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Research Paper

Analysis of Possibilities to Use Predictive Mathematical Models for Studying the Dam Deformation State

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Abstract. Long-term monitoring of the safety and reliability of large dams operation has been attracting increasing attention of researchers. Moreover, special consideration is given to the study of dam displacements that characterize its global behavior. The article discusses specifics of constructing predictive mathematical models for studying the deformation process associated with displacements of the high-head dam crest. The authors present the most successfully designed predictive mathematical models for various combinations of input effective factors, including the results of field observations and the calculated values of the component displacements. These models allow forecasting the control points of the dam body for various time stages of its operation. The advantages of using a mathematical model with separate introduction of the main effective factors into the model are shown, thereby eliminating the effect of their multicollinearity. Using the example of the Sayano-Shushenskaya hydro power plant for certain time stages of the dam operation and structures with different temperature conditions (average, warm and cold in respect to annual temperatures), the authors present the results of forecasting the dam displacements.

Keywords: Predictive mathematical model; forecasting; displacements of control points; structure deformations.

1. Introduction

The accident that occurred on August 17, 2009 at Sayano-Shushenskaya Hydro Power Plant (SSHPP) [1], resulted in human casualties, and the consequences of this accident affected the environmental, social and economic spheres of life in the entire region. Ensuring reliable and safe operation of high-head dams is one of the most important tasks in the process of their running [2]. Given that many of these structures have been in operation for a long time, the likelihood of negative, and sometimes very dangerous in terms of consequences processes, significantly increases. Therefore, numerous studies of methods to analyze monitoring data for the safe operation of dams in real time are becoming increasingly important.

In order to analyze and forecast the behavior of concrete dams based on real-time monitoring data over the past decades, numerous mathematical models have been developed. Analysis of publications in the field of theory and methods for creating models allows to identify several areas of research: the development of statistical, deterministic, and hybrid models [2–10]. In fact, various components, including those leading to irreversible changes, can be simulated – for example, plasticity, the possibility of occurrence and advancement of cracks in the dam body, filtering of water through the dam [6,11] and its irreversible displacements, rheological properties of the base [12–16] etc. Since many dams were built in mountainous regions with unfavorable geological and geotechnical conditions (faults, cracks, rock shear zones, etc.), research into stability of reservoirs' sides is mandatory [17, 18]. However, most studies focus on dam displacements that characterize its global behavior.

As of today, a considerable body of data from field observations of concrete dams operation has been accumulated. Although "pure" mathematical / statistical models in physics and engineering are regularly criticized (for example, [19]), the regression analysis is traditionally used for processing the data of dam displacement because allows to identify patterns and interdependencies in the reaction of the structure controlled parameters to applied external effects. In this case, the statistical model compares favorably with other models by providing a simpler way to establish and identify any type of correlation between the effective factors and output variables during the implementation of well-known and studied regression methods, including multiple linear regression [5,10,20], stepwise regression [21], partial regression of the least squares method [22], logistic regression [23], etc.

It should be noted that most of the developed models cannot correctly solve the forecasting problem due to the fact that the influence of the temperature factor on the dam displacement is nonlinear and has a lag effect. There is also the influence of multicollinearity between the main influencing factors (change in hydrostatic pressure and change in concrete temperature of the dam body), which introduces additional ambiguities and difficulties in the final forecasting results.

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(a) (b)

Fig. 1. Photo and Figure layout of the Sayano-Shushenskaya dam, Russia [5]

To eliminate these negative phenomena, numerous studies were carried out to improve the quality of models [7, 22, 24, 25], and methods for analyzing output parameters using models of exponential, hyperbolic, and logarithmic functions were proposed [26]. It should be noted that most conventional statistical methods cannot correctly solve this problem because the temperature factor impact on the dam displacement is nonlinear and has a delay effect [27, 28]. Therefore, the correlation-regression models can be adequately applied when, during mathematical processing, it is possible to identify and take into account the duration of the inertial delay of the dam response to the effective factors.

We managed to significantly improve the quality of mathematical models by reducing the influence of multicollinearity using machine learning methods that study approaches to constructing self-learning algorithms which focus on choosing a reasonable network structure and learning algorithm, such as a multilayer perceptron neural network (MLP) focused mainly on the accuracy and speed of convergence of models [8]; algorithm-based extreme learning machine (ELM), which combined the results of the mean prediction and the regression analysis model to improve the algorithm's ability for the generalization [29]; support vector machine (SVM) [30] which combines the advantages of solving small sample problems, nonlinear and multivariate learning, and LSM to improve the convergence rate of the algorithm; a decision tree model combined with a statistical model [31] and other fairly efficient methods for monitoring dam safety [2,32–36].

Mathematical models for assessing and predicting the strength and stability of hydrotechnical structures and their foundations are presented in literature: based on artificial neural networks in combination with a hybrid finite element method [37]; based on a dynamic analysis of concrete dams state in the frequency domain due to the change in vibrations shape [16]; as a result of solving differential equations in order to model deformations based on the data from an integrated system of combined sensors [12]; as a result of using the theory of non-stationary time series for the analysis of structural data using the algorithm for maximizing expectations with the view to finding estimates of the maximum likelihood of probabilistic models parameters [38]; using a joint approach comprising wavelet analysis technology to create an initial model of data on the dam behavior and subsequent construction of an identification model that can relate the loads and behavior of the dam [39].

Many of the predictive mathematical models discussed above yield good forecasting results. However, it is rather difficult to correctly predict the displacements of the dam body control points under various adverse temperature conditions of the structure operation (warm and abnormally cold years), as well as in emergency situations. It can be explained by the fact that the displacement magnitude is additionally influenced by factors that are difficult to take into account. In addition, as mentioned above, the influence of the temperature factor on dam displacement is nonlinear and has a lag effect, while the effect of multicollinearity between the influencing factors (change in hydrostatic pressure and change in the temperature of the concrete of the dam body) additionally introduces certain ambiguities and difficulties in the final forecasting results.

Therefore, further improvement of predictive mathematical models for ensuring reliability and safety of large dams is the important task of monitoring with the purpose to identify abnormal results in the values of the most significant (in terms of the dam safe operation) diagnosed parameter – its radial displacement.

- Earlier in [40] the prognostic models was describe developed by the authors of this manuscript, based on the theory of dynamic systems. This research devoted to improvement of these models for the more accurate prediction of the displacements of the dam body at the SSHPP.

The purpose of this research is to further improve predictive mathematical models based on the theory of dynamic systems that allow to solve a number of the set tasks:

- to eliminate the influence of the correlation dependence between input effective factors by introducing them separately into the model;
- to take into account duration of the inertial delay of the dam response to the effective factors resulting from the introduction of a correction in the form of a transport delay in the final forecasting results;
- to significantly increase the accuracy of forecasting.

2. Materials and Research Methods

The dam of the Sayano-Shushenskaya HPP (SSHPP) [41] was chosen as the object of study. The dam is one of the 25 largest dams in the world and, until a certain time, it was considered one of the most reliable structures of the type.

The arched-gravity dam of the SSHPP (Fig. 1 a,b) has a maximum height of 242 m, the total length of the dam along the ridge is 1074 m, the thickness of the dam along the ridge is 25 m, and the maximum thickness at the base of the dam is 104 m [5]. The dam is divided into 68 sections by radial seams every 15.8 m. The left-bank part of the dam is non-overflow and consists of 16 sections (1 – 16 sections) with a total length of 252.2 m along the crest of the dam. The right-bank part of the dam is also non-overflow and consists of 19 sections (sections 49 – 67) with a total length of 300 m. The spillway section of the dam consists of 12 sections (sections 37 – 48) with a total length of 190 m. The station section of the dam consists of 21 sections (sections 16 – 36) with a total length of 331 m along the ridge. Section 33 is a key section.

To ensure safety and control of the technical condition of the SSHPP dam, the radial displacements of the dam crest of the key section 33 (elevation 542 m), and the corresponding values of such parameters as hydrostatic pressure and the temperature of the dam body concrete at the lower (elevation 462 m; distance from the dam head 30.7 m; distance from the downstream side 2.85 m) and upper (elevation 461 m; distance from the dam head 2.6 m) base points, are selected as diagnostic indicators [5]. Measurements of the radial displacements of the dam are carried out using straight and reverse plumb lines located in sections 10, 18, 25, 33, 39, 45, 55 (shown by red lines in Fig. 2). These plumb lines are used to measure 69 controlled points of the dam body at elevations 344, 359, 386, 413, 440, 467, 521, 542 m. The choice of base points by temperature is explained by the fact that for the Sayano-Shushenskaya dam, it was possible to select such thermometers, whose readings generally characterize heating and cooling of the upstream and downstream faces of the dam. In particular, cooling of concrete at the lower base point by 1 degree causes an increase in the deflection of section 33 by 2.4 mm; and concrete cooling at the top base point by 1 degree leads to a decrease in deflection by 2.2 mm.

During the SSHPP dam operation, the influence of the temperature factor rates second after hydrostatic pressure; while its effect on the structure is multidirectional. In the winter season, as a result of this impact, the dam tilts towards the downstream, i.e. in the same direction as from the effect of hydrostatic load, which does not lead to deterioration of the dam body stress state due to the winter "drawdown" of the reservoir [5, 13–15]. And, on the contrary, in the summer period, due to heating of the downstream face, the dam tilts in the direction opposite to the hydrostatic load. And this poses a danger to the repaired zones of the dam resulting from violation of the certain established balance between the relevant values of temperature and hydrostatic components of displacements during the reservoir filling (May-September) and its initial drawdown (second half of September-October).

It is possible to identify components of these displacements by calculation based on a finite element analysis of the structure state with the help of ANSYS software package in the process of constructing the temperature field of the dam. So, in [42], based on the analysis for the periods of dam operation from 2004 to 2015, the authors distinguished a favorable temperature (warm) year; average and unfavorable (cold) years. It should be noted that the displacement values of the temperature components can reach relatively average values up to 5 mm for the cold year and up to 3.5 mm for the warm year, which is equivalent to a change in the hydrostatic head by 1.5 and 1.0 m, respectively. As a result of the above, it is reasonable to reduce the level of hydrostatic pressure in the winter period and to allow its increase in the summer period of the construction, while carefully controlling the amount of the dam crest displacement, in order to eliminate the threat of violating integrity of the repaired areas of the dam body and the possibility of new cracks appearance. Such recommendations formed the basis for regulating the dispatch schedule of the reservoir during the development of water use rules (WUR) at the Sayano-Shushenskaya HPP [42]. Earlier in [40], we interpreted this circumstance as the structure operation in the standard (average-temperature year) and abnormal (warm and cold years) operating conditions. This allowed, when forecasting the dam body displacements, to introduce a correction in the form of transport delay in the final forecasting results.

The least squares method (LSM) and statistical data processing were used as research methods with a view to further improving the predictive models of the dynamic type. It is well known that the degree of various factors' influence on the value of the dam displacement can be determined using various statistical models, however, their correct application is possible only in those cases where it is possible to identify and take into account during mathematical processing the duration of the inertial delay of the dam response to effective factors. In this regard, dynamic models based on the theory of dynamic systems with the properties of a number of methods for describing the structure deformations are more efficient. These models possess variable structure which is relevant to the physical nature of the process development, they take into account the inertial nature of the structure interaction with the environment and respond to temporal changes of the effective factors [40]. Moreover, the discreteness of the model, i.e. the cyclical nature of field observations can be selected with the obligatory fulfillment of the condition that the conducted field observations should refer to the basic laws describing the development of the studied process of deformation and changes in the main effective factors.

It should be noted that the prognostic models developed by the authors of this manuscript, based on the theory of dynamic systems, are used for the first time to predict the displacements of the dam body at the SSHPP.

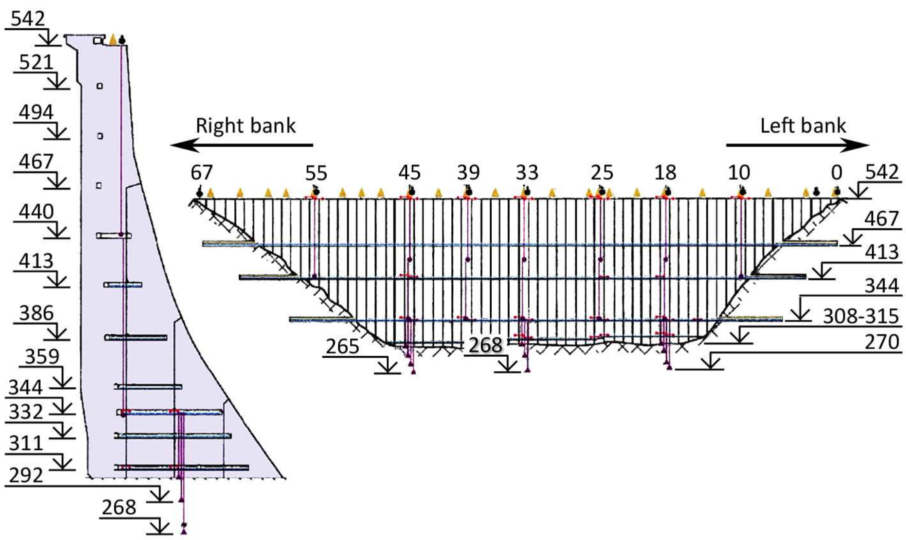

Fig. 2. Scheme of measurement points at SSHPP. Adopted from [5]

2.1 Development of dynamic models

Earlier in [40, 43], it was shown that the properties of the recurrence equation solution in the form of the first two conditional moment functions of the process of the structure observed points displacement make it a predictive model that allows to forecast displacements of the observed points. Various types of predictive mathematical models were presented in the form of first and second order recurrence equations to describe the behavior of the SSHHP dam after the 2009 accident. The choice of recurrence equations for forecasting is explained by the fact that this type of equations allows sequentially and visually, step by step, starting from the last displacement values and residual errors at the base period of the forecast, to calculate the predicted displacement values of the control points. At the same time, the following effective factors were used to construct the models: U_k – hydrostatic pressure (upstream level); T_k – the temperature of the dam concrete body at the lower and upper base points. The discreteness of the initial data was 15 and 30 days.

The most successful were mathematical models with two input effective factors of the 2nd type (1st and 2nd order) and a 3rd type model with decorrelation of these effects with the subsequent introduction of transport delay at the stage of final forecasting, which are presented below (in general form).

Dynamic models of the 2nd type with input effective factors $x(U_k, T_k)$:

1st order:

$$x_k = \phi x_{k-1} + \beta_1 U_k + \beta_2 T_k + \gamma\omega_k; \tag{1}$$

2nd order:

$$x_k = \phi_1 x_{k-1} + \phi_2 x_{k-2} + \beta_1 U_k + \beta_2 T_k + \gamma\omega_k; \tag{2}$$

Type 3 dynamic model (with decorrelation of input factors) $\Delta x(\Delta U_k, \Delta T_k)$:

$$\begin{cases} U_k = \phi_1 U_{k-1} + \beta_1 T_k + U_0 \\ x_k = \phi_2 x_{k-1} + \beta_2 T_k + x_0 \\ \Delta x_k = \phi_3 \Delta x_{k-1} + \beta_3 \Delta U_k + \Delta x_0 + \gamma\omega_k \end{cases} \tag{3}$$

Model (3) describes the dependence Δx_k on ΔU_k, where Δx_k – the differences between the actual values x_k and those found by the second equation, calculated at the base period of the forecast, and ΔU_k – the corresponding differences between the actual values U_k and those found by the first equation; ω_k – noise component. Arguments U_0, x_0, Δx_0 are the corresponding centered values.

To estimate the parameters of the model, a stepwise LSM procedure was applied by approximating the first two moment functions, i.e. at the initial stage, the model parameters were estimated by minimizing the functional expressed through the conditional mathematical expectation, and at the final stage, by minimizing the functional expressed through the conditional correlation function.

Let us show the procedure for estimating parameters of a 3rd type dynamic model $\Delta x(\Delta U_k, \Delta T_k)$ [43].

At the first stage, the parameters were estimated based on the results of observations of the input $\{T_k\}$ and output $\{U_k\}$ at the base period of the forecast k = 1, 2, ..., N.

For this, the functional $F_1(\phi_1, \beta_1) = \sum_{k=2}^{N} (U_k - \hat{U}_{k/k-1})^2$ was minimized; where $\hat{U}_{k/k-1}$ denotes the conditional mathematical expectation of the first equation, which has the form:

$$M\{U_k / U_{k-1}, T_k\} = \hat{U}_{k/k-1} = \hat{\phi}_1 U_{k-1} + \hat{\beta}_1 T_k \tag{4}$$

Estimates of the parameters $\hat{\phi}_1, \hat{\beta}_1$ were found from the solution of the corresponding systems of normal equations.

At the second stage, the dependence of displacement on the influence of temperature was determined, i.e. the parameters were estimated according to the results of observations of the input $\{T_k\}$ and output $\{x_k\}$ at the base period of the forecast k = 1, 2,..., N.

To this end, functional $F_2(\phi_2, \beta_2) = \sum_{k=2}^{N} \left(x_k - \hat{x}_{k/k-1}\right)^2$ was minimized; where $\hat{x}_{k/k-1}$ is the conditional mathematical expectation of the second equation, which has the form:

$$M\{x_k / x_{k-1}, T_k\} = \hat{x}_{k/k-1} = \hat{\phi}_2 x_{k-1} + \hat{\beta}_2 T_k \tag{5}$$

Estimates of the parameters $\hat{\phi}_2, \hat{\beta}_2$ were found from the solution of the obtained normal equations systems of the corresponding models.

At the third stage, the differences Δx_k between the actual values x_k and those calculated by the second equation were determined, and ΔU_k – the corresponding differences between the actual values U_k and those found by the first equation. Further, the parameters were estimated based on the results of observations of the input $\{\Delta U_k\}$ and output $\{\Delta x_k\}$ at the base period of the forecast k = 1, 2,..., N in the process of minimizing the functional $F_3(\phi_3, \beta_3) = \sum_{k=2}^{N} (\Delta x_k - \Delta\hat{x}_{k/k-1})^2$, where $\Delta\hat{x}_{k/k-1}$ designates the conditional mathematical expectation of the third equation of the form:

$$M\{\Delta x_k / \Delta x_{k-1}, \Delta U_k\} = \Delta\hat{x}_{k/k-1} = \hat{\phi}_3 \Delta x_{k-1} + \hat{\beta}_3 \Delta U_k \tag{6}$$

Estimates of the parameters $\hat{\phi}_3, \hat{\beta}_3$ were found from the solution of the obtained systems of normal equations for the corresponding models.

To determine the order of the autoregressive model and perform the final stage of the estimation, asymptotically unbiased estimates of the correlation function were calculated from the residual modeling errors using the expression [44]:

$$K_{\varepsilon}[m] = \frac{1}{N}\sum_{k=1}^{N-m} \varepsilon_k \varepsilon_{k+m} \tag{7}$$

where the time shift $m = 0, 1, 2 \ldots M < N$.

Residual errors ε_k were determined as the difference between the real and approximated displacement values of the constructed models:

$$\varepsilon_k = x_k - \hat{x}_{k-1} = \gamma \omega_{k-1} \tag{8}$$

The choice of the autoregression order was carried out on the basis of the constructed graphs of the correlation function. In our case, to describe the noise process, we used a model that looked like [44]:

$$\omega_k = \mu \omega_{k-1} + \eta \omega_{k-2} + \xi_k \tag{9}$$

where: μ, η – are estimated parameters by minimizing the functional of the form:

$$F_2(\mu,\eta) = \sum_{m=1}^{M} \left(\hat{K}_{\varepsilon}[m] - \mu \hat{K}_{\varepsilon}[m-1] - \eta \hat{K}_{\varepsilon}[m-2] \right)^2 \tag{10}$$

3. Results and Discussion

3.1. Improvement of mathematical models

We note that at the stage of structural identification of the model, most influence is made by the choice of input impacts, discreteness of observations, and as a result of this influence, we obtain the values of the residual errors of the noise component between the results of the measured and approximated displacements values. Based on them, their spread is estimated using the standard error m, and then graphs of the autocorrelation function are constructed. In the process of research, to construct all types of the above mathematical models, the input impacts in the following combinations were used:

- U_k (hydrostatic pressure) and $T_{k.low}$ (temperature of the dam body concrete at the lower base point), as the results of field observations;
- U_k and ΔT_{calc} (temperature component of displacements obtained on the basis of the finite element model);
- ΔU_{calc} (hydrostatic component of displacements) and ΔT_{calc}, obtained on the basis of the finite element model.

The year 2013 selected for constructing predictive mathematical models as the base period for the forecast is characterized as a “temperature average” year.

Below is the first group of mathematical models of the 2nd type of the 1st order with two input impacts $x(U_k, T_k)$ and discrete observations conducted at a 3-day interval:

$$\begin{cases} x_k = 0.3241 x_{k\text{-}1} + 1.7679 U_k - 1.9296 T_{k.low} - 836.837 + 0.6864 \omega_k \\ \omega_k = 0.6651 \omega_{k-1} + 0.0779 \omega_{k-2} + \xi_k \end{cases} \tag{11}$$

$$\begin{cases} x_k = 0.7100 x_{k\text{-}1} + 0.7504 U_k + 0.2769 \Delta T_{calc} - 357.832 + 0.4969 \omega_k \\ \omega_k = 1.1820 \omega_{k-1} - 0.2777 \omega_{k-2} + \xi_k \end{cases} \tag{12}$$

$$\begin{cases} x_k = 0.2028 x_{k\text{-}1} + 0.8036 \Delta U_{calc} + 0.7710 \Delta T_{calc} - 3.628 + 0.1431 \omega_k \\ \omega_k = 1.8780 \omega_{k-1} - 0.8926 \omega_{k-2} + \xi_k \end{cases} \tag{13}$$

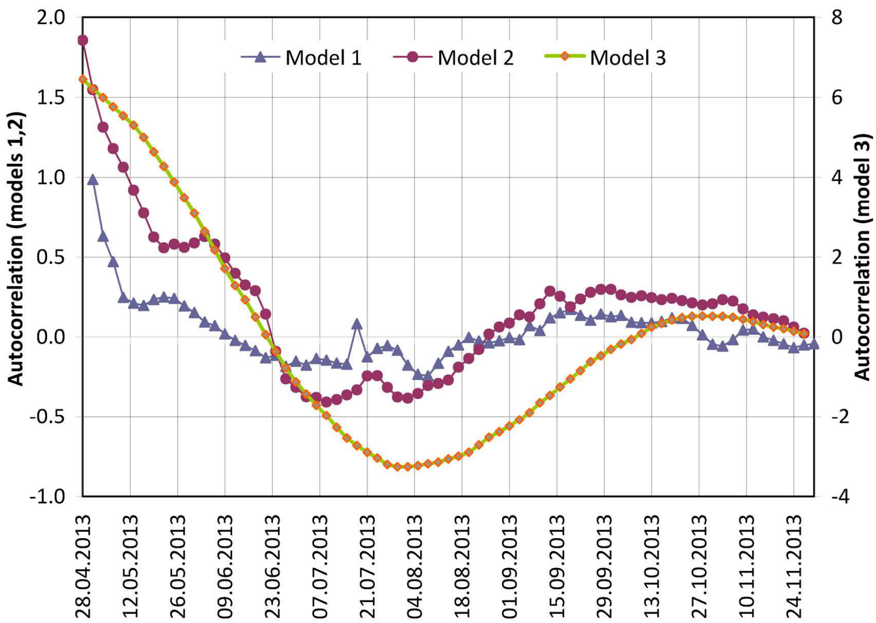

Fig. 3. Type of autocorrelation functions for the first group of models (model of the 2nd type; 1st order)

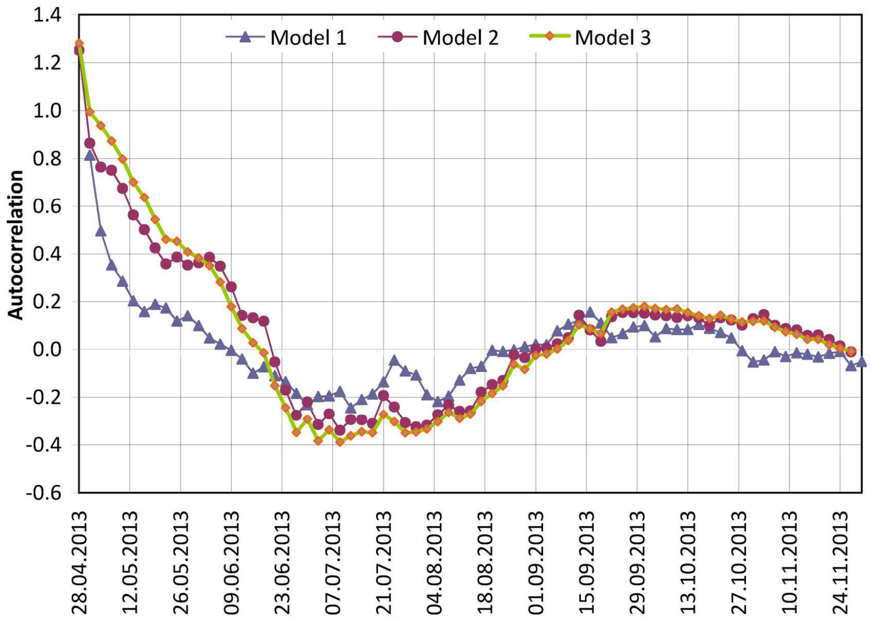

Fig. 4. Type of autocorrelation functions for the second group of models (model of the 2nd type; 2nd order)

The graphs of the autocorrelation function presented in Fig. 3, constructed from the calculated values of the correlation coefficients based on the analysis of time series, allow to draw the following preliminary conclusions at this stage.

First, the form of the autocorrelation function for models (11) and (12) indicates (see Fig. 3) that the choice of input actions combinations in the form (U_k, $T_{k.low}$) and (U_k, ΔT_{calc}) made it possible to carry out qualitative structural identification of the models under construction; the combination of hydrostatic pressure field observations with the calculated values of the displacement temperature component did not significantly impair the quality of the constructed model. The choice of the calculated values of the displacement temperature component as one of the input impacts indicates that no significant additional effects on this input parameter are observed when constructing the model.

Secondly, the choice of input impacts combinations in the form of calculated values of the hydrostatic and temperature components (ΔU_{calc}, ΔT_{calc}) significantly worsened the quality of the constructed model (13).

This indicates that the input parameter — the calculated values of the hydrostatic component — is to some extent a dependent parameter for this type of mathematical model, i.e. it is additionally affected by the impacts associated with the stress-strain state of the dam body, and which, with the help of this model, can only partially be taken into account in the future due to the introduction of the noise component in the forecast model.

The second group of mathematical models of the 2nd type with two input impacts $x(U_k, T_k)$ of the 2nd order received the following form:

$$\begin{cases} x_k = 0.4305x_{k-1} - 0.0808x_{k-2} + 1.7081U_k - 1.8268T_{k.low} - 809.273 + 0.5720\omega_k \\ \omega_k = 0.7447\omega_{k-1} + 0.0366\omega_{k-2} + \xi_k \end{cases} \quad (14)$$

$$\begin{cases} x_k = 0.9990x_{k-1} - 0.2428x_{k-2} + 0.6419U_k + 0.2313T_{k.low} - 791.491 + 0.5950\omega_k \\ \omega_k = 0.7210\omega_{k-1} + 0.1450\omega_{k-2} + \xi_k \end{cases} \quad (15)$$

$$\begin{cases} x_k = 0.8758x_{k-1} - 0.2096x_{k-2} + 0.3693U_k + 0.4367T_{k.low} + 21.823 + 0.7439\omega_k \\ \omega_k = 0.8496\omega_{k-1} + 0.0486\omega_{k-2} + \xi_k \end{cases} \quad (16)$$

With regard to this group of models, it is important to state the following.

Firstly, the quality of structural identification for all constructed models has improved in comparison with the first group of mathematical models, as evidenced by the calculated mean square errors for residual modeling errors (Table 1) and graphs of autocorrelation functions (Fig. 4), but the complexity of their construction has somewhat increased.

Secondly, the same choice of combining field observations of hydrostatic pressure with the calculated values of the displacement temperature component (U_k, ΔT_{calc}) increased the quality of the constructed model (15) in comparison with the models of the first group (11-13).

And thirdly, the combination of input impacts in the form of calculated values of the displacement hydrostatic and temperature components (ΔU_{calc}, ΔT_{calc}) reduced the quality of the constructed model (16) in comparison with models (14) and (15).

The constructed mathematical models of the 3rd type with decorrelation of input impacts $x(U_k, T_k)$ by their sequential introduction to the model and discreteness of observations conducted at a 3-day interval are presented below:

$$\begin{cases} U_k = 0.8974U_{k-1} + 0.1578T_{k.low} + 52.747 \\ x_k = 0.9342x_{k-1} + 0.1323T_{k.low} + 6.432 \\ \Delta x_k = 0.4210\Delta x_{k-1} + 1.2924\Delta U_k - 0.059 + 0.6197\omega_k \\ \omega_k = 0.1428\omega_{k-1} + 0.2383\omega_{k-2} + \xi_k \end{cases} \quad (17)$$

$$\begin{cases} U_k = 0.8866U_{k-1} - 0.0581\Delta T_{calc} + 61.299 \\ x_k = 0.9142x_{k-1} - 0.0812T_{calc} + 8.552 \\ \Delta x_k = 0.4238\Delta x_{k-1} + 1.3320\Delta U_k - 0.087 + 0.6190\omega_k \\ \omega_k = 0.1421\omega_{k-1} + 0.2471\omega_{k-2} + \xi_k \end{cases} \tag{18}$$

$$\begin{cases} U_k = 0.8822U_{k-1} - 0.1993\Delta T_{calc} + 4.476 \\ x_k = 0.9140x_{k-1} - 0.0808\Delta T_{calc} + 2.553 \\ \Delta x_k = 0.4772\Delta x_{k-1} + 0.4033\Delta U_k + 0.024 + 0.4826\omega_k \\ \omega_k = 0.5286\omega_{k-1} + 0.0910\omega_{k-2} + \xi_k \end{cases} \tag{19}$$

For this group of mathematical models, we can state the following. The quality of structural identification for all constructed models of this group has significantly improved in comparison with the first and second groups of mathematical models, as evidenced by the form of autocorrelation functions constructed from residual modeling errors (Fig. 5). At the same time, the graph of autocorrelation functions shows a sharp drop in the correlogram at the initial stage and a gradual decrease in the correlation coefficients in the form of an asymptotic approach to zero without any pronounced local trends, which indicates a decrease in the effect of multicollenarity between input effective factors. In our opinion, this happens due to the fact that at the stage of structural identification of the model, we made the correct choice of the type of mathematical model, input effective factors and discreteness of the results of observation conducted every 3d day with the possibility to sequentially introduce the main input effective factors into the model. This circumstance makes it possible to use the mathematical model (17) to predict the displacement of the dam body according to the results of field observations in standard situations of the dam operation (average temperature year). At the same time, it is possible to use the predictive model (18) for forecasting its operation in emergency situations (cold and warm years) using the results of field observations and calculated values of the displacement components or model (19) implementing the calculated values of the displacement components (hydrostatic and temperature).

Table 1 presents the results of the mean square errors m, calculated from the scatter between the measured and approximated displacements according to the all types of constructed models. Their values clearly indicate the advantage of mathematical models with decorrelation of input effective factors for subsequent forecasting.

The order of the autoregressive model for describing the noise component was determined from the analysis of the constructed graphs of autocorrelation functions for all the models under construction. All graphs presented in Fig. 3 – Fig. 5 indicate that the description of the noise process should be carried out by second-order autoregressive models of the form (9):

$$\omega_k = \mu\omega_{k-1} + \eta\omega_{k-2} + \xi_k$$

where μ and η are the estimated parameters.

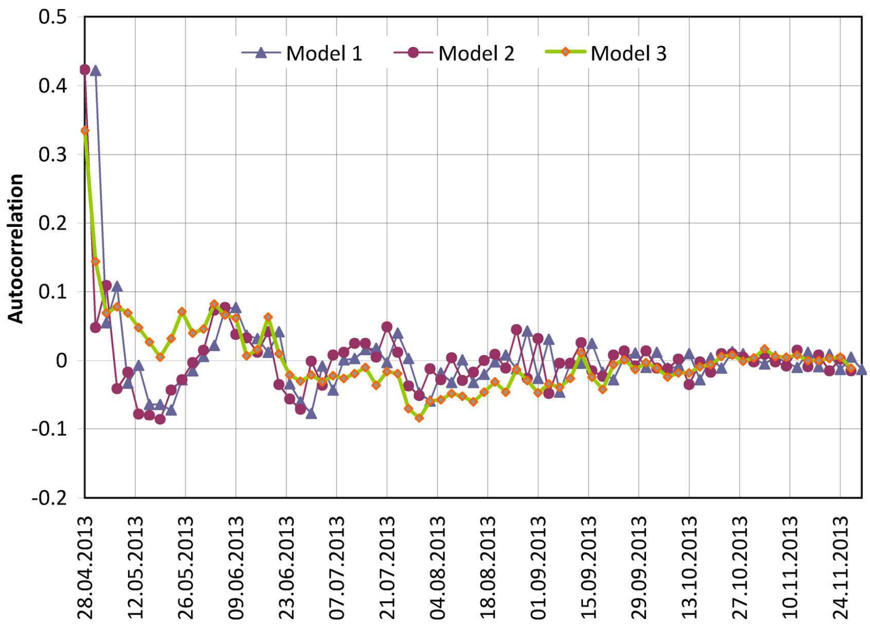

Fig. 5. Type of autocorrelation functions for the third group of models (3rd type model decorrelation of input impacts)

Table 1. Mean square errors of modeling

Model type	2nd type 1st order $x(U_k, T_k)$			2nd type 2nd order $x(U_k, T_k)$			3rd type with decorrelation $\Delta x(\Delta U_k, \Delta T_k)$		
m, mm	0.99	1.36	2.55	0.90	1.12	1.13	0.61	0.65	0.58

Table 2. The mean square errors of forecasting for 2008-2012 using mathematical model (18)

Year of the forecast	2008-2009	2009-2010	2010-2011	2011-2012
m, mm	1.40	1.54	1.80	1.68

3.2. Forecasting Results

For the subsequent forecasting of the SSHPP dam behavior, we used the constructed predictive model of the 3rd type with decorrelation of input impacts (18) with discreteness of observations conducted every 3rd day.

The following factors were used as input effective factors:

- U_k – hydrostatic pressure upstream level, as the results of field observations;
- ΔT_{calc} – temperature component of displacements (obtained on the basis of the finite element model)

Forecasting was carried out for various time stages of dam operation from 2008 to 2012. It should be noted that previous calculations for predicting displacements of the SSHHP dam control points after the 2009 accident using predictive models [40,43] allowed to obtain good forecasting results. However, in this case, it was necessary to build mathematical models that were quite complex in their structure. At the same time, such an effective factor as the temperature of the concrete body at the upper base point was practically taken into account only when calculating by models, and it was not possible to introduce it as a correction to the final forecasting results in the form of transport delay. It is explained by the fact that this factor with respect to another temperature factor (the temperature of the dam body concrete at the lower base point) has a short-period nature. In the course of the present research, when choosing the results of field observations (hydrostatic pressure) and the calculated values of the displacements temperature component (U_k, ΔT_{calc}) with data discreteness every 3 days, it was possible to build better forecast models at this stage of the study. The results of the forecasting by the model (18) in comparison with the results of previous research into the dam behavior before and after the 2009 accident, namely, in 2008-2009, 2009-2010, 2010-2011, 2011-2012, allowed to obtain more accurate forecasting results with an error of 1.4 – 1.8 mm (Table 2), which is also reflected in the graphs (Fig. 6 – Fig. 10). All the above indicates that it is possible to ensure safe operation of the dam at a higher level.

The data presented in Table 2 testify that for the selected combinations of input effective factors, the constructed forecast models of the third group with decorrelation of input impacts can be used to predict the dam displacements in almost all possible standard and non-standard situations (warm, average and cold years), as well as in emergency situations. Obviously, this can be explained by the fact that the input effect in the form of calculated values of the displacements temperature component fully reflects the temperature change throughout the dam body, including its changes at the lower and upper base points. Implementation of such methodological technique as the introduction of transport delay [40] in the final forecasting results made it possible to obtain more accurate final results.

It should be also noted that such an important parameter as the discreteness of the mathematical model every 3-day period, which characterizes the cyclical nature of field observations, provided that the basic laws governing the development of the studied process of the dam deformation and changes in the main effective factors are considered, allowed to construct predictive mathematical models that are quite correct from the point of view of structural identification.

Fig. 10 shows a graph of the measured and predicted values of the radial displacement components of the control dam points according to the model (18) for the operation period in 2016. The Fig. 10 also presents a graph of the predicted values of the radial displacement components, calculated from the hydrostatic and temperature components of the displacements based on the regression model according to the data from [42]. The calculated mean square error obtained from the differences between the predicted (according to model 18) and the measured displacement values of the dam control points amounted to 1.5 mm. The advantage of using the constructed mathematical model for forecasting is quite obvious.

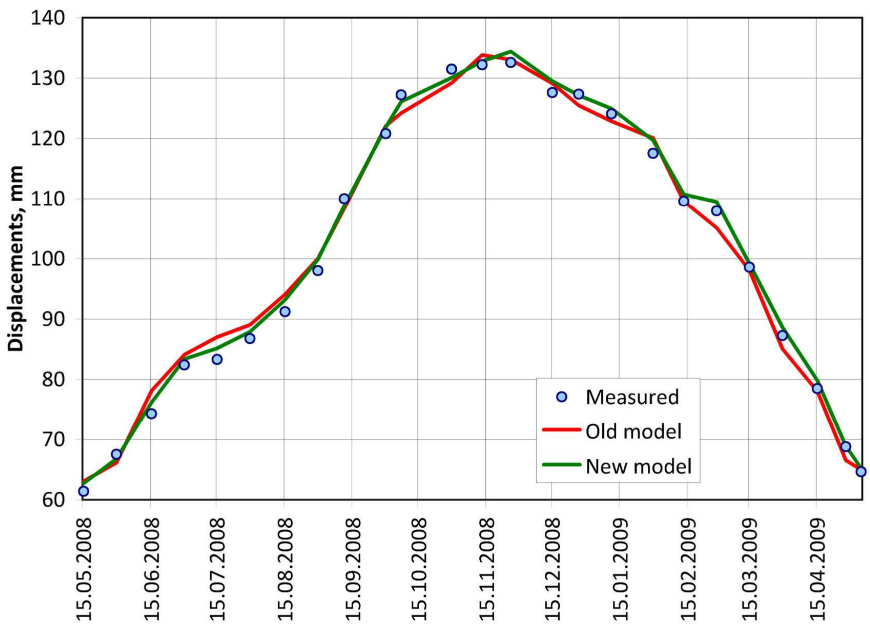

Fig. 6. Graph of changes in radial displacements and their predicted values according to model (18) and previously known forecasting results from [40, 43] for the dam operation in 2008-2009

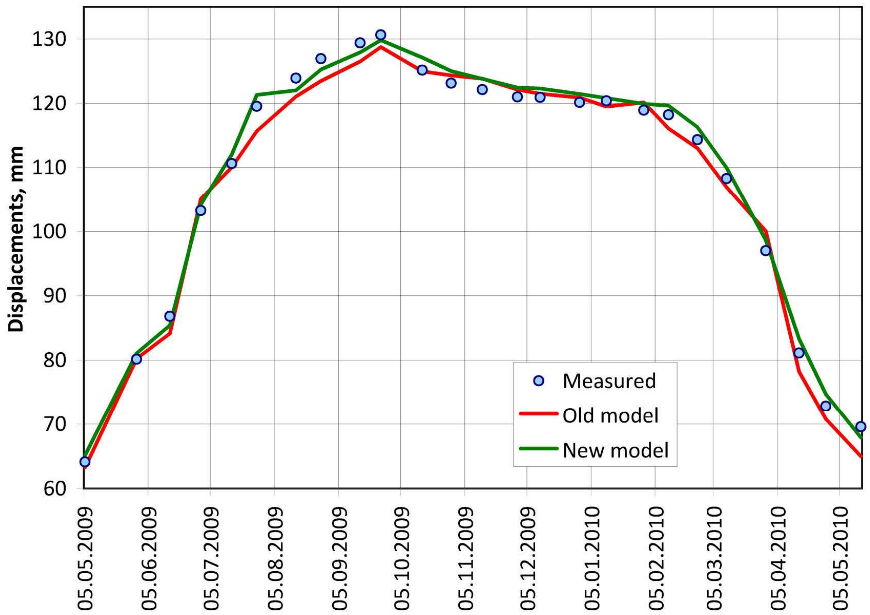

Fig. 7. Graph of changes in radial displacements and their predicted values according to model (18) and previously known forecasting results from works [40, 43] for the dam operation in 2009-2010

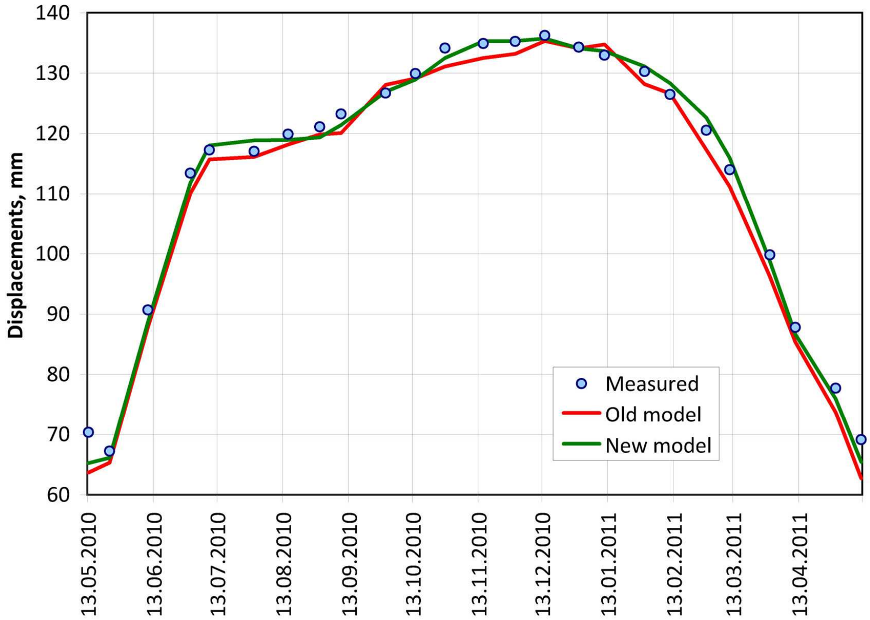

Fig. 8. Graph of changes in radial displacements and their predicted values according to model (18) and previously known forecasting results from works [40, 43] for the dam operation in 2010-2011

3.3. Results Discussion

The presented research results show that all types of the presented models can be used to predict the displacements of the dam. In this case, both the choice of initial data as influencing factors and the type of mathematical model are of great importance. The most successful (from the point of view of the correctness of the construction of the model; the form of the autocorrelation function in Fig. 3 – Fig. 5) is the model with decorrelation of the input influencing factors. It can be used to predict dam displacements in almost all possible standard and non-standard conditions (warm, medium and cold years), as well as in emergency situations. It is obvious to assume that the input action in the form of calculated values of the temperature component of displacements fully reflects the temperature change throughout the body of the dam, including its changes at the lower and upper base points. The introduction of transport lag at the final stage of forecasting made it possible to obtain more accurate final results, which is demonstrated in Fig. 6 – Fig. 10.

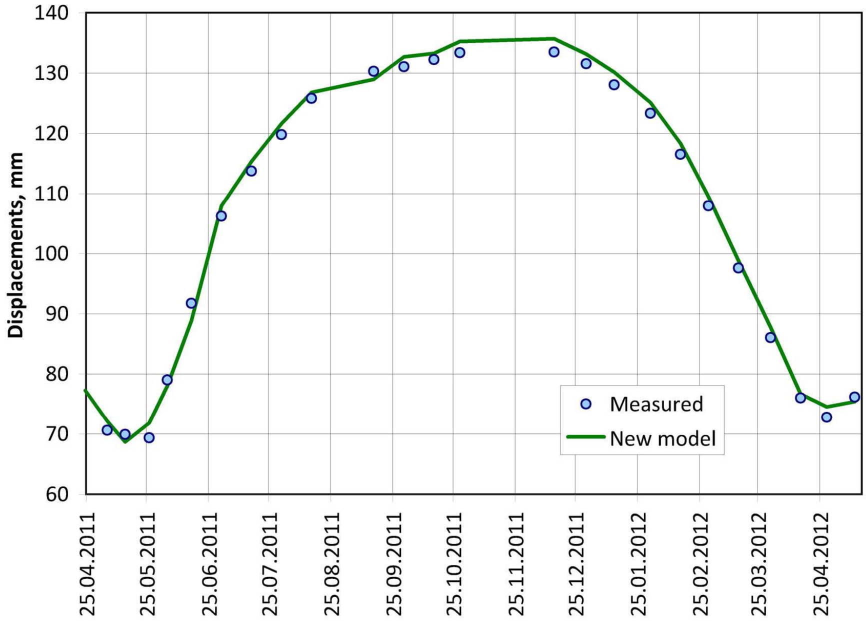

Fig. 9. Graph of changes in radial displacements and their predicted values according to model (18) for the dam operation in 2011-2012

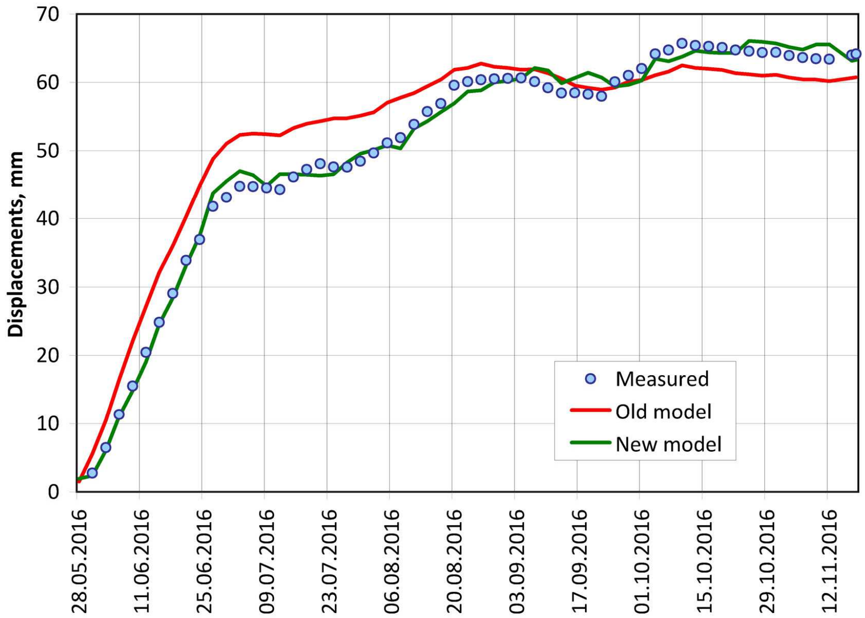

Fig. 10. Graph of changes in the measured displacement components and their forecast values calculated for the dam operation in 2016 according to the model (18)

4. Conclusions

- The performed analysis of residual error values using the standard error for designing various types of mathematical models allow to single out the most efficiently constructed mathematical models, which turned out to be mathematical models of the third group with decorrelation of input factors.
- The correct choice of input effective factors and the type of mathematical model at the stage of structural identification made it possible for this group of mathematical models to be used for prediction under various temperature conditions of the structure operation (average, warm and cold years), including emergency conditions.
- The mean square error of forecasting for 2016 based on the predictive mathematical model (18) with the introduction of transport delay in the final forecasting results when compared with the results of measured radial displacements amounted 1.5 mm, which testifies to high accuracy of the forecast. There is a real opportunity to ensure the safe operation of the dam at a higher level.
- The performed calculations based on the forecast model (18) with previously used data on effective factors for 2008-2012 allowed making predictions for these years and comparing the data with the results of previous forecasts. Hence, the results of the newly performed forecasting using model (18) showed higher accuracy in comparison with previously known ones, which indicates the possibility of the developed forecast models wider application in research into the state of high-head dams during their operation and prevention of emergency situations.

- Furthermore, it is possible to solve the inverse problem on the basis of the presented predictive mathematical models. The dam displacement in relation to the input effective factors has a delay effect (from several days to a month or more). Due to this, according to the measured values of temperature and radial displacement at a real moment of time, a short-term forecast can be made to select the safest level of hydrostatic pressure under the permissible radial displacement of the dam (in the given operating conditions), which is planned to be the object of further research.

Author Contributions

Valeriy Khoroshilov: Supervision; Project administration; Supervision; Conceptualization; Methodology; Investigation; Data Curation; Formal analysis; Software; Validation; Writing - Original draft preparation; Writing - Review & Editing; Visualization - Natalia Kobeleva: Investigation; Resources; Data Curation; Formal Analysis; Visualization - Mikhail Noskov: Resources; Formal Analysis. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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