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Research Paper

On the Analytical and Computational Methodologies for Modelling Two-wheeled Vehicles within the Multibody Dynamics Framework: A Systematic Literature Review

Camilo Andrés Manrique-Escobar[1], Carmine Maria Pappalardo[2], Domenico Guida[3]

[1]Department of Industrial Engineering, University of Salerno, Via Giovanni Paolo II, 132, Fisciano, 84084, Salerno, Italy, Email: cmanriqueescobar@unisa.it

[2]Department of Industrial Engineering, University of Salerno, Via Giovanni Paolo II, 132, Fisciano, 84084, Salerno, Italy, Email: cpappalardo@unisa.it

[3]Department of Industrial Engineering, University of Salerno, Via Giovanni Paolo II, 132, Fisciano, 84084, Salerno, Italy, Email: guida@unisa.it

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Corresponding author: Carmine Maria Pappalardo (cpappalardo@unisa.it)

Abstract. In this paper, a literature review on two-wheeled vehicle systems is methodically performed and presented. For this purpose, the principal aspects concerning the kinematic, dynamic, control, and identification features of articulated mechanical systems described within the multibody formulation approach are emphasized in this review article. First, the scientific investigations on two-wheeled vehicle modelling are chronologically described employing a historical literature review approach. This is done to set a consistent context for the subsequent developments analyzed in the paper. Then, following the systematic literature review methodology described in this work, a rich corpus of relevant documents in the time span between 2013-present is analyzed. Moreover, bibliometric methods are used to construct the conceptual structure map of the research field, which also allowed for formulating a thematic classification. Thus, considering the full-texts of the identified corpus of documents, this work presents a synthetic analysis of the fundamental issues about the multibody approaches for modelling two-wheeled vehicles. Finally, future research perspectives are pointed out in this article.

Keywords: Two-Wheeled Vehicles, Bicycles, Motorcycles, Kinematics and Dynamics of Multibody Systems, Nonlinear Control, System Identification, Systematic Literature Review, Bibliometric Analysis.

1. Introduction

The scientific community has shown great interest in the behaviour of two-wheeled vehicles ever since they were first developed [1, 2]. Therefore, the mathematical modelling of bicycles and motorcycles has been an active research area for many years [3, 4]. However, there are still many questions concerning this fascinating category of vehicles to be resolved [5, 6]. In fact, two-wheeled vehicles are known to be highly nonlinear systems with unusual non-trivial behaviours [7, 8]. However, although many researchers have addressed their modelling during the years, these systems are still challenging to be rigorously described and deeply understood [9, 10]. Thus, this introduction section deals with the fundamental problems concerning two-wheeled vehicles in general. It provides some background information and the significance of the present investigation, the identification of the research issues of interest for this study, the scope and contributions of this work, and the organization of the entire manuscript.

1.1 Background and significance of the present investigation

Today, the number of motorcycles and bicycles is in constant increase, specifically in the urban areas [11, 12], and due to the growing popularity of eBikes, two-wheeled vehicles are an integral part of the future transportation system [13, 14]. This fact contrasts with the unfortunate accident rate associated with the use of two-wheeled vehicles. Nearly a quarter of all road traffic deaths are among motorcyclists in the world [15, 16]. In general, the risk of death for a motorcycle rider is twenty-one times higher than that of other transportation modes [17, 18]. Even countries with a bicycle-friendly structure like the Netherlands have more cycling fatalities than car deaths [13], being its most dangerous means of transportation [19]. This issue has recently caught the attention of academic, industrial, and political institutions since it deserves a timely and adequate solution [11].

A peculiar characteristic of the two-wheeled vehicle dynamics is the presence of vibration modes that may become unstable [20], detrimentally affecting the rider control [21]. This feature leads to dangerous conditions since the rider cannot control high-frequency unstable oscillations [20, 22]. Ideally, because of the presence of two contact points between the vehicle wheels and the supporting road surface, there is an inherent instability, like an inverted pendulum [23], yielding the well-known stability problems [24]. These issues are even more prominent under acceleration and braking [25, 26], or while cornering [26]. Therefore, the linear and nonlinear stability analysis is a specific and vital issue in the mechanics of two-wheeled vehicles in general [20, 27]. Moreover, it is also remarkable the fact that, given the apparent straightforwardness of the geometry of two-wheeled vehicles, mainly like the bicycle, a comprehensive three-dimensional model getting their relevant dynamical features is still not well developed or analyzed, also because of the complexity associated with the collateral phenomena influencing the nonlinear dynamics and stability features

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of two-wheeled vehicles [28]. The exceptions are sophisticated multibody models that offer particularly detailed numerical results, but deep insights into the underlying physical phenomena are still difficult to be extracted from them [29–31].

1.2 Identification of the research issues of interest for this study

The in-depth analysis of the nonlinear kinematic and dynamic behaviour of two-wheeled vehicles is a challenging endeavor. Indeed, the largest majority of the models of two-wheeled vehicles developed in the literature features significant simplifications [32]. This is due to the complexity related to the multiple degrees of freedom of this family of mechanical systems and their closed-chain geometry [33]. In particular, considerable difficulties are found in analytically expressing the position and orientation of the front and rear frames [34]. In the literature, these issues are often addressed by employing linearization procedures and other kinematic simplifications [32]. Also, the studies of two-wheeled vehicles are generally confined to the linearization approaches finalized to the eigenvalue analysis [35, 36]. This is typically done to study the open-loop stability under constant speed conditions during straight running or cornering [37]. Additionally, the location of the wheel contact points is usually assumed to be in the rear frame symmetry plane. Because of these simplifying assumptions, it is apparent that there is still an open discussion in the literature on the relations between the steering angle and other critical parameters like the effective steering angle of the front fork, the trail, and the locations of the contact points [10].

The modern bicycle design and development is the final result of an evolutionary process of two centuries based on several trials and errors [19, 38, 39]. Today, therefore, designers need a sound methodology based on scientific theories to be placed side by side with current empirical methods. However, there is no agreement in the literature on how to employ the current knowledge of two-wheeled vehicles to improve and optimize their design [40]. Some works addressing this are just starting to appear in the literature [41–43]. Besides, comprehension of the essential dynamical behaviour of two-wheeled vehicles is required to unveil the mechanisms for controlling this family of systems and to identify the scenarios for instabilities that are counterproductive for the driver [21]. In particular, two-wheeled vehicles exhibit two potentially dangerous oscillatory modes known as wobble and weave [44]. The wobble mode is a high-frequency oscillation of the front wheel about the steering axis, also found in automobiles [45], the landing gears of aircraft, motorcycles and racing bicycles [22]. In recent years, little research has been published on the wobble mode of bicycles [46]. In effect, the reaction of different bicycle models is quite varied, which draws attention to their appropriate design instead of focusing on their instability [45]. On the other hand, the literature on the quality assessment of the handling characteristics of two-wheeled vehicles, until very recently, had only been interested in accident avoidance [19, 47]. Furthermore, the performance quality of either the human or the machine component alone does not determine the overall system quality. This is particularly true in the case of two-wheeled vehicles. Therefore, the performance quality of two-wheeled vehicles cannot be determined from only looking at its open-loop dynamics [48]. A common hypothesis in the literature, not proven yet, relates handling quality and self-stability in the straight running [49]. However, real vehicles are not designed to be as stable as possible since they also require manoeuvrability. In general, too little or excessive directional stability is not desirable [40]. For instance, bicycles with no self-stability are easily rideable [33, 50]. The lack of self-stability does not preclude a two-wheeled vehicle from being ride-worthy, and its presence does not prevent the handling of the system. Two-wheeled vehicles must be promptly reactive to the commands of the rider, and their stability should not be pursued without reference to other qualities [51]. Therefore, an important issue is determining what sort of handling performance is desirable and how to standardize its measurement [40]. Some works with this perspective are those found in [52, 53]. However, the assessment and the design of guidelines for the quality of handling in two-wheeled vehicles are yet in their early stages.

In the last decade, the implementation of driving assistance systems and the improvements in the road networks have increased the safety of automobile drivers. Modern automobiles feature Advanced Driver Assistance Systems (ADAS) to help the users of the vehicle to drive or park safely [54, 55]. However, these measures are still not effective for two-wheeled vehicles since similar enhancements are not standard for these vehicles, yet in development stages with only a few on mass production [56, 57]. For instance, the motorcycle industry is experimenting with high-end models, adding several electronic equipment types such as front-rear coupled braking, Electronic Stability Control (ESC) systems, airbags, and Anti-lock Braking Systems (ABS) [58]. However, accidents still occur because of inadequacy between the inherent dynamics, the driver inputs, and the infrastructure characteristics [17]. Among the others, instability is undoubtedly one of the reasons why these systems, and other similar active safety technologies, have not yet been developed for motorcycles while being available for automobiles [24]. Given the necessity to use a human rider or a control system to stabilize the motorcycle during tests, simulation is useful for evaluating new ADAS in a safe environment [59, 60]. In general, virtual testing helps in the evaluation of handling and ride qualities, as well as safety and performance issues [61]. Therefore, using a dynamical model of the rider-motorcycle system makes it possible to analyze the rider driving behavior and avoid the risk associated with the experiments [62]. For this, co-simulation is proving to be a fundamental tool [53, 63]. However, at present, there are no validated models for correctly describing the riders, hampering the proper use of virtual prototypes and leading to the necessity of an experimental verification [19, 60]. Also, modelling the vehicle without the rider can lead to poor numerical results and a lack of a practical understanding of the core physics of the problem. However, as mentioned before, although the modelling of vehicle-rider control is a popular topic with a variety of research directions, building a model for the behaviour of the rider is still a challenging task [9, 32]. Given the lack of knowledge regarding the human rider characteristics, this topic is yet in its infancy, as well as the type of control they use and the skills that distinguish their ability levels [48].

In several recent investigations, two-wheeled vehicle modelling has been of interest to ensure proper stability and controllability under a wide range of operating conditions [60, 62]. This was done through both understanding the behaviour of the system and its relationship with design and operating conditions [64–66]. In particular, one can easily identify three groups of mathematical models of two-wheeled vehicles in the literature, namely the in-plane dynamics models, the linearized inverted pendulum models and the Multibody Dynamics (MBD) models [3, 67]. The in-plane dynamics models such as [68–70], usually consider constant speeds under straight-running to investigate ride comfort, handling properties, the influence of road profiles, and the suspension system design [71]. However, by ignoring lateral dynamics, these models do not allow stability analysis. Linearized inverted pendulum models, such as the model developed in [72], oversimplify the geometry and inertial properties of the bodies and usually are not capable of exhibiting self-stability. These are of interest mostly for control purposes due to the simplicity of the resulting equations [35]. On the other hand, two-wheeled vehicle models based on the MBD approach are capable of accurately capture the dynamics of the system behaviour. However, these are not suitable for ADAS applications due to their intrinsic complexity with strong nonlinearities [15]. In contrast, simple MBD models exhibit the main modes of vibration of the system of interest, whereas the more complex models result in additional degrees of freedom and equations of motion. Therefore, the latter models are capable of exhibiting the secondary dynamic behaviour of the system, but this particular feature can obscure the main vibration modes. Consequently, given the direct correlation between the complexity of a model and the vibration modes it can identify, clear criteria are required to determine the level of detail of the model to use for a particular analysis [73].

Virtual prototyping is commonly used in the automotive industry, but it is still a developing tool for motorcycle companies [74]. Its use in motorcycle engineering can reduce design time, save the development costs, and avoid the risks associated with experiments and tests [75]. This is particularly advantageous during the development and validation of novel ideas [74]. Multibody models are typically used to optimize existing systems and to virtually test new projects [76]. This approach enables iterative simulations to reduce optimal parameter combinations to a small subset before field testing [77]. Furthermore, MBD simulations of two-wheeled vehicles can be built by the symbolic derivation of the equations of motion [36]. A plethora of different techniques can be used for implementing this approach in conjunction with the construction of general-purpose subroutines suitable for numerically solving the resulting differential-algebraic dynamic models [78–80]. Nevertheless, the symbolic development of the equations of motion is complicated and time-consuming but provides maximum flexibility in describing the model features [26]. Besides, the use of symbolic manipulation to simplify the motion equations can lead to extremely efficient simulation codes [81]. This is an important factor for some applications like hardware-in-the-loop, where computational speed is essential [75, 82]. The mathematical model in the form of a symbolic derivation can allow for performing analyses that are usually not easily carried out on commercial codes [36]. In particular, the steady-state analysis, the calculation of the eigenvalues, and the use of nonlinear dynamics methods such as numerical continuation and bifurcation analysis can be successfully employed to study the system stability [83, 84]. However, depending on the desired degree of detail to simulate the system, the resulting equations tend to extend their dimensions and complexity with increasing problem size. For instance, as emphasized in the review work of Sharp [85], the symbolic equations of the general motion of a benchmark bicycle are complicated, and recently many papers containing significantly incorrect accounts of them have been published. Also, it is not straightforward to deal with kinematic systems with closed loops. An alternative approach to developing a full nonlinear two-wheeled vehicle model is using MBD software tools based on a numerical approach. These are practically unavoidable to study complex systems, including suspensions, tyre slip models, and other non-ideal effects like frame flexibility [86]. To name a few, some of the software packages employed for two-wheeled vehicle modelling in the literature are BikeSim [87], SPACAR [88, 89], MBSymba [81], FastBike [75], and MSC ADAMS [90]. Therefore, finding proper solutions to these important problems still represents an open issue in the literature concerning two-wheeled vehicles in general.

1.3 Scope and contributions of this work

The scope and the contributions of this review article are twofold. First, the paper provides a review of the recent research works on two-wheeled vehicle dynamics and modelling. This is done by emphasizing the issues that belong to the conceptual framework of multibody system dynamics. To this end, an array of systematic literature review techniques and bibliographic analysis methodologies is employed. To the best of the authors' knowledge, this methodological approach has not been used before for analyzing the research field concerning two-wheeled vehicles. Secondly, since the latest literature review dates back to the years 2013 [19], except for some related recent works in railway and road vehicles [91], as well as in single-track vehicle dynamics and active control of safety systems [57,92,93], which only cover the two-wheeled vehicle dynamics in a complementary way, the present work provides an updated and comprehensive survey on these important issues. Thus, the paper is structured considering first a historical perspective on the dynamic modelling of two-wheeled vehicles, which refers to the time span before the year 2013, thereby summarizing the main contributions before this date considering a narrative exposition approach. Subsequently, the systematic literature review methodology considered in the paper is described and used to analyze the corpus of research papers pertaining to the time span between the years 2013-present. By doing so, the systematic method adopted in this investigation is described in detail in the manuscript. It is particularly included a tutorial example to construct the conceptual structure map of the research field of interest. This demonstrated the reproducibility of the unbiased analysis conducted in this investigation, as well as it was done to allow other researchers to employ the same approach for constructing systematic literature reviews in other affine scientific disciplines and for similar research topics. Finally, a relevant set of fundamental issues mainly related to the characteristic features of bicycles and motorcycles is identified and discussed in this review paper, paying considerable attention to the most relevant problems found in practical engineering applications associated with two-wheeled vehicles in general.

1.4 Organization of the manuscript

The organization of this paper is as follows. Section 2. contains a historical summary of the evolution of two-wheeled vehicle modelling that is presented employing a narrative approach. Section 3. describes the systematic literature review methodology employed in this work to obtain the set of relevant research papers during the time period between the years 2013-present, the corresponding bibliometric analysis methods, and the results found from the literature survey. Section 4. presents a synthesis of the documents identified and categorized with the use of the systematic review methodology employed in the paper. Finally, Section 5. includes the conclusions of this work, the current main research lines identified in this review article, some relevant future research perspectives, and a set of open issues on two-wheeled vehicle kinematics and dynamics in general.

2. Historical perspective on the kinematic and dynamic modelling of two-wheeled vehicles

In this section, the literature from the beginning of the research on two-wheeled vehicles to the year 2013 is considered. The objective is not to supersede the available literature reviews on such a temporal frame. The primary goal is, in fact, to present an overview of the main topics and introduce readers to critical studies, concepts, and methods, thereby providing a clear picture of the vastity and the complexity of this research field and to set a context for the recent developments.

2.1 Early period

The first relevant work on two-wheeled vehicle dynamics was published in 1869 by Rankine [94]. He presented a semi-quantitative observation on the stability and steering behaviour for such systems. To this end, Rankine employed an inverted pendulum model to provide a simplified analytical explanation of the steady-cornering constant lean angle. This is also the first research work that described the countersteering phenomenon. By introducing the promptitude concept, this work addressed the handling quality issue, a criterion related to the displacement and the velocity of the road contact point.

Subsequently, the most influential two-wheeled vehicle dynamics model, the so-called Whipple-Carvallo bicycle model, was indeed independently developed in 1899 by Whipple and Carvallo [95, 96]. This model is still used today and consists of four rigid bodies connected by revolute joints. The wheels are modelled as infinitely thin with a knife-edge surface. Therefore, the contact constraints with the ground are modelled by non-slipping points. Whipple derived the nonlinear equations of motion of the model for the upright and straight running condition. For this purpose, the set of Lagrange equations was employed, including a torsional spring-damper element to account for the rider inputs. Due to the lack of proper computational tools, a numerical approach to study the Whipple-Carvallo model was not available back then. Instead, assuming small roll and steering angles, an ad-hoc linearization approach was used. The result was the set of linearized differential equations of motion in the matrix form of a bicycle running along a straight path. This was the first linear model in the literature capable of predicting the non-minimum phase behaviour and the speed-dependent stability of bicycles. Whipple used the Routh-Hurwitz criterion to assess the linear stability of the system at a constant forward velocity. He predicted that the bicycle was self-stable without control of the rider within a small speed range.

Afterwards, Timoshenko and Young [97] proposed a model similar to that of Rankine [94]. They considered the balancing task of a bicycle like the balancing of an inverted pendulum, that is, a simple model with one degree of freedom, producing only the roll motion. The rider must steer the bicycle to create a centrifugal force and displace the contact point of the wheel with the ground, therefore correcting any capsize tendency. This model provided an accurate explanation of bicycle sideways capsize stability.

Since the mid-1950s, motorcycle manufacturers got involved in the present research topic. This stimulated the study of realistic physical effects to be adequately included in the models. The first work on motorcycle dynamics was carried out by Döhring [98]. It presented for the first time an entirely correct derivation of the equations of motion [89]. This model included also a phase lag between the inclination of the wheels and the reaction forces generated with the road. Moreover, a table setup based on experimental measurements was used to identify the centre of mass and the inertia moments of the rider and the motorcycle combined together. Three vehicles were considered to obtain the mechanical properties to derive the motion equations, leading to interesting numerical and experimental results.

2.2 The 1970s

In the early seventies, Jones performed a set of experiments to study the self-stability of bicycles, as well as the importance of the trail [50]. He reported experimental observations of increasing twisting torques for increasing roll angles, in agreement with the predictions of the inverted pendulum models. Also, in his work, it was concluded that the gyroscopic effect is unimportant for self-stability. Later, in their book on non-holonomic dynamics, Neimark and Fufaev presented a detailed derivation and an ad hoc linearization of the equations of motion employing the Lagrange equations [99]. The bicycle wheels were modelled as pneumatic tyres with toroidal shape. However, the derivation procedure contained a severe approximation resulting from ignoring the pitch motion of the rear frame [100]. This inaccuracy was a consequence of the closed kinematic loop due to the contact of both wheels with the road. A subsequent work proposed by Hand corrected the derivation and included a comprehensive review of the development of two-wheeled vehicle dynamics models [101]. He concluded that the literature on the topic pertaining to that period was plagued with errors since the validation by reference to the works of other authors was rarely carried out.

The subsequent work of Sharp was a significant step in the understanding of two-wheeled vehicle dynamics [102]. It consisted of the analysis of a motorcycle model with a rigidly attached rider. For the first time, the influence of a realistic tyre force model was studied. In particular, to account for the relaxation behaviour of the tyre, a first-order filter was employed. Besides, considering the absence of the control induced by the rider, it was possible to compute the eigenvalues of the linearized system as a function of the forward velocity. This approach served to identify the main three vibration modes, which the author named capsize, weave, and wobble, respectively. This investigation also studied the effect of some physical parameters on the magnitude of the modes of vibration. It was found beneficial for the stability to move the rear frame centre of mass either lower toward the bottom or forward toward the steering. Moreover, it was found that the increment of the damping coefficient in the steering caused a stabilized wobble mode but a destabilized weave mode as well. Later, a relevant experimental study of the bicycle tyre characteristics, including data for tyre force properties, was reported by Roland [103]. The subsequent work of the same author resulted in the first computer simulation of a two-wheeled vehicle based on the fully nonlinear derivation of the equations of motion [104].

After his initial groundbreaking work, Sharp extended the model to account for the rear frame flexibility of the system [105]. To this end, he included a rotational degree of freedom to the rear wheel, therefore allowing it to camber relative to the frame, although restrained with a rotational spring element to model the stiffness. This lumped element analysis approach is still used today, even in modern MBD two-wheeled vehicle models. He concluded that frame flexibility is significantly influential on the stability characteristics of two-wheeled vehicles. Although the capsize mode is not noticeably affected, a reduced stiffness deteriorates the damping of the weave mode at middle and high velocities. On the other hand, an increment of the frame stiffness above a certain level does not produce significant changes in the stability. Additionally, the frame stiffness and non-ideal tyre-force properties influence the wobble mode.

In the early seventies, the focus on structural stiffness sparked interest in the inclusion of the suspension system in the models. Jennings was the first to explore its effect on the dynamics, finding a change in the weave mode for cornering conditions [106]. In his model, the weave oscillations appeared for smaller roll angles when increasing the velocity. It was a new steering instability the author named cornering wobble. In general, the phenomenon was related to the properties of the suspension system, especially those of the rear one. Later, Sharp managed to explain the origin of this instability with a simple analysis [107]. It arose from the result of the wobble and weave modes interacting with each other due to their natural frequencies approximation when increasing the velocity or the roll angle.

An interesting series of works by Kane is worth mentioning. In [108], the author studied the kinematic formulation of a two-wheeled vehicle with toroidal wheels. He provided fundamental kinematic expressions and highlighted the complexity derived from the high nonlinearity of the constraints. Then, he opted for the minimal coordinates approach, eliminating the redundant coordinates by explicit substitution of their linearized expressions. Finally, he described a method for the development of the equations of motion. Later works considered the steady turning scenario starting from the original work of Kane [109, 110], reporting one solution family of the hand-free circular motion for the first time [111, 112]. The subsequent work of Kane included the rear frame flexibility in the developed model for high-velocity conditions [113]. For this purpose, the lumped stiffness element approach proposed by Sharp was employed [105]. In this research work, it was found a good agreement of the effects on the modal frequencies with the results of Sharp [105]. Additionally, Kane concluded that reducing the stiffness of the frame, as with wobble, affected the damping of the weave mode.

Relevant analytical and experimental works concerning the wobble mode made by Roe and his co-workers unveiled the mechanism of its origin. The first work established a general analytical framework for the castor oscillations of wheels [114]. The authors found the phenomenon related to the lateral migration of the wheel-ground contact patch. Later experimental results reported in [115] also agreed, where it was considered the frontal assembly castor for a fixed steering axis. Further work by Roe and Thorpe included the analysis of both rear and frontal assembly [116], where static load tests were performed on the front fork to study its compliance. The authors highlighted some inconsistencies between the theoretical predictions of the appearance of wobble oscillations and the measured fluctuations of the steering angle on motorcycles ridden hands-off at the onset of instability. In particular, the mathematical model of Sharp predicted wobble problems at much higher speeds [102]. At the same time, further work by Roe proved that the fork compliance was sufficient to produce wobble oscillations, without the contribution of the tyre dynamics [117]. He reported extremely close agreement with the real system of the motorcycle front wheel. Later on, Sharp provided a detailed review of the literature on two-wheeled vehicles, with an emphasis on motorcycles, up to that point [118].

2.3 The 1980s

At the beginning of the eighties, Sharp and Alstead addressed the discrepancy between the theoretical predictions and the experimental results regarding the damping of the wobble mode [119]. Employing a realistic tyre model developed in [120] that was based on the taut string theory, the authors extended the model originally developed in [102]. This model considered the frontal fork flexibility with the lumped parameter analysis. The approach consisted of adding a degree of freedom and a spring-damper system to account for the lumped physical properties. For this purpose, Sharp et al. considered three different models of the frontal fork compliance, namely (a) the lateral flexibility along the spindle axis of the wheel, (b) the torsional flexibility around an axis parallel to the steering axis, and (c) the torsional flexibility around an axis perpendicular to the steering axis and contained in the rear frame symmetry plane. The experimental results showed agreement with the model predictions based on the third approach mentioned before. To corroborate the results found by Sharp, a contemporary and independent experimental work by Verma reported similar results employing the tyre parameters measured on a Honda CB750 motorcycle [121].

Following closely, Spierings provided a detailed parametric study of the lumped analysis approach [122]. The location of the torsional axis and the spring stiffness for the vehicle were considered in this work. He found that the optimal torsional spring stiffness of the front fork is subjected to an upper and a lower limit. The low-speed wobble damping benefits from increasing stiffness, but the upper bound delimits the decreasing high-speed wobble damping. Also, the location of the torsional axis of the front fork must be as low as possible, agreeing with the experiments of Roe [116]. In fact, both works independently found that the telescopic fork have insufficient lateral stiffness to prevent instability. They mentioned that the hub-centre steering mechanism is a viable alternative to the frontal telescopic fork. Unfortunately, to date, there is little work in the literature on alternative steering mechanisms for two-wheeled vehicles.

Afterwards, for the first time, Giles and Sharp attempted to estimate through experimental measurements the lumped parameters of the front and rear frames [123,124], that is, the stiffnesses and the location of the rotation axes. For this purpose, static and dynamic load tests were used, and the latter were performed with a sinusoidally driven shaker. They found the dominant mode of deflection involving a rotation, as described by the lumped parameter analysis proposed in [119], therefore validating the approach. However, there were significant numerical discrepancies between the magnitudes of the physical properties of the lumped systems and the location of the rotation axes for static and dynamic measurements. They assumed that the best explanation for this was the difference in the mode shape and the consequent change in the moment arm length. Interested in the same research path, Raines and Thorpe focused on determining the torsional stiffness of the rear frame of two-wheeled vehicles and its relation with the position of the twist axis [125]. In particular, they employed the finite element approach for modelling a motorcycle frame made of tubes. It was found that modifying the frame to increase its stiffness may result in counterproductive results. The individual increment of the stiffness for the frame components improved the overall bending stiffness of the assembly. However, in some cases, it reduced the effective torsional stiffness. This was due to the possible migration of the twist axis away from the contact point, caused by the previous modifications, thereby increasing the resultant applied torque.

In the early eighties, a series of works by Koenen and Pacejka provided the analysis of the most complex hand-derived model of a motorcycle produced at that time [126–128]. The most significant contributions were the study of small perturbations about straight running and, for the first time, the analysis of the steady cornering conditions. The model featured the calculation of the steady-state responses and also the modal properties. For this purpose, it was derived a set of nonlinear algebraic equations for the cornering equilibrium and a set of linear differential equations with constant coefficients to account for the non-stationary response. Also, in that period, Sharp provided a detailed review up to that point of the literature on the steering behaviour of two-wheeled vehicles with an emphasis on vehicle design and analysis [129].

In the mid-1980s, the research on two-wheeled vehicles began to consider the effects of the rider body motion on the dynamics. Before this period, the rider was modelled as a rigid body attached to the rear frame. However, considering that a significant fraction of the total mass of the vehicle-rider system belongs to the rider, it became apparent that it was required to study its impact on the dynamics for generating more accurate predictions. Thus, two alternative approaches appeared in the literature to model the rider effect, referred to as, for simplicity, the passive rider and the active rider. The modelling methods based on passive rider models considered rigid bodies and spring-damper systems. On the other hand, active rider models included control elements to account for the rider actions during a specific manoeuvre. Later, a passive rider model with two degrees of freedom to account for the movements of the rider was proposed by Nishimi and his co-workers in [130]. In this work, the lateral motion of the body on the seat and its leaning motion relative to the frame were considered. The authors performed a comparison between experimental measurements and the numerical results arising from two models, and one of these did not include the rider degrees of freedom. It was found that the model featuring the rider effect showed a higher correlation between numerical and experimental results.

In the late eighties, the study of the straight running stability found the upper body parameters to mainly influence the weave mode, while those regarding the lower body influence primarily the wobble mode. Following closely, Katayama et al. in [131] employed a simpler vehicle model based on the previous results proposed by Sharp in [102], and modified the rider model proposed by Nishimi et al. in [130]. To this end, instead of considering the displacement of the lateral lower body, the rider was modelled as a double inverted pendulum. Back then, the scope of the work was to study, for the first time, the control logic employed by human riders. It was concluded that the primary control input is the steering torque, and the body leaning plays only an assisting role.

2.4 The 1990s

At the beginning of the 1990s, Pacejka et al. provided a detailed review of a large number of tyre models available in the literature up to that point [132]. The authors identified three kinds of models, that is, (a) the physically founded models (requiring computation for their solution, therefore computationally expensive), (b) the physically-based models (sufficiently simplified to have analytical solutions), and (c) the formula based empirical models. They concluded that tyre models requiring computer solutions were not as useful as the formula based empirical models employed for simulating vehicle dynamics. Of the latter, the most notable is the Magic Formula (MF) used for tyre modelling, developed by Pacejka et al. in a series of works [133–135]. This tyre model, initially conceived to work for the small camber angles presented in automobile steering systems, was later modified to fit the behaviour of motorcycle tyres [136]. For modest frequencies with a first-order system characterized by a time lag, the authors found the methodology accurate enough to model the transient behaviour of tyres.

A model containing the relevant developments available in the literature of its time was produced by Sharp [137]. It included a passive rider model with one degree of freedom. This work also employed the lumped parameter analysis to include the twist and lateral flexibility of the fork, each one described with an additional degree of freedom restrained with a spring-damper element. Similarly, the rear frame flexibility was modelled with the torsional freedom of the rear wheel around an accurate inclined axis. Besides, an improved tyre model together with the in-plane aerodynamic effects were included as well. The author then performed a parametric sensitivity analysis for the straight running linearized stability of the vehicle, finding agreement with experimental results in the literature. This model might be one of the last hand-derived in the literature on two-wheeled vehicles.

In subsequent work, Sharp reviewed the application of automated MBD software to the mathematical modelling of road vehicles [138]. The author identified two categories of MBD software, namely, the numeric and symbolic. The numeric MBD software prepares and solves the equations numerically and then post-process the results to produce outputs such as animations, graphs, or tables. Their main attribute is the generality of the problems capable of handling. This, however, at the expense of higher computational costs. Compared with hand-prepared code, a factor of fifty on computational speed is not unusual. In contrast, the symbolic MBD software derives the equations of motion in a fashion similar to hand derivation, thereby offering more choice in terms of coordinates and reference frames. Thus, the author concluded that the research field required these types of tools. The ideas originally presented in [138] were later considered in a set of works focused on motorcycle modelling and developed by Gani et al. [139–141]. The authors reproduced the hand-derived work proposed by Sharp in [137] by employing the symbolic MBD software

AUTOSIM [142–144]. It was encountered complete agreement between the results found, therefore demonstrating the feasibility of the employment of MBD software in two-wheeled vehicle modelling.

In the late nineties, Imaizumi et al., in a series of works, developed an active rider model with twelve degrees of freedom by employing an MBD computational tool [145–147]. The model featured eleven rigid bodies linked by spring-damper and active control elements. The scope was to study realistic driver actions like leaning, body pitching, and body displacement. The authors paid particular attention to the rider effect on the vibration modes of the system. It was found a destabilized behaviour when increasing the rear load weight. Therefore, it was proposed a suspension mechanism to attach the driver seat with the frame. It featured an additional lateral degree of freedom with a spring-damper guide roller. Simulations and physical experiments confirmed that the wobble mode oscillation could be well damped with the developed suspension mechanism.

Afterwards, following the kinematic analysis proposed in [108], Cossalter et al. developed a nonlinear two-wheeled computational symbolic model employing Newton-Euler equations [148]. The proposed model considered the steady cornering condition while featuring detailed geometry of the front assemble. In particular, a toroidal wheel shape was assumed. An analytical expression for the rear frame pitch angle was derived employing linearization, yet valid for large steering angles. The model included the nonlinear lateral and longitudinal forces induced by tyres, the rolling resistance, as well as the presence of aligning and twisting moments. All these physical quantities were modelled as functions of the sideslip, the longitudinal slip, and the camber angle of the tyres. The resulting simultaneous system of nonlinear algebraic equations was solved with the fixed iterative point method. The authors concluded that the front fork design and the tyre properties have a significant effect on the required steering torque to negotiate a curve. In this respect, a primary role was played by the tyre twisting torque properties, the fork caster angle, and the trail. In particular, the steering torque for a given cornering manoeuvre increased with the caster angle and decreased with the trail. Furthermore, in [149], Cossalter et al. used a simplified four degrees of freedom motorcycle model to address the manoeuvrability assessment of motorcycles. They proposed a novel approach based on optimal control methods to measure the manoeuvrability of a vehicle, in contrast to the open-loop manoeuvres usually considered. In this work, a numerical quantification employing the optimal control penalty function was used. This approach provided a metric being a function of only the intrinsic properties of the vehicles without the bias related to using rider models. The results showed a general improvement with higher tyre stiffness. However, the authors concluded that there is a need for balance in the rear and front tyres stiffness to obtain the fastest manoeuvre.

2.5 The 2000s

By the early 2000s, given the increasing level of detail required for two-wheeled vehicle modelling, the manual derivation of the motion equations was considered impractical and error-prone. The idea was reaffirmed in [65], an update of the AUTOSIM model originally developed in [139], where a thirteen-degree-of-freedom system considering the front and rear frame compliances and the rider leaning was considered. Additionally, the model included a detailed geometrical description of the frontal fork, the suspension system, and the rear swingarm. The main improvement was the cornering analysis, used to investigate the inconsistency between the experimental measurements of Jennings [106] and the predictions of Koenen [128]. The latter found a negligible effect of the suspension system in cornering stability. The authors attributed this discrepancy to some inaccuracies in the development of the original model [128]. The latest model of Sharp would subsequently be the reference for the development of the BikeSim complement of the software AUTOSIM [65].

The first relevant simulation of a two-wheeled vehicle under straight accelerating conditions was performed by Limebeer et al. in [25] employing the AUTOSIM model developed in [65]. The results showed equivalence between accelerating/breaking and downhill/uphill conditions. Therefore, the stability of a motorcycle accelerating/braking was assessed by analyzing the eigenvalues of the system at constant speed going uphill/downhill. Moreover, a destabilizing effect was found on the wobble mode for the descending or breaking on a level surface scenario. In contrast, ascending an inclined level surface at a constant speed or acceleration conditions showed stability improvements. A subsequent tutorial/review article of Sharp is noteworthy since it condensed the relevant information on two-wheeled vehicle dynamics up to that point [66]. It provided a summary of his two previous literature reviews [118, 129], as well as recent outcomes of the time, with an introductory perspective. An update was published a few years later in [150]. Subsequently, employing the model developed by Sharp in [65], Limebeer et al. studied the effects of road profiling on the steering behaviour of motorcycles [64]. They found regular low amplitude road undulations to be detrimental for rider control when cornering. In particular, they observed the instabilities of the wobble mode and weave mode in low and high velocity conditions, respectively. Therefore, they associated the resonant responses to the accidents caused by the rider loss of control. Given that wobble resonances were found to be front-wheel dominated, it was necessary to include in the model an effective steering damper.

Another relevant MBD vehicle model was published by Cossalter and Lot [75]. The authors presented a nonlinear motorcycle model with eleven degrees of freedom derived with the symbolic MBD code called MBSymba that employed the natural coordinates method [81]. This approach produced simple equations with high computational efficiency. Here the resulting system contained redundant coordinates and a set of algebraic equations to account for the kinematic constraints. Also, the authors included a novel comprehensive tyre model with proper geometric tyre shape. It consisted of implicit differential equations with an approach similar to that of the relaxation models [151]. Experimental results showed agreement with the numerical simulation regardless of not considering frame and fork compliance or the rider movements. This model served as the basis for the subsequent development of the commercial software FastBike, a tool for the multibody simulation of motorcycles that allows performing stability, maneuverability, and comfort analysis. In particular, said tool features structural and suspension compliances, nonlinear spring-shock, and multiple types of suspension linkages, to name a few.

Further work of Cossalter led to the publication of the first engineering book exclusively treating motorcycle dynamics [152]. Afterwards, an experimental study of motorcycle tyre models by Tezuka correlated experimental data to the Magic Formula (MF) and the Carpet Plot (CP) tyre models [153]. The MF model showed a higher correlation for straight running and equal correlation for cornering conditions. Besides, the author mentioned advantages related to the change of tyre parameters when working with the MF tyre model. Then, he concluded by highlighting the need for the development of a tool for fast tyre parameter calculation. Later, Pacejka published a book summarizing the developments and reporting improvements of the MF tyre model [154].

The posterior work of Cossalter et al. presented a large rotating disk test machine to measure motorcycle tyre properties, as well as experimental results showing the comparison between different tyres and their influence in operating conditions [155]. In [26], the authors used modal analysis techniques to identify, with laboratory tests, the structural modes of vibration of two-wheeled vehicles. In this work, there were considered scooters and sport motorcycles. The first structural vibration mode of two different sport motorcycle models was significantly higher than the wobble frequencies. This proved the structural compliance to be of little influence on stability for this kind of vehicle. In contrast, the structural compliance of scooters showed to have a significant influence on the stability and handling properties. The first structural mode of vibration of these vehicles was mainly a torsional deformation near the steering head. This was in agreement with the established model for the analysis of the fork compliance in the literature. Cossalter et al. continued the work focusing on scooter vehicles and their wobble stability [156]. In this work, a new two-wheeled vehicle model, featuring a passive rider, structural compliance, and tyre modelling, was devised. The authors found a significant influence of structural flexibility on system stability. Two antagonist effects were highlighted: (a) the destabilization due to fork flexibility and (b) a wobble stabilizing gyroscopic torque around the steering axis produced by the fork bending and wheel spin. The velocity defined the predominance, where the low-velocity range evidenced the adverse effect. They concluded that a solution for the instability is strengthening the vehicle frame or increasing the steering inertia of the front assembly.

An update of the model originally developed in [65], with the available advances to that date, was presented in [51]. The main features were: (a) the accuracy of the tyre-road contact geometry, (b) the analytical model for the mono-shock rear suspension, and (c) the inclusion of the MF tyre model. The authors employed experimentally measured physical parameters of the Suzuki GSX-R1000K1 motorcycle. The measurement of the inertias was done with time oscillations of the suspended parts. The stiffness-damping structural properties were estimated with indirect measurements. Also, the work included the MF tyre model coefficients for a variety of motorcycle tyres. In [51], the authors included a rider model as the one proposed in [130]. It is found a stabilizing effect of the wobble mode while increasing the weight of the rider. Besides, cornering stability analysis showed the in-plane and out-of-plane modes couple, as predicted in the literature. In particular, the roll angle of about 15 (deg) was the minimum lean exhibiting modal coupling. It was also the less damped configuration in the presence of regular road undulations. This issue was subsequently addressed in [157, 158], where the authors, based on the model originally presented in [51], proposed a design of a motorcycle steering compensator system and validated it experimentally.

As already mentioned above, after the late 1980s, research interest in bicycles was overshadowed by motorcycles, with only a few works on bicycle dynamics [159–162], and its control [163, 164]. In the mid-2000s, however, the interest was re-sparked thanks to a series of works starting with the investigation proposed by Astrom et al. [33]. The paper presented a review of bicycle dynamics modelling and some experiments performed from a control perspective. Furthermore, it explained analytically and numerically the self-stability phenomenon. Similarly, Limebeer et al. included a historical account of the evolution of two-wheeled vehicles in his work [165]. Besides, they analysed the inverted pendulum bicycle models, together with the counter-steering phenomenon, and complemented the oversimplified explanation of self-stability provided in [33]. Modelling aspects of motorcycles were analyzed for bicycles as well, namely, tyre models and structural compliance. Afeterward, Meijaard et al. published in [89] a rigorous literature review of the linearized bicycle dynamics models and self-stability updating the work in [101]. They presented the linearized equations of motion for the Whipple model in closed form. It was the first standard journal publication in English to include an entirely correct derivation. This considering the small typographical errors the authors found in [95]. Additionally, the bicycle benchmark problem was formulated as a tool to verify the correctness of a two-wheeled vehicle model. The authors provided a dataset of the system physical parameters and its open-loop eigenvalue calculations for the straight running condition. Four different methods verified the correctness of the data with fourteen digit, including MBD simulations in the software SPACAR [88] and AUTOSIM [144]. Further work experimentally validated the model with measured eigenvalues [166], thereby concluding that the flexibility of the frame, the rim, and the accurate tire modelling were negligible for bicycles at low-speed riding. In [111, 112], Basu-Mandal et al. also studied the benchmark bicycle in the hand-free circular motion condition. He performed the symbolic derivation of the nonlinear bicycle equations of motion by two different approaches, namely Newtonian and Lagrangian, producing eigenvalues in agreement with [89]. The authors then addressed the steady cornering of a bicycle without driver inputs, that is, with no steering torque or body lean, finding four different one-parameter families of solutions. It was, therefore, highlighted that previous works in the literature only found one of the family solutions [33, 109, 148, 160].

To analyze the wobble and weave instabilities, Katayama et al., in a series of works [167, 168], employed an analytical approach called energy flow in contrast to the dominant trial-error approach in the literature. This is based on the analysis of the kinetic energy of the main dynamic degree of freedom of the vibration mode to be studied. In particular, an instability condition is considered when the incoming energy is greater than the output energy in a time window. In this way, the authors determine that the wobble mode is activated mainly by the yaw and roll rates and suppressed by lateral accelerations and the front tire side force. Instead, the weave mode is stabilized by the force of rolling acceleration in lateral movement and the force of yaw rate. In particular, the authors find that activating the steering motion stabilizes the weave mode but destabilizes the wobble. Similar to the effect of employing a steering damper, which stabilizes the wobble mode but delays the phase of the front tire side force, thus destabilizing the weave mode.

2.6 From 2010 to 2013

Further work by Sharp showed a path-following controller for the benchmark bicycle employing the discrete-time optimal linear control theory [169]. This was later extended with a timely literature review, including a discussion on the physical explanation of the self-stability [85]. Additionally, there were considered the effects of acceleration, tyres cross-section, frame compliance, and rider-control modelling. These were subsequently implemented in [45] to model the wobble instability of bicycles, proving, therefore, the fundamental findings on motorcycles to be transferable to bicycles. Later, Moore and Hubbard performed a parametric study to test the effects of physical properties in the self-stability region of bicycles [170]. The main findings included the determination of the frontal assembly properties as the main factor. In particular, the front wheel diameter, the fork caster angle, the trail, and the wheelbase were considered. In a set of works, Kooijman et al. presented a comprehensive description of the self-stability of bicycles [171]. Moreover, the rider control mechanisms were studied using motion capture technology, and Principal Component Analysis (PCA) [172]. The authors found indications that the steering was the main control action of the rider. In contrast, the upper body leaning was only complementary, appearing with the pedalling frequency. This is further reaffirmed in [173], where two commonly observed rider poses are studied. For this purpose, a passive rider model was added, and its effect on self-stability was studied. It was observed that the upright rider position with the hands on the handlebar eliminated the self-stability mechanism. In contrast, a forward leaned rider with stretched arms and hands on the handlebar displaced the self-stability region at a higher speed. Finally, the authors studied the modal controllability of the bicycle-rider system. It is found that the steering torque had excellent modal controllability in contrast with the low one of the body leaning. This indicated the primary role of steer control in lateral balancing.

The availability of highly detailed motorcycle models provided the researchers with fundamental tools to address other intricate aspects of two-wheeled vehicles. For instance, Lot focused on developing an active rider model employing the optimal manoeuvre method with a highly detailed MBD motorcycle model [174]. Cossalter et al. studied the chattering behaviour of motorcycles, which is a detrimental auto-excited vibration [175]. Here, the authors employed their previous vehicle models and tyre models to address the reproduction of the phenomenon [75, 151]. Based on professional riders anecdotal scenarios, it was achieved a numerical simulation replicating the chattering behaviour. They found a strong modal coupling with the unsprung masses movement, this being the reason why braking preceded its onset. A posterior work of Cossalter et al. included an advanced motorcycle model with twenty-nine degrees of freedom, accounting for all the features available in the literature up to that point [36]. The model was then employed for a detailed study of the passive rider effects on motorcycle stability [176]. Evangelou et al. used the motorcycle model developed in [51] to study the effect of the road camber on the stability of motorcycles [177]. To evaluate steady-state behaviour, the vehicle was simulated running on a conical surface. They concluded that the camber road effect was detrimental to system stability. In particular, the desirable configuration for the low-speed regime was that of perpendicularity with the road. The opposite applied to the high-speed regime. Other instability sources considered were the acceleration and high-speed cornering, causing burst oscillations [178]. A mechanical compensator to control this instability was later proposed in [179].

In a recent work [87], Sharp et al. continued the research on chatter vibrations based on the observations provided in [175]. The authors focused on the appearance of chattering behaviour at a constant speed running through the corner-apex with the tyres near the friction limits. The work considered two semi-independent models. One was an update of the motorcycle model in [51], including a highly detailed chain-drive and a controller to negotiate the manoeuvre. An extension with extra features, datasets for physical properties and rider controllers, part of the BikeSim commercial software, was used as the second model. Both independent simulations predicted the existence of a vibration mode fitting on the chatter qualitative descriptions. The corresponding damping decreased systematically when increasing the cornering effort, while the front-fork bending stiffness broadly defined natural frequency. Possible mitigating measures for this instability included the addition of energy dissipation in the frame structure. Subsequently, Lake et al. presented a comprehensive review of the literature on the effects of the material flexibility of motorcycle components on system stability [73]. The work provided a synthetic summary of the intricate research findings since the initial application of the lumped-parameter analysis. It additionally included a discussion regarding the feasibility of using advanced materials like carbon fibre to manufacture motorcycle structural elements. To conclude, the readers interested in delving into the research works collocated in the present time frame are referred to the reviews available up to this point on two-wheeled vehicle rider control [19], handling qualities [23], and rider modelling [21].

3. Systematic literature review methodology

Literature reviews have a crucial role in synthesizing past research findings to effectively use the existing knowledge base and formulate future research perspectives. For many years, researchers have sought expert opinions from historical reviews to update their knowledge of a specific research topic or a given fundamental issue [180]. In this paper, historical review refers to the narrative literature review approach. As explained in [181–183], there is a substantial difference between the systematic literature review approach and the narrative method in the sense that “systematic reviews differ from traditional narrative reviews by adopting a replicable, scientific and transparent process, in other words a detailed technology, that aims to minimize bias through exhaustive literature searches of published and unpublished studies and by providing an audit trail of the reviewer’s decisions, procedures and conclusions” (Tranfield et al., 2003). In fact, the traditional ad hoc approach of literature reviews may risk missing important information and introducing biased interpretations. Therefore, this section describes the systematic literature review methodology used in this work to objectively obtain the set of relevant papers during the time frame 2013-present and their corresponding bibliometric analysis.

In general, the historical literature review approach can be biased by the reviewer experience, prior beliefs, and overall subjectivity [184]. This is reflected in the inclusion-exclusion criteria for documents, which are typically not described in the background of the review papers, offering, therefore, an arbitrary selection of evidence that may not correctly represent the state of existing knowledge. Besides, selecting some studies over others ultimately leads to the so-called sample selection bias [181]. Moreover, the data-extraction process is informal, in the sense that it is not standardized or systematic, and the synthesis of these data is generally a narrative juxtaposition of evidence [184]. On the other hand, the use of systematic literature review approaches, based on proper methods specifically devised for this purpose, such as the techniques considered in this investigation, allows for overcoming the aforementioned limitations [181]. This systematic approach incorporates methods to reliably extract factual and quantitative information from the literature [180], thereby differing from traditional narrative reviews by adopting a replicable, scientific, and transparent process [181]. Thus, this section describes the standardized approach employed for collecting, refining, processing, and analyzing the publications concerning two-wheeled vehicles. This is followed by a methodological bibliometric analysis that allows for quantitatively studying the collected literature material to identify relevant authors and publications, hot topics, and research clusters.

The systematic process employed in this investigation for the identification of the relevant publications in the period 2013-present is shown in Figure 1.

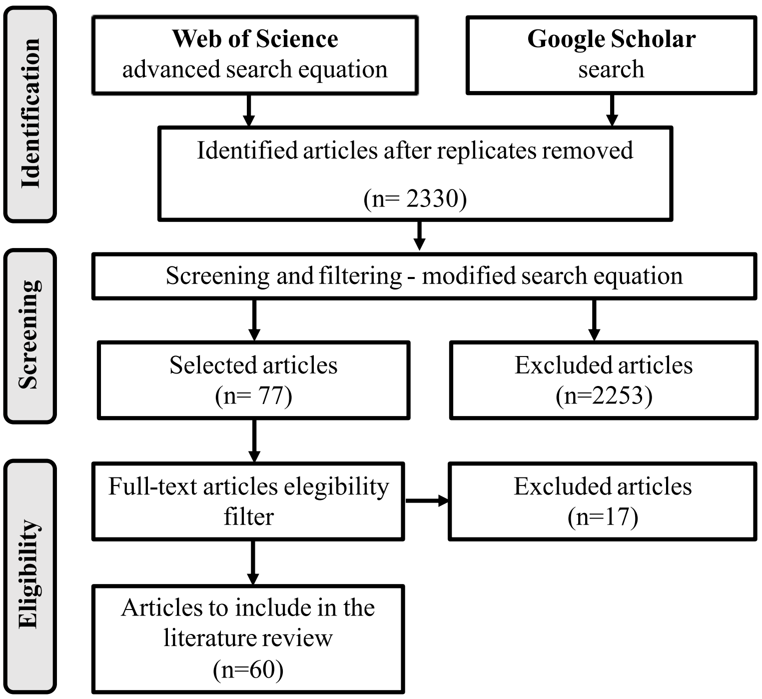

Figure 1. Systematic literature review methodology - PRISMA flow diagram.

The data reported in Figure 1 are obtained through the use of the standard procedure called PRISMA [185]. PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) is a standard process that guides systematic review reporting. More specifically, the formulation of an advanced search string with boolean operators served as input to the Web of Knowledge database as follows:

((TS = ((Motorcycle OR Bicycle OR Two-wheeled vehicle OR single-track vehicle OR powered two wheeler) AND (Dynamic* OR Stability OR Model* OR "Multibody Dynamics" OR Kinematic*))) AND (WC = (ENGINEERING MECHANICAL)) NOT (TS=(arthroplasty OR bicycle sharing system OR CVT DYNAMICS OR car* OR automobile OR bicycle vehicle model OR Waveboard OR Thermal analysis OR uncertainty modeling OR vehicle dynamics OR vehicle control OR ("bycicle model" NEAR vehicle) OR Continuously Variable Transmissions OR helmet OR Differential brakes OR Waveboard OR Thermal analysis OR uncertainty modeling OR Computational Fluid Dynamics OR CFD OR Aerodynamic OR Catalytic converter OR Fault diagnosis OR additive manufacturing OR Cavitation OR fluid OR Stress State OR planetary gear train OR Crash NEAR/5 model* OR Chain Drive OR Street-Style)))

where 'TS' represents the topic of a publication, 'WC' is the Web of Science category, and the '*' wildcard means a fuzzy search. Additionally, only journal articles in English were considered. Complementarily, the Google Scholar search engine was used for a further search with the same basic keywords. The combined resulting documents were then screened and filtered. Finally, a full-text review by the authors allowed excluding the remaining documents that were not within the scope of this review paper.

The metadata of all documents indexed in Web of Science was used for a bibliographic analysis using the Bibliometrix tool in R software [186]. Tables 1, 2, and 3 provide the preliminary information about the recollected literature, namely, the number of journals, an average year from publication, average citation per document, the total number of authors, to name a few.

Table 1. Main information about the recolected documents.

Main information about data	Results
Timespan	2013 : *present*
Sources (Journals, Books, etc)	28
Documents	60
Average years from publication	3.44
Average citations per documents	4.383
Average citations per year per doc	0.9948
References	1152

Table 2. General information on document types and content.

Document types	Results
article	51
article; early access	4
article; proceedings paper	2
proceedings paper	3
Document contents	**Results**
Keywords Plus (ID)	95
Authors Keywords (DE)	213

Table 3. General information on authors and author collaboration.

Authors	Results
Total authors	142
Author Appearances	184
Authors of single-authored documents	4
Authors of multi-authored documents	138
Authors collaboration	**Results**
Single-authored documents	5
Documents per Author	0.423
Authors per Document	2.37
Co-Authors per Documents	3.07
Collaboration Index	2.51

The most relevant articles in the systematic selection of literature are mentioned in Table 4. Similarly, the most influential references for the documents in the systematic review are listed in Table 5.

Figure 2 shows the scientific production of the top 10 authors in the time span considered in this study.

In Figure 2, the bubble size indicates the number of documents published. The small bubbles represent one publication while the large ones two. The colour intensity is proportional to the total citations per year. The relevance of the authors decreases as they move down the list. The number of publications in the time interval established for the analysis is considered to evaluate the relevancy.

Table 4. The 10 most cited documents in the review collection.

Most cited documents	Total citations	Total citations per year
Corno M, 2015, IEEE TRANS CONTROL SYST TECHNOL [187]	22	3.667
Doria A, 2013, VEH SYST DYN [188]	22	2.75
Barbagallo R, 2016, PROC INST MECH ENG PT K-J MULTI-BODY DYN [76]	14	2.8
Corno M, 2015, IEEE-ASME TRANS MECHATRON [92]	15	2.5
Bulsink VE, 2015, ADV MECH ENG [90]	12	2
Klinger F, 2014, VEH SYST DYN [46]	12	1.714
Dabladji MEH, 2016, CONTROL ENG PRACTICE [189]	11	2.2
Sequenzia G, 2015, PROC INST MECH ENG PT K-J MULTI-BODY DYN [190]	10	1.667
Wang EX, 2015, IEEE TRANS INTELL TRANSP SYST [191]	8	1.333
Doria A, 2014, PROC INST MECH ENG PART C-J ENG MECH ENG SCI [192]	8	1.143

Table 5. The 10 most cited references in the review collection.

Most cited references	Total citations
Meijaard JP, 2007, P ROY SOC A-MATH PHY [89]	26
Kooijman JDG, 2011, SCIENCE [171]	15
Whipple F. J. W., 1899, Q J PURE APPL MATH [95]	15
Sharp RS, 2004, MULTIBODY SYST DYN [51]	14
Astrom KJ, 2005, IEEE CONTR SYST MAG [33]	13
Sharp RS RS, 1971, J MECH ENG SCI [102]	13
Limebeer DJN, 2006, IEEE CONTR SYST MAG [165]	11
Cossalter V, 2002, MOTORCYCLE DYNAMICS [152]	10
Cossalter V, 2011, MECCANICA [36]	10
Plochl M, 2012, VEHICLE SYST DYN [45]	10

At this stage, a factorial analysis is performed using the Multiple Correspondence Analysis (MCA) technique with the authors' keywords in all the collection articles [186]. The MCA is the equivalent of the principal components and principal coordinates for qualitative variables. It is a data analysis technique useful to summarize the information contained in a contingency table. It is used to detect and represent underlying structures in a data set. More specifically, the MCA algorithm is a variant of the PCA algorithm for dimensionality reduction, and this is a well-known machine learning algorithm used in unsupervised learning, bibliometrics, and comprehensive science mapping analysis. In this work, the MCA algorithm takes as input a matrix $\boldsymbol{A}$ constructed with all the systematically collected articles and all the authors' keywords of those articles. The i-th row of $\boldsymbol{A}$ correspond to i-th article and the j-th column to the j-th keyword. The component a_{ij} of $\boldsymbol{A}$ is equal to 1 if the j-th keyword is included in the authors' keywords of the i-th article, otherwise $a_{ij} = 0$. Therefore, the structure of $\mathbf{A}$ has the form $[\text{Article} \times \text{Keyword}]$. From it, it is computed the co-occurrence matrix $\boldsymbol{B}_{coc}$ as follows

$$\boldsymbol{B}_{coc} = \boldsymbol{A}^T \boldsymbol{A} \tag{1}$$

Then, the relative frequency matrix $\boldsymbol{F}$ is obtained as

$$\boldsymbol{F} = N^{-1} \, \boldsymbol{B}_{coc} \tag{2}$$

where N is the total number of appearances of all the keywords in all the articles. Namely, the summation of all the elements of the matrix $\mathbf{A}$ is equal to N. Thus, N is given by

$$N = \sum_{i=1}^{N_a} \sum_{j=1}^{N_k} a_{ij} \tag{3}$$

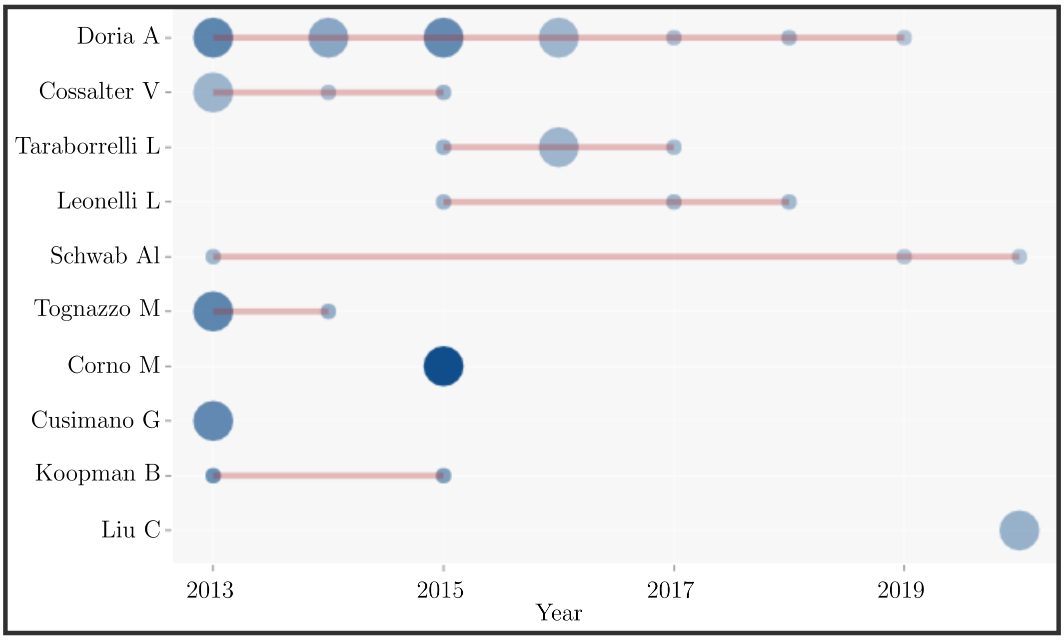

Figure 2. The top 10 authors production over time. Bubble size indicates the number of documents published. Small ones represent one publication and the large ones two. The colour intensity is proportional to the total citations per year.

where a_{ij} is the component of the matrix **A** in the i-th row and j-th column, while N_a and N_k respectively represent the total numbers of articles and authors' keywords of the collection. Therefore, N represents the sample size. Subsequently, the vectors $\boldsymbol{r}$ and $\boldsymbol{c}$ can be extracted from the construction of the matrix $\boldsymbol{F}$. These correspond to the totals by rows and columns, respectively. Hence, each element of the vectors $\boldsymbol{r}$ and $\boldsymbol{c}$ is computed as

$$r_i = \sum_{j=1}^{N_k} f_{ij} \ , \quad c_j = \sum_{i=1}^{N_k} f_{ij} \tag{4}$$

where f_{ij} is an indexed element of matrix $\boldsymbol{F}$. Then, the diagonal matrices $\boldsymbol{D}_c$ and $\boldsymbol{D}_r$ are respectively constructed as follows

$$\begin{cases} \boldsymbol{D}_c = \text{diag}(\boldsymbol{c}) \\ \boldsymbol{D}_r = \text{diag}(\boldsymbol{r}) \end{cases} \tag{5}$$

Subsequently, the matrix of relative frequencies $\boldsymbol{Z}$, standardized by the root of the row and column totals, is defined as

$$\boldsymbol{Z} = \boldsymbol{D}_r^{-1/2} \ \boldsymbol{F}^T \ \boldsymbol{D}_c^{-1/2} \tag{6}$$

From the mathematical expressions considered above, one seeks to maximize the weighted squared sum denoted with m obtained by projecting the matrix **Z** along the direction of a vector $\boldsymbol{\phi}$. Considering the constraint $\boldsymbol{\phi}^T\boldsymbol{\phi} = 1$ describing the orthonormality of the eigenvectors, this process can be expressed as

$$m = \boldsymbol{\phi}^T \ \boldsymbol{Z} \ \boldsymbol{Z}^T \ \boldsymbol{\phi} \tag{7}$$

The optimal value of m is found when $\boldsymbol{\phi}$ is an eigenvector of the matrix $\boldsymbol{M} = \boldsymbol{Z}\boldsymbol{Z}^T$. This for a set of eigenvectors associated to a set of eigenvalues $\lambda < 1$ to avoid the trivial solution of the projection problem. Because of how the problem is defined, the maximum eigenvalue of the matrix $\boldsymbol{M}$ is equal to one. Then, it is possible to project the matrix $\boldsymbol{F}$ onto the direction defined by the vector $\boldsymbol{\phi}$ using the following form

$$\boldsymbol{y}(\boldsymbol{\phi}) = \boldsymbol{D}_c^{-1} \ \boldsymbol{F}^T \ \boldsymbol{D}_r^{-1/2} \ \boldsymbol{\phi} \tag{8}$$

where $\boldsymbol{y}$ is the vector of projections of the matrix of co-occurrence $\boldsymbol{B}_{coc}$ in the new vector space. This projection is extended for two dimensions of the form

$$\boldsymbol{C} = \boldsymbol{D}_c^{-1} \ \boldsymbol{F}^T \ \boldsymbol{D}_r^{-1/2} \ \boldsymbol{\Phi}_2 \tag{9}$$

where $\boldsymbol{\Phi}_2 = [\boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2]$ is the matrix composed of the two eigenvectors associated with the two largest eigenvalues λ_1 and λ_2, both having modulus strictly minor of one. The rows of the matrix $\boldsymbol{C}$ correspond to the best projection onto a two-dimensional space of the keywords represented in the matrix $\boldsymbol{B}_{coc}$. Thereby, each row of matrix **C** represents an authors' keyword in the new two-dimensional space. Due to the discarded eigenvectors, there is a loss of variance in the projected data **C**. The preserved variance by each dimension is given as follows:

$$\Delta Var_1 = \frac{\lambda_1}{\left(\sum_{i=1}^{N_k-1} \boldsymbol{\Lambda}_{i,i}\right)} \quad , \quad \Delta Var_2 = \frac{\lambda_2}{\left(\sum_{i=1}^{N_k-1} \boldsymbol{\Lambda}_{i,i}\right)} \tag{10}$$

where $\boldsymbol{\Lambda}$ is the diagonal matrix of eigenvalues of the matrix $\boldsymbol{M}$ containing the eigenvalues sorted in increasing order. Subsequently, to build the conceptual structure map of the research area, the K-means clustering algorithm [186], explained in Algorithm 1 by employing a pseudocode, is used to label the row vectors of the matrix **C**. This makes it possible to identify research topics in the area of interest.

Algorithm 1: Pseudocode of the K-Means clustering algorithm.

Input:

- $\{\boldsymbol{t}_1,\ \boldsymbol{t}_2, ...,\ \boldsymbol{t}_{N_k}\} \in \mathbb{R}^2$: the N_k row vectors of the output matrix of the MCA algorithm, **C**
- K: number of clusters

Output:

- **m**: vector of cluster labels $[m_1,\ m_2, ...,\ m_{N_k}]$ of each $\mathbf{t}_i$ element

```
Randomly initialize the K cluster centroids χ_1, χ_2,..., χ_K ∈ ℝ^2
while convergence criteria has not been met do
    for i=1 to N_k do
        Assign to t_i a cluster label with the closest cluster centroid
        Update m_i with the cluster label assigned to t_i
    for j=1 to K do
        Update the χ_j cluster centroid by computing the average of the vectors {t_1, t_2, ..., t_{N_k}} assigned to the cluster j
return m
```

Table 6. Dataset for the numerical example. The rows and columns respectively represent the articles and the authors' keywords in the dataset of the example.

	K_1	K_2	K_3	K_4	K_5	K_6
A_1	1	0	1	0	1	0
A_2	0	0	1	1	0	1
A_3	1	0	0	0	1	0
A_4	0	1	0	1	0	1
A_5	0	0	1	1	1	0

A small demonstrative example is presented below to clarify the numerical procedure used to perform an objective and systematic literature review. For this purpose, it is employed a dataset composed of five articles and six authors' keywords, as shown in Table 6. In this example, the matrix **A** consists of the rows and columns of the Table 6 and, therefore, it is explicitly defined as

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{11}$$

The structure of matrix **A** has the form [Article $\times$ Keyword]. Each element a_{ij} of the matrix **A** contains the binary information if or not the article i includes the keyword j in its text. From the definition of the matrix **A**, employing Equation (1), the matrix $\mathbf{B}_{coc}$ can be computed as follows

$$\mathbf{B}_{coc} = \boldsymbol{A}^T \boldsymbol{A} = \begin{bmatrix} 2 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 3 & 1 & 2 \\ 2 & 0 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 2 & 0 & 2 \end{bmatrix} \tag{12}$$

The structure of matrix $\mathbf{B}_{coc}$ has the form [Keyword $\times$ Keyword]. Each element b_{ij} of the matrix $\mathbf{B}_{coc}$ represents the number of times the keywords i and j are included as authors' keywords in the same article. In contrast, the diagonal elements represent the number of times a keyword is included in the whole collection of articles. Subsequently, the sample size N is computed from Equation (3) yielding

$$N = \sum_{i=1}^{N_A} \sum_{j=1}^{N_K} a_{ij} = 14 \tag{13}$$

where $N_A = 5$ and $N_K = 6$ are respectively the number of articles and keywords in Table 6. Then, the matrix **F** is computed employing Equation (2) that yields

$$\boldsymbol{F} = N^{-1}\boldsymbol{B}_{coc} = \begin{bmatrix} 0.1429 & 0 & 0.0714 & 0 & 0.1429 & 0 \\ 0 & 0.0714 & 0 & 0.0714 & 0 & 0.0714 \\ 0.0714 & 0 & 0.2149 & 0.1429 & 0.1429 & 0.0714 \\ 0 & 0.0714 & 0.1429 & 0.2143 & 0.0714 & 0.1429 \\ 0.1429 & 0 & 0.1429 & 0.0714 & 0.1429 & 0 \\ 0 & 0.0714 & 0.0714 & 0.1429 & 0 & 0.1429 \end{bmatrix} \tag{14}$$

Subsequently, the vectors $\boldsymbol{r}$ and $\boldsymbol{c}$ can be computed from Equation (4) as follows

$$\boldsymbol{r} = \begin{bmatrix} 0.3571 & 0.2143 & 0.6429 & 0.6429 & 0.5714 & 0.4286 \end{bmatrix}^T$$
$$\boldsymbol{c} = \begin{bmatrix} 0.3571 & 0.2143 & 0.6429 & 0.6429 & 0.5714 & 0.4286 \end{bmatrix}^T \tag{15}$$

Given the nature of the problem that produces a symmetric $\mathbf{F}$ matrix, the vectors $\boldsymbol{r}$ and $\boldsymbol{c}$ are equal. Then, the diagonal matrices $\boldsymbol{D}_c$ and $\boldsymbol{D}_r$ are constructed following Equation (5), which yields

$$\boldsymbol{D}_c = \boldsymbol{D}_r = \begin{bmatrix} 0.3571 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2143 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6429 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6429 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5714 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4286 \end{bmatrix} \tag{16}$$

Subsequently, the matrix of relative frequencies $\boldsymbol{Z}$ is computed employing Equation (6), yielding

$$\boldsymbol{Z} = \boldsymbol{D}_r^{-1/2}\, \boldsymbol{F}^T\, \boldsymbol{D}_c^{-1/2} = \begin{bmatrix} 0.1429 & 0 & 0.0714 & 0 & 0.1429 & 0 \\ 0 & 0.0714 & 0 & 0.0714 & 0 & 0.0714 \\ 0.0714 & 0 & 0.2143 & 0.1429 & 0.1429 & 0.0714 \\ 0 & 0.0714 & 0.1429 & 0.2143 & 0.0714 & 0.1429 \\ 0.1429 & 0 & 0.1429 & 0.0714 & 0.2143 & 0 \\ 0 & 0.0714 & 0.0714 & 0.1429 & 0 & 0.1429 \end{bmatrix} \tag{17}$$

Then, it is addressed the problem of maximizing the weighted squared sum projection of the matrix $\mathbf{Z}$ along an unit vector $\boldsymbol{\phi}$, as described in Equation (7). For this purpose, the eigenvalue problem associated with the matrix $\boldsymbol{M}$ must be solved, where

$$\boldsymbol{M} = \boldsymbol{Z}\boldsymbol{Z}^T = \begin{bmatrix} 0.0459 & 0 & 0.0459 & 0.0204 & 0.0612 & 0.0051 \\ 0 & 0.0153 & 0.0153 & 0.0306 & 0.0051 & 0.0255 \\ 0.0459 & 0.0153 & 0.0970 & 0.0817 & 0.0817 & 0.0459 \\ 0.0204 & 0.0306 & 0.0817 & 0.0970 & 0.0510 & 0.0663 \\ 0.0612 & 0.0051 & 0.0817 & 0.0510 & 0.0919 & 0.0204 \\ 0.0051 & 0.0255 & 0.0459 & 0.0663 & 0.0204 & 0.0510 \end{bmatrix} \tag{18}$$

Such problem can written as

$$\boldsymbol{M}\boldsymbol{\Phi} = \boldsymbol{\Phi}\boldsymbol{\Lambda} \tag{19}$$

where the matrix $\boldsymbol{\Phi}$ is the matrix of column eigenvectors of the matrix $\boldsymbol{M}$ and the matrix $\boldsymbol{\Lambda}$ is the diagonal matrix of eigenvalues of the matrix $\boldsymbol{M}$. These two matrices are respectively equal to

$$\boldsymbol{\Lambda} = \begin{bmatrix} -9.01 \times 10^{-17} & 0 & 0 & 0 & 0 & 0 \\ 0 & 6.89 \times 10^{-4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0089 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0730 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5150 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} 0.4226 & 0.0459 & -0.4298 & 0.4917 & -0.5175 & 0.3536 \\ -2.98 \times 10^{-14} & -0.4863 & 0.3440 & 0.6131 & 0.4408 & 0.2739 \\ -3.54 \times 10^{-14} & -0.6558 & -0.1512 & -0.5518 & -0.1328 & 0.4743 \\ 0.5669 & 0.4341 & 0.2966 & -0.2676 & 0.3249 & 0.4743 \\ -0.5345 & 0.2549 & 0.4867 & 0.0581 & -0.4572 & 0.4472 \\ -0.4629 & 0.2791 & -0.5910 & 0.0540 & 0.4534 & 0.3873 \end{bmatrix} \tag{21}$$

Then, the reduced eigenvector matrix $\boldsymbol{\Phi}_2$ is defined by constructing a matrix with the two column vectors of the full eigenvector matrix $\boldsymbol{\Phi}$ that correspond to the two largest eigenvalues of the eigenvalue matrix $\boldsymbol{\Lambda}$ satisfying the constraint $\lambda_1,\ \lambda_2 < 1$. Therefore, the matrix $\boldsymbol{\Phi}_2$ is equal to

$$\boldsymbol{\Phi}_2 = \begin{bmatrix} \boldsymbol{\phi}_1 & \boldsymbol{\phi}_2 \end{bmatrix} = \begin{bmatrix} -0.5175 & 0.4917 \\ 0.4408 & 0.6131 \\ -0.1328 & -0.5518 \\ 0.3249 & -0.2676 \\ -0.4572 & 0.0581 \\ 0.4534 & 0.0540 \end{bmatrix} \tag{22}$$

where the vectors $\boldsymbol{\phi}_1$ and $\boldsymbol{\phi}_2$ are the eigenvectors associated with the eigenvalues $\lambda_1 = 0.5150$ and $\lambda_2 = 0.0730$, respectively. Subsequently, the projection on the two-dimensional plane of the matrix $\mathbf{B}_{coc}$, defined in Equation (12), is calculated. To do this, it is

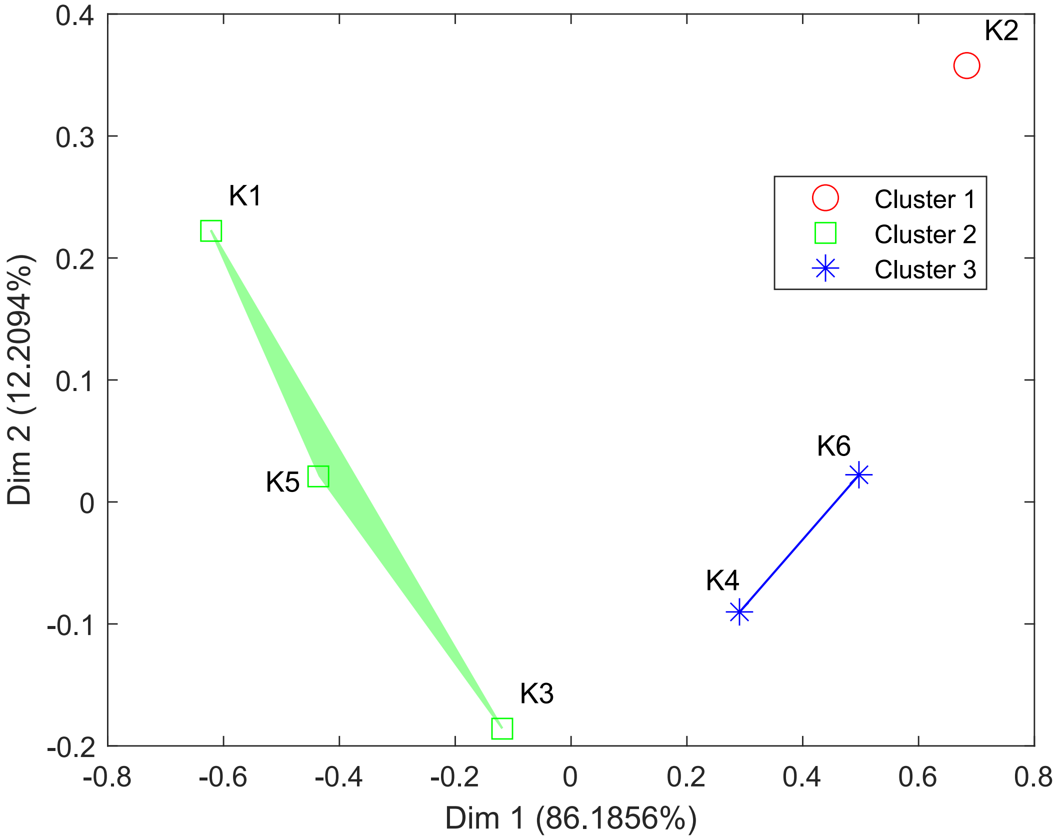

Figure 3. Keywords of the numerical example reported in Table 6 projected in the two-dimensional space in the matrix $\boldsymbol{C}$ defined in Equation (23). The cluster labels reported in the vector $\boldsymbol{m}$ are included using symbols.

employed Equation (9), yielding

$$\boldsymbol{C} = \boldsymbol{D}_c^{-1}\,\boldsymbol{F}^T\,\boldsymbol{D}_r^{-1/2}\,\boldsymbol{\Phi}_2 = \begin{bmatrix} -0.6215 & 0.2222 \\ 0.6834 & 0.3577 \\ -0.1188 & -0.1859 \\ 0.2908 & -0.0901 \\ -0.4341 & 0.0208 \\ 0.4971 & 0.0223 \end{bmatrix} \tag{23}$$

where the i-th row vector of matrix $\boldsymbol{C}$ represent the best projection on the two-dimensional space of the i-th keyword. The preserved variance by each dimension of the matrix $\boldsymbol{C}$ is computed by using Equation (10) as follows:

$$\begin{aligned} \Delta Var_1 &= \frac{\lambda_1}{\left(\sum_{i=1}^{N_k-1} \boldsymbol{\Lambda}_{i,i}\right)} = \frac{0.5150}{0.5975} = 0.8618 \\ \Delta Var_2 &= \frac{\lambda_2}{\left(\sum_{i=1}^{N_k-1} \boldsymbol{\Lambda}_{i,i}\right)} = \frac{0.0730}{0.5975} = 0.1220 \end{aligned} \tag{24}$$

Finally, the row vectors of matrix $\boldsymbol{C}$ are clusterized employing the K-Means method setting the parameter $K = 3$, as shown in Algorithm 1. This yields the vector $\boldsymbol{m}$ of cluster labels of each keyword represented by each column vector of the matrix $\boldsymbol{C}$ given by

$$\boldsymbol{m} = \begin{bmatrix} 2 & 1 & 2 & 3 & 2 & 3 \end{bmatrix}^T \tag{25}$$

Figure 3 is the projection of the keywords in Table 6 in the two-dimensional space.

In particular, each keyword is associated with a row of the matrix $\boldsymbol{C}$ and, therefore, the two elements of the i-th row of this matrix represent the coordinates of the keyword i represented onto the projection plane shown in Figure 3. Thus, the Dim 1 and Dim 2 entries per keyword correspond to the first and second elements of the row vectors of matrix $\boldsymbol{C}$. Moreover, each keyword features its corresponding cluster label from vector $\boldsymbol{m}$. The percentage reported in each dimension of Figure 3 is computed by multiplying by one hundred the variance parameters ΔVar_1 and ΔVar_2 obtained from Equation (24).

A process homologous to the one previously described allows the keywords of the collection of articles systematically selected in the literature review to be visualized in Figure 4. That is, the conceptual structure map of the research topics reviewed in the present work is shown in Figure 4.

The proximity between words shown in Figure 4 is related to the proportion of documents that simultaneously includes them as keywords, thereby exhibiting a similarity. In Figure 4, the origin of the axis represents the average position of all column profiles, thus being the centre of the research field [186]. The percentage reported in the Dim 1 (30.8%) and Dim 2 (20.99%) axes are the total data variation due to the dimensionality reduction. The colours are the three clusters identified by the K-means algorithm, the red

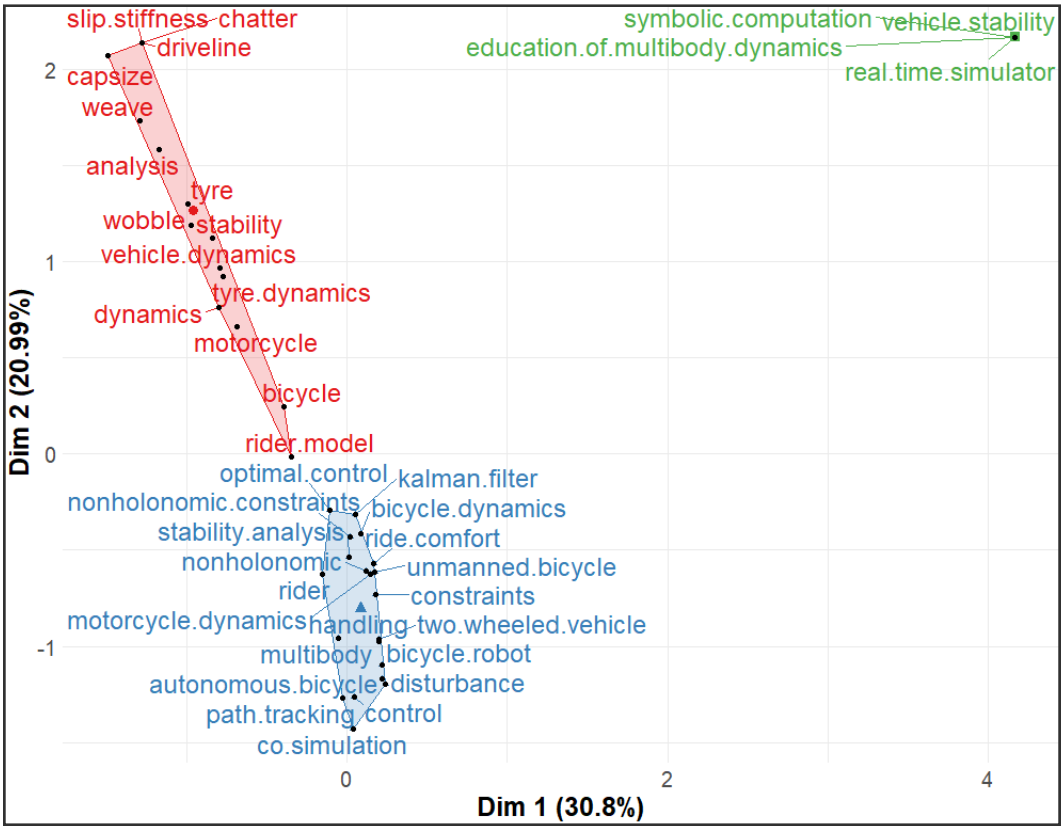

Figure 4. The conceptual structure map created with the MCA algorithm and the K-Means clustering algorithm. Red: multibody modelling of two-wheeled vehicles; Blue: control engineering and system identification; Green: the benchmark bicycle modelling and analytical dynamics.

one related to multibody modelling of two-wheeled vehicles, the blue one addresses control engineering and system identification studies, and the green one concerns studies related to the benchmark bicycle model and analytical dynamics.

On the other hand, Figure 5 shows the social structure of the research field. That is, the co-authorship network showing the association between the authors of the documents systematically collected in this review paper is represented in Figure 5.

Each author is referred by its surname and the initial letter of the first name, as shown in Figure 5. The size of the label is proportional to the number of works of the author. The edge size is proportional to the number of shared works between the connecting authors. The colors represent the identified clusters of common co-authorship. Figure 5 allows identifying influential research groups and authors. Here, Doria Alberto shows up as the most influential author in the time span of the study.

In summary, the systematic methodology employed in this work for performing the review process enabled identifying the principal research work available in the literature, thereby allowing for recognition of the fundamental issues related to the analysis of two-wheeled vehicles in general. Thus, these fundamental problems of engineering interest are thoroughly discussed in the next section.

4. Fundamental issues

In this section, the fundamental issues that emerged from using the systematic review methodology applied to the literature on two-wheeled vehicles are analyzed in detail. To this end, based on the conceptual structure map created with the MCA and K-means algorithms [180–186], as shown in Figures 4 and 5, further full-text analysis led to systematically group the collected literature into six categories. Each one of the identified categories describes a particular research line for the two-wheeled vehicle dynamics. The remaining parts of this section, therefore, report the essence of the systematically collected and categorized literature.

It is important to note that the resurgent interest in bicycle dynamics in the mid-2000s allowed researchers to employ previously developed methodologies for studying motorcycle dynamics. However, despite the topological similarity of the two vehicles, the achievement of bicycle detailed modelling and design criteria requires studying the system particularities. For instance, among the others, the magnitude of the dynamic variables of operation, the high rider-to-vehicle mass ratio, and the physical properties of its components are important features that distinguish the dynamics of bicycles from motorcycle dynamics. Therefore, when pertinent, a subdivision for bicycle-related and motorcycle-related works is set in this section.

4.1 Tyres and structural elements

Aiming at the development of models with a high level of accuracy, interest in the estimation of tyre properties, as well as in the use of lumped structural flexibility of the frame and fork, is found in the literature. This is timely, given, for instance, the lack of a standardized methodology for estimating the lumped structural properties used in motorcycle and bicycle modelling, as discussed in detail below.

4.1.1 Bicycles

In [188], Doria et al. performed experimental measurements to identify bicycle tyres properties. For this purpose, the rotating disk machine presented in [155] was employed. The experimental data were fitted with the MF tyre model [135]. The work proved feasible the use for bicycles of the experimental measurement methods originally devised for motorcycle tyres. Also, it was found

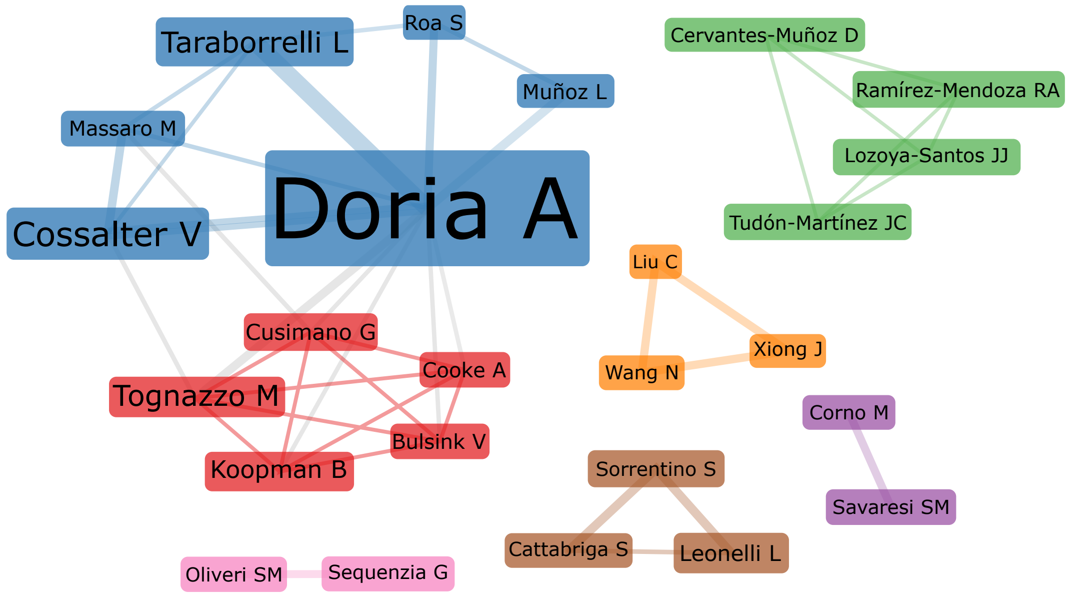

Figure 5. Authors collaboration network. Each author is mentioned in the format "surname + first names' initial letters". It was employed the Kamada-Kawai network layout, normalization with Salton index, and clustering with the Louvain algorithm.

similar behaviour in bicycle and motorcycle tyres. In [193], Souh took the logical subsequent step by addressing the influence of tyre side forces on bicycle self-stability. Due to the dataset shortage of bicycle wheel properties, the author employed the cornering and camber stiffnesses borrowed from [85] and the relaxation length obtained from [165]. The bicycle model considered was the benchmark bicycle complemented with the tyre model. The self-stability assessment took place by employing modal analysis and dynamic simulation of the bicycle. It was found that tyre sideslip forces slightly changed the weave vibration mode. On the other hand, the wobble mode and the rear wobble mode were considerably affected.

As far as the stability problem is concerned, a series of works by Doria et al. aimed to study the effect on the bicycle stability of tyres and the properties of the structural components [194–196]. In particular, the influences of the geometry, inertia, and flexibility of the front fork were investigated. The works were based on the bicycle model of Klinger [46]. The authors employed the factorial experimental design approach to assess the correlation of the tyre properties and the fork lateral stiffness and damping with the stability. To this end, correlation coefficients with the stability indexes were defined. The authors used, among others, the area under the eigenvalue plot for the self-stability region. The available experimental measurements of bicycle properties were also employed. It was found that the main influential parameters that affected the weave mode were the fork stiffness, together with the tyre cornering and twisting stiffness. In contrast, wobble mode was primarily influenced by the height of the twist axis and the fork damping, as well as the tyre cornering stiffness and the sideslip relaxation length.

4.1.2 Motorcycles

In [197, 198], Massaro and Cossalter studied the effect of inflation pressure on tyres properties and motorcycle stability. The authors considered significant variations of the internal tyre pressure in experimental tests for steady-state and transient properties. An experimental measurement method was proposed, and the data for different kinds of tyres were reported. Then, considering the MF tyre model, the coefficient dependence with the internal tyre pressure was analysed. Results included a significant camber thrust since self-aligning and twisting torques were produced by tyre deflation. Also, considering a transient regime, state measures found an increased relaxation length. Regarding stability, it was found beneficial for wobble and weave modes to increase the inflation pressure of the rear tyre. In contrast, an increase in the inflation pressure of the front tyre stabilized the wobble for low and medium velocity but was detrimental for high velocity. Later, Doria et al. proposed a methodology to characterize the lateral tyre properties based on modal analysis [199]. To this end, they identified four out-of-plane mode shapes, with increasing natural frequencies for higher inflation pressures. Then, estimations of the stiffness and damping properties of the lateral vibration based on the experimental modal analysis were described. Additional predictions of the relaxation length were also contrasted with experimental measurements showing satisfactory agreement.

In [200], Ballo et al. developed an analytical in-plane rigid-rim tyre model for motorcycles featuring explicit expressions for the resultant rim radial and axial forces. The model consisted of a rigid rim suspended by springs surrounded in the other end by a deformable curved beam. In this work, an agreement between the predicted radial deformations, the numerical results arising from the use of the finite element method, and experimental measurements was reported. In [201], Cossalter et al. presented a timely work proposing an experimentally-based lumped-system approach to model front frame compliance, with posterior complementation of Doria regarding the twist axis definition [202]. For this purpose, they introduced the MotoStiffMeter, an experimental apparatus designed to measure the physical properties of motorcycle components under static and dynamic conditions. The analysis of two different kinds of motorcycle forks showed that the fork twist axis measured was not perpendicular to the steering axis, contrary to the typical literature assumptions. A further modal analysis unveiled the existence of three modes of vibration of the front assembly, where the lateral mode was the one in the frequency range of interest, thereby proving accurate the perpendicular twist axis approximation. Furthermore, Doria and Taraborrelli focused on the rear frame lumped-system analysis, particularly the method to define the location and orientation of the twist axis [202]. Aware of the experimental inconsistencies reported in [123], the authors considered static and dynamic measurements. Finally, they proposed a method to identify the structural stiffness properties based on the experimental axes of the static and dynamic twist axes.

4.2 Innovative improvements for two-wheeled vehicles

Due to the emphasis of the present work is on MBD modelling, the developments of ADAS, ESC, and active safety systems are merely mentioned herein in connection with the main topic of interest. Indeed, coverage in detail is outside the scope of this work.

Interested readers are referred to the works of Savino et al. [57] and Schroter et al. [203]. On the other hand, the methodologically selected works presented in this review paper are specifically focused on improving the stability, handling, and design features of two-wheeled vehicles. These investigations are described in the remaining.

4.2.1 Bicycles

Considering the minor maintenance requirements and higher reliability of rear hub bicycle transmissions, Wu and Chang proposed a design methodology for these systems [204]. To this end, a five-links, two degrees of freedom epicyclic gear mechanism for the speed-changing task was considered in this work. Furthermore, the clutching mechanism designs and the three hub transmissions, including the gear-teeth of all elements, were described. The authors concluded that the proposed design methodology was suitable for similar mechanisms with more velocity relations. Similarly, another alternative mechanism with potential for improvement to traditional designs is presented in [205]. There, Corno et al. proposed an alternative to the usual pneumatic tires of bicycles that features in-wheel suspension. This contrasts with the traditional approach of installing suspension on the front fork and rear swingarm. The authors formulated an in-plane bicycle model featuring rear and front airless tires, with the spring-damper in-wheel mechanism proposed. Using a data-driven approach, they identified the spring and damper coefficients of the front and rear wheels, the energy dissipation properties, and the road disturbance filtering. In particular, the latter, which is directly related to the system's ride comfort, presents between 17% and 26% improvement. However, this is accompanied by a pedaling power dissipation of around 14% at cruising speed. Overall, the mechanism is a viable alternative to improve ride comfort in systems where power dissipation is not an issue.

In [206], Maier et al. presented an MBD model of the bicycle braking to develop a braking dynamics assistance (BDA) system. The proposed bicycle model featured front suspension and a passive rider with ten degrees of freedom. It was reported a good correlation of the numerical simulations with the experimental results. However, they mentioned the need for a higher level of detail. Moreover, Klug et al. delved into the front fork structural compliance effects in brake control systems [13]. The authors emphasized that flexibility altered the dynamics of the system and produced errors in the measurement signals, therefore affecting the estimation of the wheel slip that depends on the longitudinal tyre velocity measurements. By placing a gyroscope in the fork, an error compensation remedy was also proposed in this work.

Typically, intelligent passive bicycles are traditional bicycles with enhanced functionality given by the installed mechatronic systems. In this respect, Corno et al. proposed such a system focused on controlling the driver heart rate (HR) [187]. By regulating the ratio of a continuously varying transmission, this work aimed to keep the driver HR at a value specified by the user. This was done to distribute the user effort throughout the trip evenly. A simplified linear model of the bicycle was used to formulate a transfer function, while experimental measurements were used to identify the driver pedalling cadence. With the formulated control-oriented model of the entire system, a suboptimal sliding mode controller proved satisfactory results.

4.2.2 Motorcycles

In [207], Lozoya-Santos et al. proposed a rear semi-active suspension system for an off-road motorcycle. Three different control algorithms were studied in this paper. Employing a quantitative handling-dependent index, the authors compared the effect of each control scheme to the passive suspension system. Ultimately, performance increment was found, however, limited to particular conditions of velocity and road properties. Similarly, Khadr et al. proposed a design of a semi-active suspension system for a motorcycle [208]. The two-wheeled vehicle dynamic model, with moderate complexity, was derived following the Modified Denavit-Hartenberg method. To this end, two systems were developed using the skyhook algorithm. The control parameters were then optimized using the NSGA-II algorithm and an objective function in terms of the vertical acceleration of the chassis and the dynamic wheel loads. Numerical simulations showed superior vehicle performance with an active suspension system. In [24], Lot and Fleming studied the use of actively controlled gyroscopes to stabilize the vehicle in the braking task as well as its complete range of operating velocity. It was employed a mathematical model of the vehicle featuring the effect of the gyroscope to assess its stability. In particular, the effects on the weave, wobble, and capsize modes arising from the passive and active stabilization methodologies were studied. An optimal setup was that where the gyroscope spinning and swinging were parallel to the wheel rotation and the yaw of the vehicle, respectively. Finally, the author mentioned the need for further investigation to account for the time-varying aspects of the system.

Research related to design improvements includes a set of works of Moreno et al. In [209–211] the effect of implementing a front-rear suspension interconnected through a spring-damper system in a sports motorcycle is studied. Starting from a modification of the model [51], the in-plane dynamics is analyzed to evaluate comfort and the efficiency of the suspension. The authors defined the latter as the normalized difference between the value achieved by the variable after a bump input with and without interconnection forces and moments. Through a parametric analysis, it is concluded that the stiffness coefficient hardly affects the dynamic response. Thus, it is considered only a damping coefficient. In particular, the latter turned out to have a region of convenience, resulting in a trade-off problem between ride comfort and handling, which the authors analyzed through numerical optimization by proposing an objective function for a linear combination of handling and ride comfort efficiencies. The results showed that a speed varying damper will allow better performances, mainly for low and medium speeds. A subsequent out-of-plane stability analysis confirms that the interconnection mechanism proposed does not represent any stability issue for all the possible configurations. Later, In [211, 212], Moreno et al. evaluated the impact on motorcycle behaviour when replacing the traditional front suspension system with an alternative design, for instance, the Girder and Hosack mechanisms. For this purpose, it was employed a numerical simulation approach based on modifications of the model presented in [51]. The performance evaluation relayed on comparing the eigenvalue stability of two alternative systems with the telescopic fork mechanism. The authors found that the geometrical configurations of both the alternative suspension systems studied did not affect the stability or shock absorption properties. However, the anti-immersion qualities of alternative systems made them worthy of consideration for future research.

In [22], Griffin and Popov assessed the energy efficiency improvements of the all-wheel-drive scheme used in four-wheeled vehicles on motorcycles. It was proposed a thirteen-degree of freedom model implemented in SIMSCAPE Multibody, featuring the tyre model taken from the reference [84] and two proportional-integral controllers for the driveline torque and the steering. The authors found a modest energy efficiency improvement, estimated around 0.2%. However, the negotiation of a curve with all-wheel-drive active required a smaller steer angle, glimpsing a possible improvement in handling and stability. This idea was later considered in [213]. In this work, the authors presented the equations of motion of a two-wheeled vehicle with an all-wheel-drive scheme. The model considered small roll and steering angles. Comparisons with nonlinear simulations carried out in MSC ADAMS served as validation. The authors confirmed the findings reported in [22] regarding improved maneuverability in cornering. However, in-depth research is still required to establish the advantages and disadvantages of this power transmission type.

4.3 Benchmark bicycle studies

In the last decade, the non-holonomic nature of two-wheeled vehicles has drawn attention, not only from engineers for their practical applications but from mathematicians and physicists. In fact, for instance, the Whipple model has been recently included numerous times as an example in studies of analytical dynamics, geometric mechanics, and differential geometry [99, 214–216]. A detailed description of the Whipple model and the ad hoc derivation of its linearized equations of motion via Newton-Euler angular momentum balance arguments is presented in [217], and employing Kane's method in [218]. Considering nonlinear phenomena, Escalona et al. structured a multibody dynamics engineering course based on the Whipple nonlinear bicycle model, as presented in a series of work [219, 220]. The authors documented in detail the procedure for formulating the system equations of motion, including the contact holonomic constraints through non-generalised coordinates. Using the Lagrange equations of the first kind, a DAE system was obtained, later converted into an ODE using the generalised coordinate partitioning method. By doing so, the linear stability analysis performed employing the benchmark bicycle parameters agreed with the work of Meijaard et al. [89]. In [220], Escalona et al. extended the project by including an experimental validation of the work with an instrumented bicycle and an inverse dynamic experiment fed with the signals coming from the instrumented bicycle.

Among the others, nonlinear kinematics analysis of the Whipple model was studied in [10], in which a relevant approach and a viable solution to the problem were proposed. The position and the orientation of the bicycle bodies were obtained in this paper as explicit functions of the steering and roll angles. To this end, linearization in the remaining variables was used. Finally, the author obtained the wheels contact points expressions. It is also worth mentioning that two significant works concerning the development of the equations of motion for the Whipple model with control purposes are those of Wang et al. [191] and Turnwald and Liu [221]. Both models were formulated with a minimum coordinates approach, however, following different analytical dynamics formalism. For instance, in [191], Wang et al. employed the Lagrange equations of the first kind for non-holonomic systems, followed by the coordinate partitioning method to obtain the ODE with minimum coordinates. In [221], Turnwald and Liu, on the other hand, followed the approach originally developed in [222], that was based on geometric mechanics. Overall, the former nonlinear derivations of the Whipple model showed potential for control schemes and ADAS development, being somewhere in between oversimplified inverted pendulum models and highly complex MBD models.

Considering the stability problem, Xiong et al. studied the stability of the Whipple model in [223]. A detailed derivation of the contact constraints and the use of the Gibbs-Appell equations for non-holonomic systems resulted in the equations of motion as an ODE system. In this paper, the verification of the results presented in [112] on the hand-free circular motion stability was provided. Additionally, in contrast with the work of Meijaard et al. [89], the authors obtained the linearized equations of motion of the Whipple bicycle model in symbolic form without recurring to the ubiquitous ad hoc linearization procedures. Also, a subsequent work proposed by Xiong et al. presented an analytical study of the Whipple model performing a circular motion on a surface of revolution [224]. The authors symbolically derived the equations of motion using the Lagrange equation of the first kind. The formulation of contact constraints required particular treatment, given the intricate geometry. Employing Lagrange equations of the first kind, they obtained a DAE system of nine differential equations, six contact equations, two holonomic, and four non-holonomic constraints. Thus, a result similar to that proposed in [112] was found, namely four families of one parameter-dependent solution.

4.4 Two-wheeled vehicles multibody modelling

Where the Whipple model becomes unsatisfactory, the advances in MBD modelling of bicycles focus on considering the structural compliance of the system and improving the high-speed regime predictions. This fact is not surprising, given the similarity of the last decade bicycles dynamic models with the one originally developed in [102]. The latter is well known to have discrepancies in predicting the wobble mode instability. On the other hand, due to the highly detailed MBD motorcycle models produced during the last decade, most of the recent work on motorcycle modelling has focused on studying secondary vibration modes such as chatter or wheel patter. Additionally, there is an emerging interest in the development of compatible models for control applications. That said, this subsection is devoted to the multibody analysis techniques employed for modelling bicycles and motorcycles.

4.4.1 Bicycles

In [46], Klinger et al. studied the stability of wobble mode on racing bikes. Based on a previous model developed in [45], it was considered the presence of a passive driver with his hands on the handlebar described by three generalized coordinates. Moreover, in this work, there was an account of the front fork flexibility by lumped-system analysis. Experimental observations and numerical simulations showed that it was challenging and probably ineffective to damp the wobble just with the driver reaction. Later, Tomiati et al. [28] extended the model of Klinger et al. [46] by adding nonlinear tyre characteristics in order to obtain the correct behaviour of the shimmy mode. This was a non-divergent oscillation that was hidden by linear stability analyses. Therefore, the authors employed a bifurcation analysis to assess the phenomenon correctly. Ultimately, a parametric analysis of the effect of tyre properties on the shimmy mode, as well as a rigorous experimental validation of the model, were reported. Later, experimental data records of linear accelerations and angular velocities of six points of a racing bicycle experimenting hands-on shimmy were reported in [225]. To the best of the authors knowledge, this is the most detailed bicycle model available in the literature.

In an attempt to summarize the diverse research efforts, Table 7 shows some of the main contributions to bicycle dynamics modelling.

In summary, Table 7 follows a chronological order to identify the main contributions in bicycle dynamics modelling ranging from the year 1989 to the year 2019.

4.4.2 Motorcycles

In [11], Nehaoua et al. presented an eleven-degrees of freedom MBD motorcycle model featuring rear and front suspensions, as well as tyre-generated forces. On the other hand, there was no account of the structural compliance or the rider body effect in this model. A medium level of complexity was chosen to present the symbolic step by step derivation of the motion equations using Jourdain formalism. In particular, the authors developed a controller based on the Lyapunov method to perform longitudinal and cornering tests. They highlighted the advantage, from a control perspective, of using a model with medium complexity. With a similar perspective, Leonelli and Mancinelli presented a motorcycle model with high computational efficiency [226]. In this model, no accounts of structural compliance or passive rider effect were considered. The model included a novel rigid tyre model with dynamic response capacity to road profiles. The reason to employ a particular set of coordinates was explored [1], namely mixed relative coordinates, reference points, and the resulting kinematic analysis based on that. Experimental validations showed the capability of the model to accurately reproduce in-plane and out-of-plane frequency responses with high computational efficiency, thereby having the potential for the development of electronic active control systems.

Ajmi et al. studied the handling quality of two-wheeled vehicles in cornering manoeuvrers [227]. It was considered an eleven

Table 7. Main contributions in bicycle dynamics modelling.

Year	Authors	Main Contributions
1989	Whipple [95]	Derived the equations of motion of the bicycle and studied its linear motion in straight-running conditions.
2007	Meijaard et al. [89]	Performed a rigorous literature review on the equations of motion. Proposed the 'benchmark bicycle' case study.
2008	Sharp [85]	Considered, with some simplifications, the effects of acceleration, tyres cross-section, and frame compliance.
2012	Ploch et al. [45]	Included the tyre-slip and passive rider effects to model and experimentally validated the wobble mode of bicycles.
2014	Klinger et al. [46]	Extended the model of Ploch et al. for racing bicycles considering the effect of the rider hands on the handlebar.
2019	Tomiati et al. [28]	Included nonlinear tyre characteristics to accurately simulate the shimmy vibration mode.

degrees of freedom model with MF tyre forces. The system kinematics were modelled following the Modified Denavit-Hartenberg approach to obtain equations of motion in terms of the degrees of freedom. An equivalent implementation of the system in ADAMS took place for validation. To this end, LC, U-turn, and steady turning manoeuvrers were simulated, and manoeuvrability indices were evaluated. Moreover, the authors performed a sensitivity analysis of the Koch index [228], finding that the wheelbase relevance could make it a design parameter for handling improvement. In [229], Sorrentino and Leonelli addressed the issue of the driving mechanism of the self-excited chatter vibration. In their work, it was considered the finding of Cossalter et al. [175], where the chattering behaviour was reproduced in a braking manoeuvre in straight running condition. Therefore, the authors employed a simplified two-degree of freedom model to analyse the phenomenon. The paper included the wheel-swingarm suspension and the chain transmission, which were connected to a constant velocity translating frame. An exhaustive analysis of linear stability and parametric sensitivity was also performed, considering multiple equilibrium configurations.

As discussed in several investigations, the gradient of the nonlinear characteristic slip was found to be relevant, along with two dimensionless parameters, in the onset of tyre instability. In particular, an energy balance aided with phase diagrams explains the self-exciting nature. Then, Leonelli et al. extended the work with a planar motorcycle model made of eleven rigid bodies, including chain transmission and MF flat rigid ring tyres [230]. Findings related to the importance of tyre parameters were explored by time-domain simulations and frozen time eigenvalue analysis. Furthermore, experimental data and a numerical simulation showed agreement on the emergence of the unstable oscillation. Almost in contemporary works, stability maps were created based on the identified driving parameters showing agreement with the work of Sorrentino et al. [229]. Therefore, the crucial role of the tyre nonlinear slip function was confirmed. Also, a self-excited vibration phenomenon related to the front wheel locking due to heavy braking, know as front-wheel patter, was addressed in [231]. In this paper, using a methodology similar to the one developed in [230], the critical parameters of the self-excited vibration were identified, while a ground longitudinal/downforce dependency parameter defined the switching mechanism to instability. Besides, front suspension damping was recognized as the most influential manoeuvre-independent parameter.

Ride comfort analysis was studied in [232]. The authors proposed an in-plane ten degree of freedom motorcycle model, similar to the well-known half-car model. However, physical properties corresponding to a motorcycle were used together with an improved model of the wheels. These were considered as rigid disks suspended by two mass-spring systems perpendicular to each other, in addition to a rotational spring-damper element. These elements represented the tyre stiffness due to the internal pressure. Experimental results were in agreement with the model predictions for different kinds of wheel non-uniformities. Finally, the authors mentioned that, with the proposed tyre model, tyres could be optimized for rider comfort. In contrast to the analytical model developed in [232], Doria et al. presented an experimental-numerical method to assess the rider comfort of road bicycles [233]. Employing frequency response function measurements of the system, the model of the authors predicted the vibrations experienced by the driver. In addition, two worth mentioning works are those of Ferretti et al. [234], and Sequenzia et al. [190], both concerning the development of a multibody dynamics motorcycle model in Modelica and ADAMS/View, respectively. These are, to the knowledge of the authors, the first research works featuring structural elements modeled as flexible bodies. In particular, the deformation of the swingarm [234] and the flexibility of the rear frame [190] were considered.

In an attempt to summarize the diverse research efforts, Table 8 shows some of the main contributions to motorcycle dynamics modelling.

In synthesis, Table 8 follows a chronological order to identify the main contributions in motorcycle dynamics modelling.

4.5 Rider modelling

Modelling the human control logic to ride a two-wheeled vehicle is an emerging topic with several applications in the handling and manoeuvrability analysis of two-wheeled vehicles. Given the considerable high rider-to-vehicle mass ratio of two-wheeled vehicles, it is required to consider the effect of the rider body on the system dynamics. Today, through the use of computational MBD tools, detailed modelling of the rider-vehicle system is feasible. In the literature, the rider is often modelled using linked rigid bodies. In general, passive rider models include spring-damper elements to preserve body posture. In contrast, active rider models include actuators to mimic the pose of real riders during a manoeuvre. Interested readers in the rider posture while driving a two-wheeled vehicle are referred to the work of Arunachalam et al. [235].

As a pioneering work, Schwab et al. investigated the logic of the human rider control of bicycles [236]. The authors collected experimental data from bicycle riders on a narrow treadmill while applying an intermittent lateral impulsive force. Then, a rider control model with the experimentally identified rider control parameters employed in the Whipple bicycle was proved to stabilize the system. Similarly, Wang et al. presented a model of a bicycle controller emulating the human rider steering and body leaning control strategies based on human control experimental measurements [237]. A proportional-derivative feedback control law with time delays was proposed in this work. By the quasi-polynomial mapping-based root finder, the roots of the closed-loop systems were identified.

A work on the dynamic response of bicycle rider based on experimental identification was carried out in [238]. It considered all rider-vehicle interfaces and all three displacement directions, including, in particular, frequency response functions of the rider body interaction with the handlebars, seat, and footpegs. These were expressed through the Apparent Mass (APMS) and the Seat-

Table 8. Main contributions in motorcycle dynamics modeling.

Year	Authors	Main Contributions
1971	Sharp [102]	Studied the stability of a motorcycle with no rider control as a function of the forward velocity. Identified and named the main modes of vibration: the capsize, weave, and wobble.
1974	Sharp [105]	Studied the effect of the rear frame flexibility on the out-of-plane modes of vibration employing the lumped parameter analysis.
1980	Sharp and Alstead [119]	Considered the frontal fork flexibility with the lumped parameter analysis correcting the discrepancies on the onset velocity of the wobble mode.
1988	Katayana et al. [131]	Proposed a double inverted pendulum model to account for the human rider model and study the control logic of the human rider.
1998	De Vries and Pacejka [136]	Proposed an adapted version of the MF tyre model originally developed in [135] for high roll angles to be used in motorcycle models.
2001	Sharp and Limebeer [65]	Developed a thirteen-degrees-of-freedom model in AUTOSIM considering the front and rear frame compliances and the rider body leaning. Additionally, it considered cornering conditions analysis.
2002	Cossalter and Lot [75]	Developed an MBD motorcycle model for real-time applications featuring a proper tyre model.
2004	Evangelou et al. [157,158]	Developed and experimentally validated a steering compensator for motorcycles to address an instability identified in [51].
2008	Cossalter and Massaro [175]	Reproduced in a numerical simulation an instability anecdotally reported in racing motorcycles referred to as chattering behaviour.
2015	Sequenzia et al. [230]	Presented an MBD motorcycle model featuring the rear frame as flexible body employing the Craig-Bampton method.
2018	Leonelli et al. [230]	Identified the physical mechanism of the onset of the chattering behavior.

To-Sternum Transmissibility (STST). The authors reported that, for most frequencies and perturbation directions, the rigid body assumption is inadequate, and a parametric model is required to account for the rider dynamics. Moreover, Doria et al. [192, 239] developed testing equipment able to generate roll oscillations, like motorcycles during operation, to study the rider body response to roll oscillations. Then, by experimental measurements, the authors identified the stiffness and damping model parameters of a passive rider torso with three degrees of freedom. The benchmark bicycle equations were computed for the resultant rider-bicycle system, and its open-loop stability was analysed. In this work, particular detrimental effects of the rider arms stiffness were highlighted. Later, Bulsink et al. complemented the work of Doria et al. [192, 239] by including a tyre model, and a more elaborate five degrees of freedom bicycle rider [90]. Self-stability modifications were studied by combining nonlinear numerical results in ADAMS and system identification methods. In this investigation, the damping and stiffness of the torso were found to have a negligible effect. Besides, agreeing with the findings reported in [192], the authors concluded that the stiffness of the rider arms was destabilizing.

To conclude, a highly detailed motorcycle model was developed by Sequenzia et al. in [190]. With the use of ADAMS/View, the authors implemented a rider model with twenty-eight degrees of freedom and anthropomorphic physical properties. Moreover, the motorcycle frame was modelled as a flexible body employing the Craig-Bampton method. Later, Barbagallo et al. addressed a co-simulation of the system rider-motorcycle with ADAMS/View-MATLAB/Simulink [76]. In this work, an active control system was implemented to consider the rider posture. Additionally, the authors successfully simulated an entire lap of the Monza circuit.

4.6 Nonlinear control and system identification

Given the unstable nature of two-wheeled vehicles when operating outside their range of self-stability, a controller that provides stability is required to study the system dynamics in general conditions. Furthermore, features such as the non-holonomic constraints or being an underactuated non-minimum phase system have recently attracted the attention of researchers in control engineering. In particular, trajectory tracking, feedback stabilization, dynamic stability, and system identification are some topics addressed. Due to the emphasis of the present work is on two-wheeled vehicles modelled within the multibody formulation approach, the research dealing with nonlinear control and system identification, vast fields on their own, is merely mentioned herein in connection with the topic of interest. A complete review of control and identification issues concerning two-wheeled vehicles is indeed out of the scope of this work. Interested readers are referred to the detailed work of Corno et al. [92].

The control of the unmanned benchmark bicycle model was addressed by Shafiekhani et al. in [240] by employing fuzzy critic-based learning with the steering torque as a control signal. In [9], Chu and Chen considered an extended benchmark bicycle model, including a degree of freedom for the rider lean. They developed a controller based on model predictive control theory for lateral stability. Similarly, Baquero-Suarez et al. addressed the lateral stability control with the active disturbance rejection control (ADRC) approach [241], thereby performing an experimental implementation of the model that proved its robustness and performance. In

[242], Leonelli and Limebeer studied the lap-time minimization of a motorcycle with an optimal control approach. The modelled vehicle was an extended Whipple bicycle system with toroidal wheels and MF tyre forces. Experimental validation of the model based on professional driver test data was also reported in this work.

A robotic approach for the control of a bicycle, using a moment-gyroscope control element, is found in the work of Kim et al. [243]. To this end, two controllers were considered, one model-based controller with the gyroscope reaction for the balance task and a PID control to handle the speed and path tracking. Likewise, Park and Yi considered an active stabilizing controller with two moment-gyroscopes and a linear quadratic regulator algorithm to balance the system [244]. A scissor-pair scheme allowed for the cancellation of the unwanted component of the gyroscopic torque resulting in a structure similar to the work of Kim et al. [243]. In a similar vein, Tofigh et al. considered one additional degree of freedom of a twin flywheel system in an aligned setup [245]. In particular, a novel configuration was introduced with a fractional sliding mode control scheme with a set of constraints imposing orthogonality between the flywheels spin axis and an equal angular speed.

Regarding system identification, primary interest concerned the estimation of the two-wheeled vehicle roll angle and the lateral forces, given the direct measurement difficulties. In [189], Dabladji et al. employed a Takagi-Sugeno (TS) observer estimation of the lateral dynamics and the lateral forces for time-varying longitudinal velocities. In [246], Zhang et al. proposed an attitude estimation approach along with a partial drift-free attitude estimation method, and agreement with several bicycle riding examples was found in this research work. In [15], Damon et al. presented a nonlinear Luenberger observer capable of simultaneously estimate the rider action and the lateral dynamics. Validation with numerical simulations with a motorcycle model in BikeSim, as well as numerical comparisons with the work of Dabladji et al. [189] via the root mean squared error (RMSE), proved the superior performance. Sanjurjo et al. proposed an absolute roll angle estimator based on measurements of a three-axis angular rate sensor and the wheel speed sensor [56]. These were then fed to a Kalman filter, whereas an instrumented bicycle test was used for validation. Similarly, Romualdi et al., employing the Extended Kalman filter fed with measurements of an inertial unit, estimated in real-time the roll angle of motorcycles [247]. In [248], the authors aimed at the open issue of motorcycles passive suspension systems optimization by proposing an estimator of tire-road contact forces. This allows verifying suspension optimization tasks objectively, in contrast with the usual approach of subjective evaluation of a driver. Verification against simulation showed that the estimation is in good agreement.

4.7 Further Developments

In this subsection, further developments found in the literature on the analytical and computational methodologies for modeling two-wheeled vehicles regarding the recent electrification trends of bicycles and motorcycles are analyzed. In particular, the works focusing on the influence of electric drive trains on two-wheeled vehicles dynamics are considered herein. Due to the emphasis of the present work is on MBD modelling, the developments focusing on other areas of research are merely mentioned herein in connection with the main topic of interest. Indeed, coverage in detail is outside the scope of this work. Interested readers are referred to the works of Hung and Lim [249].

Some research efforts towards the dynamic analysis for the dimensioning of the components of the electric drive trains include the work of Ba Hung et al. [250]. Ba Hung et al. presented a parametric analysis as an optimal design tool of the nominal power of the pedaling assistance motor, the crank length, and the diameter of the wheels of an e-bike. The authors employed an in-plane longitudinal dynamic model to estimate the required traction power when considering a slope, air drag, and rolling resistance. From this, they proposed a PID control system to guarantee a constant speed of advance accompanying the power input of the rider with the electric motor. A similar approach is presented in [251], where the sizing of the drivetrain of an electric motorcycle is studied using a longitudinal in-plane motorcycle model. In particular, the procedure to determine the nominal power of the motor and the design of the battery pack considering the power and autonomy restrictions were performed. Experimental tests are used to verify the feasibility of the system operation with the electric motor power estimated analytically. The battery pack's dynamical performance is analyzed via a data-driven approach. In [252], a sensitivity analysis is employed to study how the system parameters affect the dynamic characteristics and electric consumption characteristics of an electric motorcycle. The authors considered a detailed electric motor model and determined through simulations the potential to optimize the system by adjusting input parameters. In particular, a detailed report of the sensitivity of the parameters in the system's energy consumption is presented.

Given the space and geometry restrictions related to motorcycles, some studies focus on the sizing of the battery pack for these systems. Two case studies for the design of the battery pack for electric motorcycles are presented in [253,254]. In particular, Drummond et al. in [253] detailed the workflow to adapt a traditional motorcycle to an electric traction system considering as a restriction the conservation of the system's center of mass location. In contrast, a general graphical tool-based method for sizing motorcycle battery packs is presented in [255]. This is through the series-parallel assembly of ion-lithium battery single cells. All these works consider the traction power of the electric motor as a design input parameter and estimate the battery pack dynamical model via equivalent circuit firs-order models. In contrast, to estimate and forecast the state of charge (SOC) and output voltage of a battery pack of an electric motorcycle system, Caliwag et al. proposed data-driven approach [256]. The estimator is a combined statistical and time-series neural network approach, namely the VARMA/LSTM. The model is trained with experimental data of an electric motorcycle during the CVS-40 drive cycle and the experimental tests prove superior performance compared to the traditional approaches.

Hanifah et al. presented an electric motorcycle dynamics model based on [165], featuring two degrees of freedom considering leaning and wheels rotation [257]. The authors also included the electric motor and battery dynamics via a transfer function and a parametrized first-order model. The experimental results show an evident ability for the developed electric motorcycle model to predict the driving range and track the speed profile during the simulation of two different driving cycles, namely, the WMTC and NEDC. In a similar vein, a multibody model of an electric motorcycle featuring a steady-state in-plane longitudinal dynamics together with a steady cornering dynamics model based on [152] is presented in a set of papers by Farzaneh et al. [258, 259]. An energy management system optimizes energy consumption by imposing optimal steady speed given a trajectory segmented in steady forward and steady cornering conditions. The optimal steady-state forward speed is identified for each trajectory track via optimization with dynamic programming [258], and heuristic optimization with the Cuckoo algorithm [259].

Due to the hybrid drive nature of e-bikes, some researchers have adopted methodologies developed for the energy optimization of hybrid cars to these systems. In [260], it is proposed a rule-based algorithm to optimize the start-stop, and the specific operating point of a small range extender system of an electric bicycle. In particular, the architecture considered features a small internal combustion engine with an electric generator. The numerical experiments performed showed that the system improves fuel efficiency and keeps the right battery SOC level. In contrast, a more refined approach is that of Guanetti et al. [261], where the Equivalent Consumption Minimization Strategy (ECMS) is applied to the energy management of series-hybrid electric bicycles. That is, an architecture where the pedaling power is not mechanically linked to the wheels but the electric generator. The authors employed an in-plane longitudinal analysis to model the bicycle dynamics and considered an objective function that minimizes the effort perceived by the cyclist and the difference between the SOC at the end of a trip with a target value defined by the user.

In general, the literature on two-wheeled vehicles featuring electric drive trains uses low-dimensional dynamic models mainly as a tool to estimate the power demand given an operating condition. Longitudinal in-plane models are mainly considered. However, the effect of out-of-plane dynamics on energy consumption is not negligible, as shown in [258, 259]. Considering this, the implementation of multibody out-of-plane models with a higher degree of refinement is recommended. Additionally, the use of electric drive trains notably modifies the rider-to-vehicle mass ratio of these vehicles, so the analysis of the effect of the driver's body dynamics is another recommended line of research.

5. Summary, conclusions, and research perspectives

In this work, a review of the literature on two-wheeled vehicle systems was performed, paying particular attention to the aspects concerning the kinematics and dynamics of multibody systems. Given the vast availability of literature reviews on this topic until the year 2013, the present study was divided into two parts. First, given the lack of a unified database and high-quality metadata of the early works, the scientific developments until the year 2013 were chronologically discussed by employing a historical literature review approach. Therefore, the preliminary part of the present work is aimed to set an appropriate context for the later developments and does not intend to supersede the available literature reviews on such a temporal frame. Then, in the second part of this investigation, employing the systematic literature review methodology described in the paper, the collection of documents in the time frame between the years 2013-present was identified for a more detailed review. Subsequently, the research field was statistically analysed using bibliometric methods, thereby identifying the relevant documents, references, and authors. Additionally, the conceptual structure map that graphically describes the topics of interest for this investigation was drawn. The construction of the social network of the researchers involved in this field was also performed. Finally, the analysis of the documents full-texts was carried out to categorize and synthetically describe the set of collected documents.

In this review article, five categories of fundamental issues were identified in the literature analysis concerning the research on two-wheeled vehicles, as reported in detail below.

- Research on methods for estimating mechanical properties of the components of two-wheeled vehicles.
- Innovations in the design and improvements in the functionality extension of two-wheeled vehicles.
- Analytical dynamics studies on the Whipple-Carvallo bicycle model and other related benchmark models.
- Development of accurate multibody models of two-wheeled vehicles.
- Identification and modelling of the rider-vehicle system dynamics.
- Studies of the control problem and investigations in system identification of two-wheeled vehicles.

Due to the current maturity achieved in the area of two-wheeled vehicle modelling, to make further improvements in the safety, handling, and comfort of this category of vehicles, a global and multidisciplinary insight of the system kinematic and dynamic features is essential. To this end, the design exploration of different solutions and the optimization of the system geometric and structural parameters are the natural following research topics. In the literature, there is also a renewed interest in the effect of the mechanical properties of structural components on the stability of two-wheeled systems. However, research is still needed to evaluate and understand the phenomenon entirely. This is because the nonlinear interactions that affect the system stability properties are intricate and usually antagonistic to various modes of vibration [73]. Therefore, the assessment of the optimal mechanical properties of the structural components should be a topic of future research for two-wheeled vehicles. Further studies are needed to optimize other properties that influence the load capability, reliability, and comfort of two-wheeled vehicles. This to develop in future guidelines for the design of two-wheeled vehicles in general.

In recent years, studies regarding the compliance of structural components of two-wheeled vehicles have been carried out by employing a lumped parameter approach. To date, this is still the dominant method in the literature. Nonetheless, the number and position of the lumped stiffness and the best tests for their identification is still an important issue. Research works like [201] and [202] can be considered as first-attempt methods that give an estimation of the properties in question. However, no general methods for estimating the lumped stiffness and damping properties have been developed yet in a standard fashion. This should be a priority in future investigations. An alternative to consider the compliance of the structural components of two-wheeled vehicles is the flexible multibody analysis based on modern finite element approaches, which allow for consistently describing finite rotations and large deformation problems. This is just beginning to be considered in the literature and can be considered a viable research path. Works like [234] and [190] are first attempts towards it.

Tyre modelling for motorcycles and bicycles is still a relevant topic of research. For instance, the use of more appropriate tyre models for the simulation of vehicle dynamics traveling on uneven roads or rough surfaces is required. Some works addressing this issue were discussed in the present review, but it is apparent from the current literature survey that further research on this topic is required. For bicycle dynamics modelling, the use of motorcycle analysis methods for tyre properties estimation proved feasible, as discussed in [188]. However, although this aspect is important for manoeuvres involving significant longitudinal acceleration, there has been limited application of tyre models in bicycle modelling described in recent research. Besides, few works addressed the relationship between the wheel stiffness, the tyre properties, the slip values, and the tyre forces. Furthermore, until today, the characterization of bicycle tyre properties under combined longitudinal and lateral friction conditions remains to be investigated. Tomiati et al. reported an apparent potential in tyre properties to prevent the onset of bicycle shimmy oscillations [28]. However, a more systematic theoretical and experimental analysis is needed to assess it.

Examples of the design exploration of two-wheeled vehicles are present in the literature. For instance, one can refer to the works related to the optimization of the final drive geometry [262], rear hub transmission [204], or the study of alternative front suspension systems [212]. Research work in this area is still in its early stages but shows high potential, for which further investigations are recommended. In particular, despite their promising improvements concerning the ubiquitous frontal telescopic fork, few works in the literature studied mechanisms such as the Girder, Hosack, or hub-centre steering systems. Further research could also lead to enhance motorcycle stability at high velocity and improve handling by investigating the design space of alternative mechanisms. To perform a thorough performance comparison, investigations employing the traditional and innovative models under homologous operating conditions are an interesting starting point to assess the quality of each design configuration. Other works aim to include additional features to improve the safety and manoeuvrability of the systems from an intelligent control perspective. For instance, active and semi-active damping systems [207,208], Braking Dynamics Assistance (BDA) systems [13,206], or stability control systems [24] are appropriate examples for achieving these goals.

Research in the areas discussed above depends on the availability of suitable multibody models. This is the topic of most significant maturity in literature since the available dynamic models of two-wheeled vehicles are detailed enough to exhibit the known vibrational instabilities as well as to predict further instability behaviours. However, the appropriate level of detail needs to be identified in advance for each application. This is because excessive detail can result in modelling that is incompatible with a particular application and hinder the analysis and understanding of the specific problem of interest. In this context, the literature highlights that, for describing their open-loop stability, proper modelling of two-wheeled vehicles requires at least one lumped stiffness for the front frame and one for the passive motion of the rider. Therefore, additional modelling details could be omitted depending on the application, as shown in some application-oriented two-wheeled vehicle models found in the literature [11, 226, 227].

Recently, bicycle modelling research has focused on adapting the techniques developed in the previous decade in motorcycle models. However, it is still difficult to take into account the implications of the intrinsic features of bicycles. Further research is needed in this area, in particular, due to the growing interest in improving driver safety alongside the emergence of smart bicycles. On the other hand, the study of motorcycle multibody models mainly focused on two instability phenomena identified in racing motorcycles, namely the motorcycle chatter (a sudden auto-excited vibration of the rear and front unsprung masses of a motorcycle during braking) [229, 230] and wheel patter (a sustained self-excited vibration of the motorcycle front wheel that precedes its locking during heavy braking in a straight motion) [231]. The analysis of the onset mechanism and the potential mitigation method for instability phenomena such as these require more research to identify improvements in the design or inclusion of additional mechanisms for counteracting the instabilities.

Several research works in the literature pointed out the need to consider the dynamics of the passive rider body in the analysis of two-wheeled vehicles. Since the last decade, studies aiming to identify the rider have appeared, and the effect of the upright and relaxed position of the rider on the system dynamics are known. However, it remains to be determined the appropriate level of modelling detail for the rider model. Besides, research on the estimation of the physical properties of the rider multibody model and the dynamic effect of its pose are still required. Finally, research aimed at understanding the control logic behind the human rider remains open despite some recent advances. In general, the development of a validated rider model is advocated in the literature, and research with this objective is necessary, given its versatile applicability for virtual prototyping, handling, and comfort analysis.

To conclude, the dynamic properties of two-wheeled vehicles make them a challenging platform to control. Thus, this problem has recently attracted the attention of the control engineering research community. In this review paper, a cluster for research works was identified using the K-means algorithm in the conceptual structure map created using the Multiple Correspondence Analysis (MCA) method. It was found that the works addressing this issue are mainly focused on developing control systems for path tracking, optimal control, and system identification of two-wheeled vehicles. However, these works use dynamic models having great simplifications. In perspective, further research focused on the development of exhaustive multibody models of two-wheeled vehicles with more degree of detail for the applications mentioned above is recommended.

Author Contributions

This research paper was principally developed by the first author (Camilo Andrés Manrique-Escobar) and by the second author (Carmine Maria Pappalardo). The detailed review carried out by the third author (Domenico Guida) considerably improved the quality of the work. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

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The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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ORCID iD

Camilo Andrés Manrique-Escobar https://orcid.org/0000-0002-9917-0215
Carmine Maria Pappalardo https://orcid.org/0000-0003-3763-7104
Domenico Guida https://orcid.org/0000-0002-2870-9199

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