$

where $\bar{v}$ is the dimensionless transversal displacement and t^* is dimensionless time.

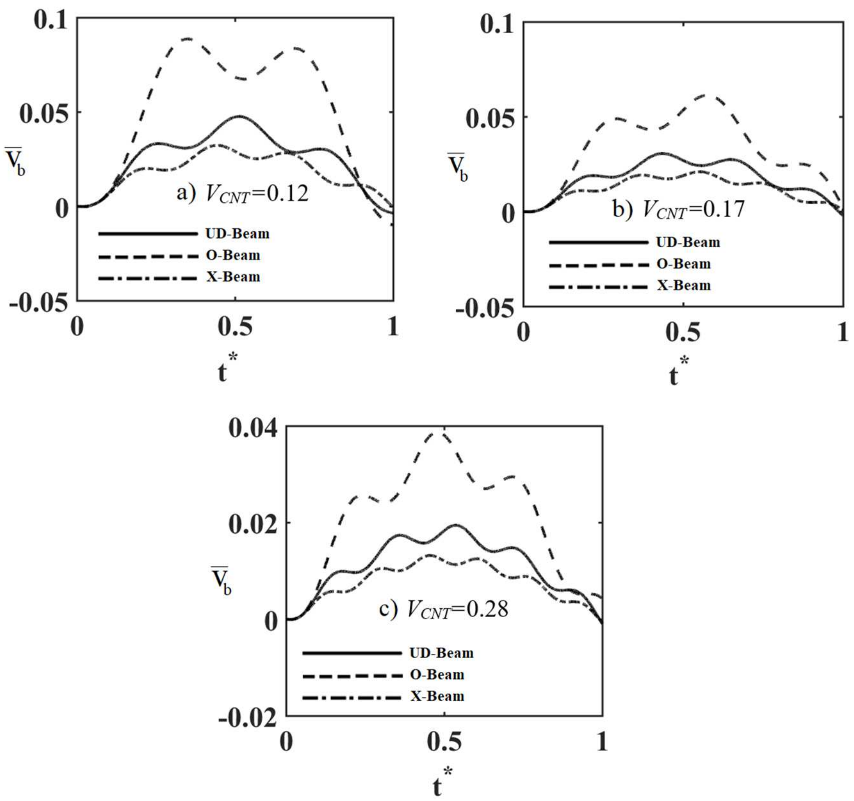

Fig. 2. Time history of dimensionless dynamic displacements at midspan of CNTRC beam with different reinforcement patterns and volume fraction of CNTs.

3. Numerical Results

In this section, the effects of carbon nanotube volume fraction, patterns of CNTs and velocity of moving load on the dynamic displacements of CNTRC beam are perused and discussed. The five-point Gauss rule is employed to calculate the integration. In the numerical results, number of the series term is taken 10.

In this study, the following material properties for reinforcement and matrix constituents are used as [44, 48]: $E_{11}^{CNT} = 600\,GPa$, $E_{22}^{CNT} = 10\,GPa$, $G_{12}^{CNT} = 17.2\,GPa$, $\upsilon^{CNT} = 0.19$, $\rho^{CNT} = 1400\,kg/m^3$, $E^p = 2.5\,GPa$, $\upsilon^p = 0.30$, and $\rho^p = 1190\,kg/m^3$. It is also notable that the shear correction factor (K_s) is chosen as 5/6 in the analysis. The geometry parameters of the beam are selected as b=0.1 m, h=0.1 m and L=4 m and the magnitude of load is chosen as Q=100 kN.

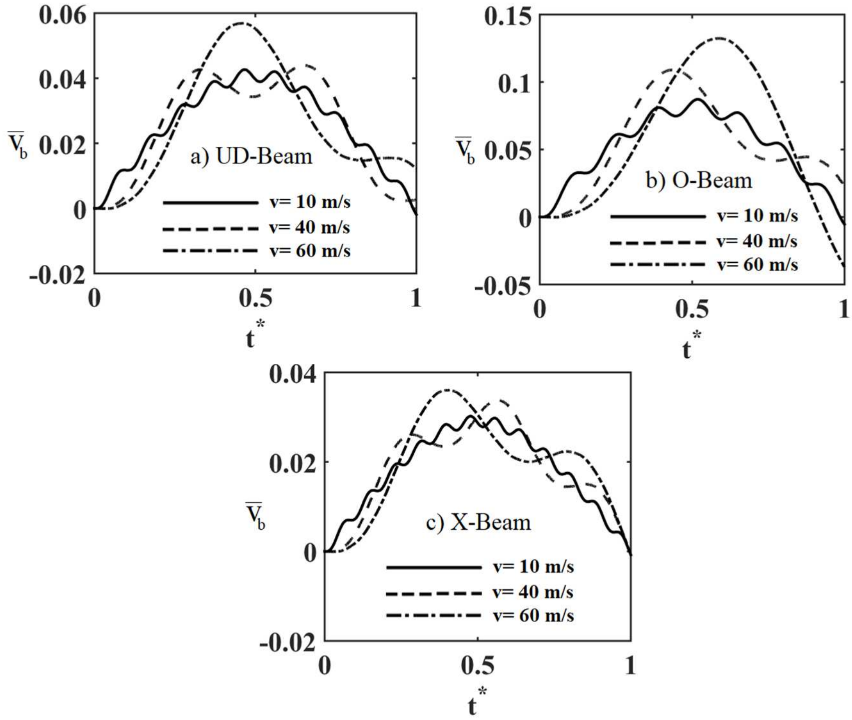

Fig. 3. Time history of dimensionless dynamic displacements at midspan of CNTRC beam with different reinforcement patterns and the velocities of load for volume fraction of CNTs V_{CNT}=0.12.

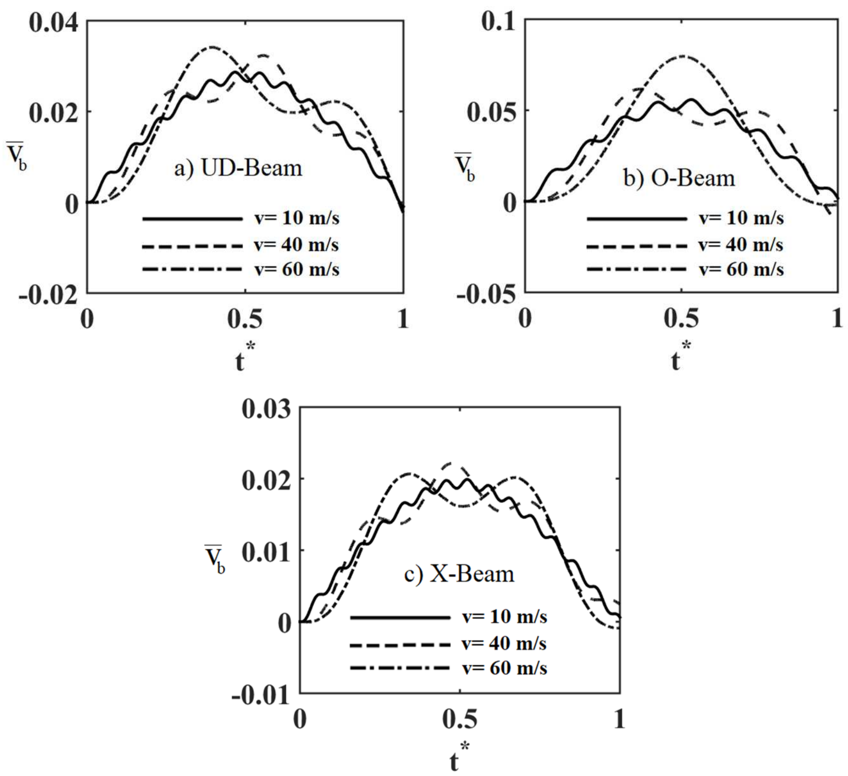

Fig. 4. Time history of dimensionless dynamic displacements at midspan of CNTRC beam with different reinforcement patterns and the velocities of load for volume fraction of CNTs V_{CNT}=0.17.

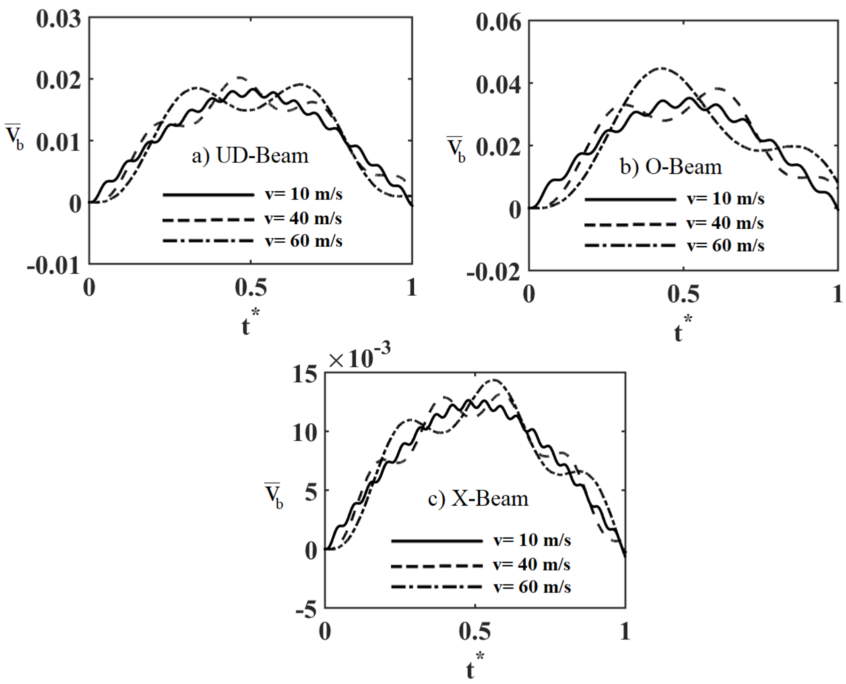

Fig. 5. Time history of dimensionless dynamic displacements at midspan of CNTRC beam with different reinforcement patterns and the velocities of load for volume fraction of CNTs V_{CNT}=0.28.

In Fig. 2, time histories of dimensionless dynamic displacements at midspan of CNTRC beam are presented with different reinforcement patterns and volume fraction of CNTs for V_{CNT}=0.12 , V_{CNT}=0.17 , and V_{CNT}=0.28. In this figure, the velocity of load is taken as $v_f = 30\,m/s$. It is seen from Fig. 2 that the dynamical displacements of O-Beam are the biggest of all. It is known that the upper and lower surfaces of the beam have high axial stresses and strains. So, O-Beam model has lowest stiffness. Consequently, more deflections occur in O-Beam model. Also, the dynamical displacements of UD-Beam are bigger than those of X-beam. This is because; however, the reinforcements spread at bother surfaces in UD- and X-Beams, X-beam has the biggest specific strength in all patterns. So, dynamic responses of X-Beam are the lowest of all.

Figures 3-5 depict the effects of velocities of load on the dynamic displacements at midspan of CNTRC beam with different reinforcement patterns for volume fraction of CNTs for V_{CNT}=0.12, V_{CNT}=0.17 and V_{CNT}=0.28, respectively. It is observed from these figures that the reinforcement patterns and volume fraction of CNTs are very effective on the behavior of the moving load. With different load velocity, the dynamic responses of different reinforcement patterns change considerably. For example, the dimensionless displacements at midspan for different velocity of moving load are $\bar{v}_{60\,\mathrm{m/s}} > \bar{v}_{10\,\mathrm{m/s}} > \bar{v}_{40\,\mathrm{m/s}}$ for t^*=0.5 for UD-beam. However, this situation is very different for O-beam and X-beam such as $\bar{v}_{60\,\mathrm{m/s}} \cong \bar{v}_{40\,\mathrm{m/s}} > \bar{v}_{10\,\mathrm{m/s}}$ for t^*=0.5 for X-beam.

4. Conclusion

Dynamic response of Carbon nanotube-reinforced composite beams under a moving load is investigated. The composite beam is considered as made of polymeric matrix and reinforced the single-walled carbon nanotubes. Three different distribution patterns are taken into consideration. Lagrange procedure is used to evaluate the governing equations on the basis of Timoshenko beam theory. The resulting equation is solved by utilizing Newmark average and Ritz methods. The effects of patterns and volume fractions of CNTs, and moving load's velocity on the dimensionless dynamic displacements of CNTRC beam are examined. The main findings of the present study can be outlined as:

- The volume fraction of CNTs are quite effective.
- The dynamic displacements decrease by increasing the volume fraction of CNTs.
- The effect of distribution pattern is also considerable.
- X-Beam is the most rigid model while O-Beam model has the lowest rigidity.

Author Contributions

Ö. Civalek planned the scheme, initiated the project, and suggested the numerical simulation; S.D. Akbas, H.M. Numanoglu and B. Akgöz developed the mathematical modeling and examined the theory validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Not applicable.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

a_i, b_i, c_i	Unknown coefficients	u_0	Axial displacement
b	Width of beam cross-section	U_e	Potential energy of external loads
E	Elastic modulus	U_i	Strain energy
{F(t)}	Load vector	$\bar{v}$	Dimensionless transversal displacement
G	Shear modulus	v_0	Vertical displacement
h	Height of beam cross-section	v_f	Velocity of load
I	Lagrangian functional	V_{CNT}	Volume fraction of carbon nanotube
K	Kinetic energy	V_p	Volume fraction of polymer matrix
K_s	Shear correction factor	α_i, β_i, γ_i	Coordinate functions
[K]	Stiffness matrix	η	Efficiency parameter
L	Length of beam	υ	Poisson's ratio
[M]	Mass matrix	ρ	Density
Q	Magnitude of load	$\emptyset$	Rotation
t^*	Dimensionless time	ω_b	Dimensionless frequency

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