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Research Paper

A Generalized Identification of Joint Structural State and Unknown Inputs Using Data Fusion MKF-UI

Lijun Liu[1], Jiajia Zhu[2], Ying Lei[3]

[1] Department of Civil Engineering, Xiamen University, No.182 Daxue Road, Xiamen, 361005, China, Email: liulj214@xmu.edu.cn

[2] Department of Civil Engineering, Xiamen University, No.182 Daxue Road, Xiamen, 361005, China, Email: jiaj.zhu@foxmail.com

[3] Department of Civil Engineering, Xiamen University, No.182 Daxue Road, Xiamen, 361005, China, Email: ylei@xmu.edu.cn

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Corresponding author: Y. Lei (ylei@xmu.edu.cn)

Abstract. The classical Kalman filter (KF) can estimate the structural state online in real time. However, the classical KF presupposes that external excitations are known. The existing methods of Kalman filter with unknown inputs (KF-UI) have limitations that require observing the acceleration response at the excitation point or assuming the unknown force. To surmount the above defects, an innovative modal Kalman filter with unknown inputs (MKF-UI) is proposed in this paper. Modal transformation and modal truncation are used to reduce the dimensionality of the structural state, and the accelerations at the excitation positions do not need to observe. Besides, the proposed MKF-UI does not require the assumption of unknown external excitation. Therefore, the proposed approach is suitable for the generalized identification of dynamic structural states and unknown loadings. The effectiveness and feasibility of the proposed identification approach are ascertained by some numerical simulation examples.

Keywords: Kalman filter, modal transformation, unknown inputs, limited measurements, data fusion.

1. Introduction

The state estimation of a dynamic system is important for both structural health monitoring (SHM) and structural vibration control [1-2]. In this respect, the classical Kalman filter (KF) can estimate the structural state in real-time. However, the classical KF presupposes that external excitations are known. In practice, external excitations are crucial information in structural monitoring. In many structural identification methods, it is assumed that the external excitations on structures are known. However, due to the boundedness of technical methods or field conditions, the external excitations acting on the structures, especially dynamic loads, are hard to be directly measured or impossible to be accurately measured [3-5]. Nevertheless, it is relatively easy to measure the dynamic responses of the structure under dynamic load. So, the indirect inversion method of identifying dynamic load information using the structural dynamic response provides a foundation for the health monitoring of engineering structures.

In the past explorations and researches, many scholars studied the identification methods of unknown excitations [6-10]. Especially, to circumvent the weakness of the classic KF approach, some researchers have developed some reformative KF with unknown excitation. Based on the least square estimation, Kalman filter with unknown input (KF-UI) has been developed [11-14]. Zhang and Xu [15] used the method in [13] to simultaneously identify the response and unknown inputs. They also studied the optimal strategy of multi-type sensor configuration for unknown outside inputs reconstruction [16]. Based on the scheme of the classical KF, He *et al.* [17] and Liu *et al.* [2] proposed the improved KF with unknown inputs. But in the above methods, the accelerations at unknown external inputs needed to be measured. Lourens et al. [18] developed an augmented Kalman filter with unknown forces, where the unknown excitations were introduced in the state vector and detected in combination with the structural states. This method needed to assume the increase item of the unknown forces was the zero-mean white Gaussian random process.

Modal transformation offers an efficient approach to reduce the dimension of the structural system. Structural modal characteristics can be identified by modal Kalman filter approach (MKF). To reduce the dimensionality of the motion equation and computational complexity, Li *et al.* [19] proposed the identification of the earthquake ground motion using KF technique and the modal expansion method. Zhi *et al.* [20-21] proposed the reconstruction of modal wind loads on high-rise buildings using Kalman filter-based algorithm. Azam *et al.* [1] demonstrated a dual implementation of the Kalman filter to estimate the full states of a linear state-space model with unknown inputs. But, in the above methods, the external modal loads also were assumed to be stochastic sequences with zero mean.

To overcome the above limitations of existing KF-UI based identification approaches, an innovative modal Kalman filter with unknown inputs (MKF-UI) is proposed in this paper. Concurrent identification of structural modal coordinates and unknown

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modal forces are carried out by combing KF-UI developed by authors [2] with the modal truncation technique. Then, the unknown external excitations are estimated by transforming to the identification of unknown modal forces. Because the structural modal coordinates are independent of the position of external excitations, the proposed approach does not need the acceleration information at the excitation positions. Moreover, the proposed MKF-UI does not require the random walk assumption of unknown external excitation. So, the proposed method is suitable for the generalized identification of dynamic system states and unknown external loads.

Similar to the previous KF-UI, the proposed MKF-UI also has a low-frequency drift in the identification of the external excitation and the structural displacement. Azam *et al.* [1] introduced the expert guess on the covariance of the unknown input to avoiding the drift effect. Maes *et al.* [22] studied a recursive smoothing algorithm to detect the input force and structural states which assumed a time delay to reduce the uncertainty in the identification results with considering non-collocated response and inputs. There were also some common methods to deal with the drift, such as regularization approaches [23], post-signal processing schemes [24], increasing dummy measurements [25], and data fusion [2]. In this paper, data fusion of partial displacements and acceleration measurement were adopted to eliminate the drifts problem in the identification results. Some numerical simulation examples were used to demonstrate the performance and feasibility of the proposed approach.

The rest of the paper is organized as follows: In Section 2, an innovative MKF-UI is derived for the identification of joint structural state and unknown excitation, and data fusion of partial displacement and acceleration is adopted. In Section 3, two numerical examples are utilized to verify the flexibility and applicability of the proposed MKF-UI. Finally, Section 4 summary some conclusions of the research work.

2. The Proposed Identification by MKF-UI

The equation of motion for an *n*-DOF linear structural system can be expressed as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(\mathrm{t})+\boldsymbol{C}\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{\eta}^{u}\boldsymbol{f}^{u}(\mathrm{t}) \tag{1}$$

where **M**, **C** and ***K*** are the mass, damping and stiffness matrix of the structures, respectively. $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$ and $\ddot{\boldsymbol{x}}$ are the vectors of the displacement, velocity and acceleration responses, respectively. $\boldsymbol{f}^{u}(\mathrm{t})$ is an external excitation vector, and $\boldsymbol{\eta}^{u}$ is the impact matrix linked with the input.

When the response at the excitation is not observed, the state equation and the observation equation in discrete form can be written as:

$$\boldsymbol{X}'_{k+1}=\boldsymbol{A}'_{k}\boldsymbol{X}'_{k}+\boldsymbol{B}'_{k}\boldsymbol{f}^{u}_{k}+\mathbf{w}_{k} \tag{2}$$

$$\boldsymbol{Z}_{k+1}=\boldsymbol{D}'_{k}\boldsymbol{X}'_{k+1}+\boldsymbol{v}_{k+1} \tag{3}$$

in which $\boldsymbol{X}'_{k+1}$ and $\boldsymbol{X}'_{k}$ is the state vector at $t=(k+1)\Delta t$ and $t=k\Delta t$, $\boldsymbol{A}'_{k}$ is the state transformation matrix, $\boldsymbol{f}^{u}_{k}$ is the unknown force vector at time $t=k\Delta t$ with impact matrix $\boldsymbol{B}'_{k}$, and $\boldsymbol{w}_k$ is the model noise with zero mean and a covariance matrix $\mathbf{Q}_k$. $\mathbf{Z}_{k+1}$ is the observed response vector, $\boldsymbol{D}'_{k}$ is the known measured matrix associated with structural state, $\boldsymbol{v}_{k+1}$ is the measured noise vector, which is assumed a Gaussian white noise vector with zero mean and a covariance matrix $\boldsymbol{R}_{k+1}$.

2.1 The Equations in Modal Space

Based on the employment of modal transformation $\boldsymbol{x}(\mathrm{t})=\boldsymbol{\Psi}\boldsymbol{q}(\mathrm{t})$, where $\boldsymbol{\Psi}$ is the orthometric mode shape matrix, $\boldsymbol{q}(\mathrm{t})$ denotes the modal coordinate vector, Eq.(1) can be rewritten in modal coordinate as:

$$\boldsymbol{M}\,\boldsymbol{\Psi}\ddot{\boldsymbol{q}}(\mathrm{t})+\boldsymbol{C}\,\boldsymbol{\Psi}\dot{\boldsymbol{q}}(\mathrm{t})+\boldsymbol{K}\,\boldsymbol{\Psi}\boldsymbol{q}(\mathrm{t})=\boldsymbol{\eta}^{u}\boldsymbol{f}^{u}(\mathrm{t}) \tag{4}$$

Left-multiply both sides of the Eq. (4) with $\boldsymbol{\Psi}^{\mathrm{T}}$, Eq.(4) can be expressed by

$$\boldsymbol{I}\ddot{\boldsymbol{q}}(\mathrm{t})+\boldsymbol{\Lambda}\boldsymbol{q}(\mathrm{t})+\boldsymbol{\Gamma}\dot{\boldsymbol{q}}(\mathrm{t})=\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\eta}^{u}\boldsymbol{f}^{u}(\mathrm{t}) \tag{5}$$

in which,

$$\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{M}\,\boldsymbol{\Psi}=\boldsymbol{I}\quad;\quad\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{K}\,\boldsymbol{\Psi}=\boldsymbol{\Lambda}\,;\quad\boldsymbol{\Gamma}=\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{C}\,\boldsymbol{\Psi} \tag{6}$$

and $\boldsymbol{\Lambda}=diag(\omega_1^2,\,\omega_2^2,\,\ldots\,,\omega_i^2,\,\ldots,\,\omega_n^2)$, ω_i is the ith order undamped natural frequency of the structure. Under the assumption of Rayleigh damping, $\boldsymbol{\Gamma}=diag(2\zeta_1\omega_1,2\zeta_2\omega_2,\,\ldots\,,2\zeta_i\omega_i,\,\ldots\,,2\zeta_n\omega_n)$, ζ_i is the ith modal damping ratio, $2\zeta_i\omega_i$ is the ith order modal damping.

Let the state vector $=(\boldsymbol{q}\ \ \dot{\boldsymbol{q}})^{\mathrm{T}}$, then

$$\dot{\boldsymbol{X}}=\begin{pmatrix}\dot{\boldsymbol{q}}\\ \ddot{\boldsymbol{q}}\end{pmatrix}=\begin{pmatrix}\dot{\boldsymbol{q}}\\ -\Lambda\boldsymbol{q}-\Gamma\dot{\boldsymbol{q}}+\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\eta}^{u}\boldsymbol{f}^{u}(\mathrm{t})\end{pmatrix} \tag{7}$$

So, Eq. (5) can be expressed as state equation by

$$\dot{\boldsymbol{X}}+\bar{\boldsymbol{A}}\boldsymbol{X}=\bar{\boldsymbol{B}}\boldsymbol{F}^{u}(\mathrm{t}) \tag{8}$$

where,

$$\bar{\boldsymbol{A}}=\begin{bmatrix}0 & -I\\ \boldsymbol{\Lambda} & \boldsymbol{\Gamma}\end{bmatrix}\quad,\qquad\bar{\boldsymbol{B}}=\begin{bmatrix}0\\ \boldsymbol{I}\end{bmatrix},\qquad\boldsymbol{F}^{u}=\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\eta}^{u}\boldsymbol{f}^{u} \tag{9}$$

$\boldsymbol{F}^{u}$ is the unknown modal force vector. Then, the discrete state equation can be transformed as

$$\boldsymbol{X}_{k+1} = \boldsymbol{A}_k\boldsymbol{X}_k + \boldsymbol{B}_k\boldsymbol{F}_k^u + \mathbf{w}_k \tag{10}$$

in which,

$$\boldsymbol{A}_k = \boldsymbol{I} - \Delta t\begin{bmatrix} \boldsymbol{0} & -\boldsymbol{I} \\ \boldsymbol{\Lambda} & \boldsymbol{\Gamma} \end{bmatrix} \qquad \boldsymbol{B}_k = \Delta t\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \tag{11}$$

and $\mathbf{X}_k$ is the state vector at $t = k\Delta t$ with the state transformation matrix $\boldsymbol{A}_k$, $\boldsymbol{X}_{k+1}$ the state vector at time $t = (k+1)\Delta t$. $\boldsymbol{F}_k^u$ is the unknown modal force vector at time $t = k\Delta t$ with impact matrix $\boldsymbol{B}_k$, and $\boldsymbol{w}_k$ the model noise with zero mean and a covariance matrix $\mathbf{Q}_k$.

The discrete observation equation can be expressed as:

$$\boldsymbol{Z}_{k+1} = \boldsymbol{D}_k\boldsymbol{X}_{k+1} + \boldsymbol{H}_k\boldsymbol{F}_{k+1}^u + \boldsymbol{v}_{k+1} \tag{12}$$

where $\mathbf{Z}_{k+1}$ is the observed response vector, $\boldsymbol{D}_k$ is the known measured matrix associated with structural state, $\boldsymbol{H}_k$ is the matrix associated with unknown modal forces vectors, respectively.

2.2 The Proposed MKF-UI

Similar to the classical KF, there are two steps in the proposed MKF-UI. The first one is the time prediction process, and the predicted state $\boldsymbol{X}_{k+1}$ is predicted as:

$$\tilde{\boldsymbol{X}}_{k+1|k} = \boldsymbol{A}_k\hat{\boldsymbol{X}}_{k|k} + \boldsymbol{B}_k\hat{\boldsymbol{F}}_{k|k}^u \tag{13}$$

where $\tilde{\boldsymbol{X}}_{k+1|k}$ is the predicted $\boldsymbol{X}_{k+1}$ at the time $t = (k+1)\Delta t$, $\hat{\boldsymbol{X}}_{k|k}$ and $\hat{\boldsymbol{F}}_{k|k}^u$ is the estimated $\boldsymbol{X}_k$ and unknown outside inputs $\boldsymbol{F}_k^u$ at time $t = k\Delta t$, respectively.

The second step is the measurement correction process, and the estimated $\boldsymbol{X}_{k+1}$ is derived as:

$$\hat{\boldsymbol{X}}_{k+1|k+1} = \tilde{\boldsymbol{X}}_{k+1|k} + \boldsymbol{G}_{k+1}(\boldsymbol{Z}_{k+1} - \boldsymbol{D}_{k+1}\tilde{\boldsymbol{X}}_{k+1|k} - \boldsymbol{H}_{k+1}\hat{\boldsymbol{F}}_{k+1|k+1}^u) \tag{14}$$

in which $\hat{\boldsymbol{X}}_{k+1|k+1}$ is the estimated $\boldsymbol{X}_{k+1}$ at time $t = (k+1)\Delta t$, and $\boldsymbol{G}_{k+1}$ is the Kalman gain matrix, $\boldsymbol{G}_{k+1} = \tilde{\mathrm{P}}_{k+1|k}^X\boldsymbol{D}_{k+1}^{\mathrm{T}}(\boldsymbol{D}_{k+1}\tilde{\mathrm{P}}_{k+1|k}^X\boldsymbol{D}_{k+1}^{\mathrm{T}} + \boldsymbol{R}_{k+1})^{-1}$.

When the number of measured responses is larger than the number of unknown modal forces, $\hat{\boldsymbol{F}}_{k+1|k+1}^u$ can be estimated by minimizing the errors $\boldsymbol{\Delta}_{k+1}$ between actual observations $\boldsymbol{Z}_{k+1}$ and observations calculated by Eq.(12).

$$\boldsymbol{\Delta}_{k+1} = \boldsymbol{Z}_{k+1} - \boldsymbol{D}_{k+1}\hat{\boldsymbol{X}}_{k+1|k+1} - \boldsymbol{H}_{k+1}\hat{\boldsymbol{F}}_{k+1|k+1}^u \tag{15}$$

Based on least-squares estimation, $\hat{\boldsymbol{F}}_{k+1|k+1}^u$ could be estimated as:

$$\hat{\boldsymbol{F}}_{k+1|k+1}^u = \boldsymbol{J}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}(\mathrm{I} - \boldsymbol{D}_{k+1}\boldsymbol{G}_{k+1})(\boldsymbol{Z}_{k+1} - \boldsymbol{D}_{k+1}\tilde{\boldsymbol{X}}_{k+1|k}) \tag{16}$$

in which $\boldsymbol{J}_{k+1} = \left[\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}(\mathrm{I} - \boldsymbol{D}_{k+1}\boldsymbol{G}_{k+1})\boldsymbol{H}_{k+1}\right]^{-1}$.

The error of the estimated $\hat{\boldsymbol{F}}_{k+1|k+1}^u$ can be derived as

$$\hat{\boldsymbol{e}}_{k+1|k+1}^{\mathrm{F}} = \boldsymbol{F}_{k+1}^u - \hat{\boldsymbol{F}}_{k+1|k+1}^u = -\boldsymbol{J}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}(\mathrm{I} - \boldsymbol{D}_{k+1}\boldsymbol{G}_{k+1})(\boldsymbol{D}_{k+1}\tilde{\boldsymbol{e}}_{k+1|k}^X + \boldsymbol{v}_{k+1}) \tag{17}$$

Then, the error $\hat{\mathrm{e}}_{k+1|k+1}^X$ and error covariance matrix $\hat{\mathrm{P}}_{k+1|k+1}^X$ can be estimated respectively as

$$\hat{\mathrm{e}}_{k+1|k+1}^X = (\mathrm{I} + \boldsymbol{G}_{k+1}\boldsymbol{H}_{k+1}\boldsymbol{J}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}\boldsymbol{D}_{k+1})(\mathrm{I} - \boldsymbol{G}_{k+1}\boldsymbol{D}_{k+1})\tilde{\mathrm{e}}_{k+1|k}^X - \boldsymbol{G}_{k+1}\left[\mathrm{I} - \boldsymbol{H}_{k+1}\boldsymbol{J}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}(\mathrm{I} - \boldsymbol{D}_{k+1}\boldsymbol{G}_{k+1})\right]\boldsymbol{v}_{k+1} \tag{18a}$$

$$\hat{\mathrm{P}}_{k+1|k+1}^X = (\boldsymbol{I} + \boldsymbol{G}_{k+1}\boldsymbol{H}_{k+1}\boldsymbol{J}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}\boldsymbol{D}_{k+1})(\boldsymbol{I} - \boldsymbol{G}_{k+1}\boldsymbol{D}_{k+1})\tilde{\boldsymbol{P}}_{k+1|k}^X \tag{18b}$$

Also, the error covariance matrix $\hat{\mathrm{P}}_{k+1|k+1}^{\mathrm{F}}$ and $\hat{\mathrm{P}}_{k+1|k+1}^{XF}$ can be derived respectively as

$$\hat{\mathrm{P}}_{k+1|k+1}^{\mathrm{F}} = \boldsymbol{J}_{k+1} \tag{19a}$$

$$\hat{\mathrm{P}}_{k+1|k+1}^{XF} = \left(\hat{\mathrm{P}}_{k+1|k+1}^{XF}\right)^{\mathrm{T}} = \mathrm{E}\left[\hat{e}_{k+1|k+1}^X\left(\hat{e}_{k+1|k+1}^F\right)^{\mathrm{T}}\right] = -\boldsymbol{G}_{k+1}\boldsymbol{H}_{k+1}\boldsymbol{J}_{k+1} \tag{19b}$$

and the error $\tilde{\mathrm{e}}_{k+1|k}^X$ and error covariance matrix $\tilde{\mathrm{P}}_{k+1|k}^X$ are derived respectively as

$$\tilde{\mathrm{e}}_{k+1|k}^X = \boldsymbol{A}_k\hat{\mathrm{e}}_{k|k}^{\mathrm{X}} + \boldsymbol{B}_k\hat{\boldsymbol{e}}_{k|k}^{\mathrm{F}} + \boldsymbol{w}_k \tag{20a}$$

$$\tilde{\mathrm{P}}_{k+1|k}^X = [\boldsymbol{A}_k \;\; \boldsymbol{B}_k]\begin{bmatrix} \hat{\mathrm{P}}_{k|k}^{\mathrm{X}} & \hat{\mathrm{P}}_{k|k}^{\mathrm{XF}} \\ \hat{\mathrm{P}}_{k|k}^{\mathrm{FX}} & \hat{\mathrm{P}}_{k|k}^{\mathrm{F}} \end{bmatrix}\begin{bmatrix} \boldsymbol{A}_k^{\mathrm{T}} \\ \boldsymbol{B}_k^{\mathrm{T}} \end{bmatrix} + \boldsymbol{Q}_k \tag{20b}$$

Finally, the identified modal forces can be used to estimate the unknown external excitations by:

$$\hat{f}^{u}_{k+1|k+1}=\left[\left(\Psi^{T}\eta\right)^{T}\left(\Psi^{T}\eta\right)\right]^{-1}\left(\Psi^{T}\eta\right)^{T}\hat{F}^{u}_{k+1|k+1} \tag{21}$$

Since the modal coordinates are applied to classical KF, the degrees of freedom can be reduced by modal truncation. In the existing KF-UI, for a linear structure with n degrees of freedom, the unknown number needed to be identified is $2n+p$ (p is the number of unknown force). However, in the proposed MKF-UI, the identified unknown number is $3r$ when the first r modes are excited ($r<n$).

To overcome the drift problem, data fusion of partial measurement of displacement and acceleration responses are adopted in the observation equation, because displacement and acceleration have low and high frequencies vibration behaviors respectively. So, Eq.(12) can be expressed as

$$\boldsymbol{Z}_{k+1}=\begin{Bmatrix}\boldsymbol{Z}^{a}_{k+1}\\ \boldsymbol{Z}^{d}_{k+1}\end{Bmatrix}=\begin{bmatrix}\delta^{a} & \varepsilon^{a}\\ \mu_{s} & 0\end{bmatrix}\begin{Bmatrix}\boldsymbol{q}_{k+1}\\ \dot{\boldsymbol{q}}_{k+1}\end{Bmatrix}+\begin{Bmatrix}\boldsymbol{S}\\ 0\end{Bmatrix}\boldsymbol{F}^{u}_{k+1}+\boldsymbol{v}_{k+1}=\boldsymbol{D}_{k+1}\boldsymbol{X}_{k+1}+\boldsymbol{H}_{k+1}\boldsymbol{F}^{u}_{k+1}+\boldsymbol{v}_{k+1} \tag{22}$$

in which, Z_k is the measured response vector including acceleration and displacement, δ^{a}_{k} and ε^{a}_{k} are measurement matrices associated with the measured accelerations, μ_s is the locations of measured displacements, and $\boldsymbol{S}$ is the influence matrix of unknown inputs on the acceleration responses.

3. Numerical Example

Numerical simulation examples are utilized to demonstrate the performance and feasibility of the proposed MKF-UI approach. Using the data fusion of partially measured responses of displacement and acceleration, the structural state and unknown outside input are jointly identified in the numerical simulation examples.

3.1 Identification of Shear Frame Structure and Unknown Outside Inputs

A thirty-story shear frame structure shown in the Fig.1 is used as a numerical example to validate the proposed method. The structural parameters are: $m_i=60$ kg, $k_i=12\times10^{5}$ N/m, $c_i=1.0\times10^{3}$ Ns/m, $i=1\ldots30$. An unknown external excitation of wide-banded white noise is employed on the top of the building.

As mentioned, the purpose of introducing modal transformation and modal truncation is to reduce the unknown number that needs to be identified. For the identifying results, it is desirable that the more modes are included, but this will increase the calculating costs. So, a balance must be found between identifying results and calculating costs. At the same time, the first r modes selected in the modal state space should have the greatest contribution to the identification inverse problem. At this point, the first 4 modes in the modal state space are selected to estimate the structural system.

Only 10 story acceleration responses $\ddot{x}_3$, $\ddot{x}_4,\ddot{x}_6,\ddot{x}_{10},\ddot{x}_{13},\ddot{x}_{17},\ddot{x}_{18},\ddot{x}_{22},\ddot{x}_{24}$ and $\ddot{x}_{29}$ are observed. It is noted that acceleration response at the position of unknown excitation (the top of the frame building) is not observed. As mentioned above, data fusion of partial displacements and acceleration was adopted to eliminate the drifts problem. The limited displacement x_5, x_{10},x_{15},x_{20} and x_{25} are measured. Data fusion of 5% noisy measurement data of the above 10 accelerations and 5 displacements are adopted in the observation equation Eq. (22). The concurrent identified velocity, displacement, and unknown inputs using the proposed MKF-UI are shown in Fig.2. It is clear that the estimated structural state and unknown external excitations are consistent with the true values.

3.2 Identification of a cantilever beam and unknown inputs

A cantilever beam is used to further verify the applicability of the proposed approach. The cantilever beam is long 3.75 m and is divided into 15 elements as shown in Fig.3. The structural parameters are: $EI_i=2.75\times10^{4}\text{Nm}^2, i=1\cdots15$, Rayleigh damping is adopted with damping ratios equal to 0.03. An unknown input $f^{u}=2000\sin(2\pi t)+800\sin(7\pi t)$ is employed to be applied on the beam. In this example, the first 5 modes in the modal state space are selected to estimate the structural system.

The partial nodal acceleration responses and displacements are observed as shown in Fig. 3. It is also noted that the acceleration response at the location of unknown input is not observed. Fig.4 shows all the estimation values using MKF-UI with data fusion of 5% noisy measurement data. It is shown that the concurrent identified results including structural state and unknown inputs agree with the exact values.

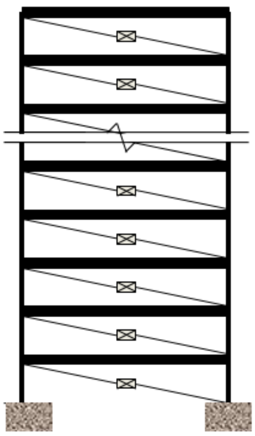
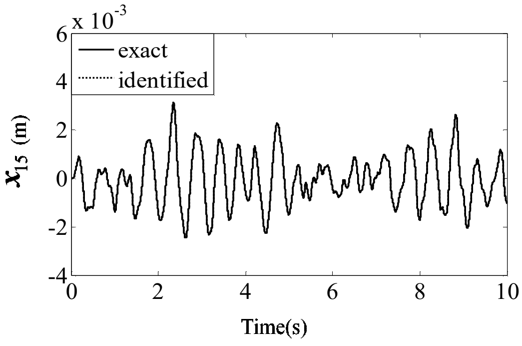
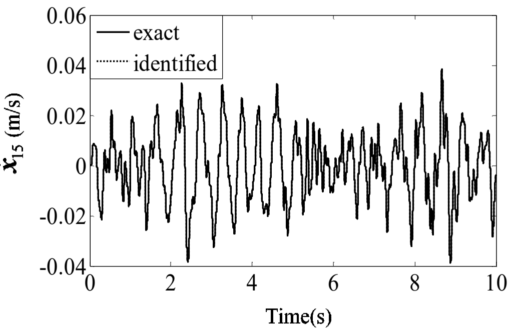


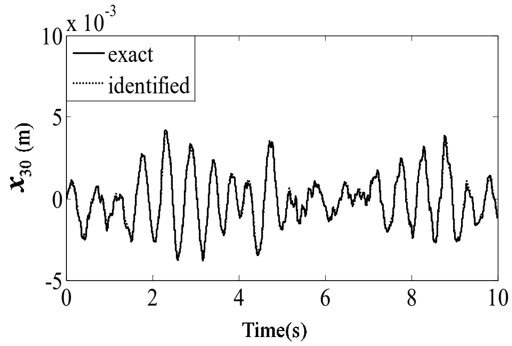
Fig. 1. A thirty-story shear frame structure



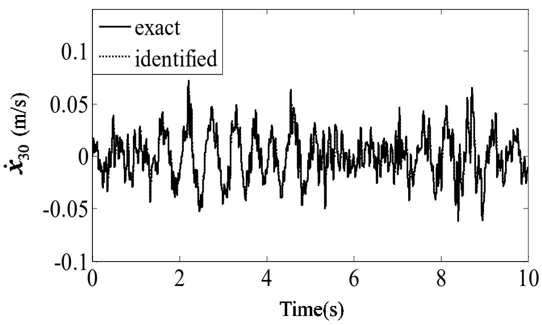
(a) Comparison of identified displacement of 15th floor



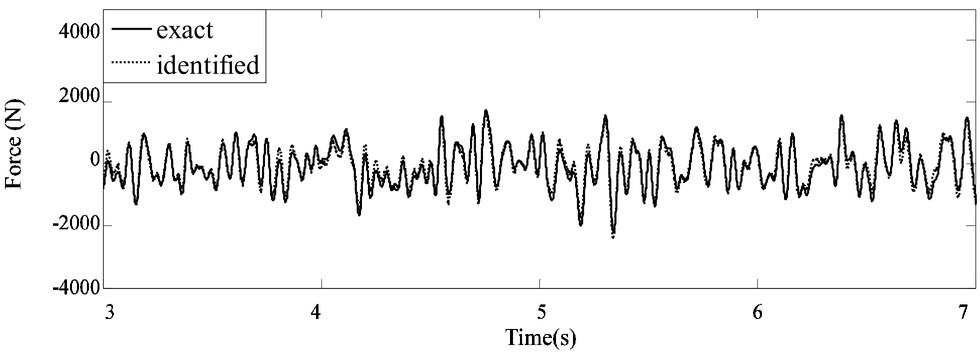
(b) Comparison of identified velocity of 15th floor



(c) Comparison of identified displacement of 30th floor



(d) Comparison of identified velocity of 30th floor



(e) Comparison of identified unknown inputs

Fig. 2. Comparisons of identified results with data fusion

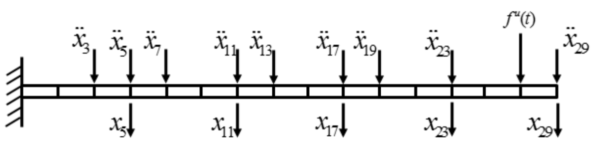
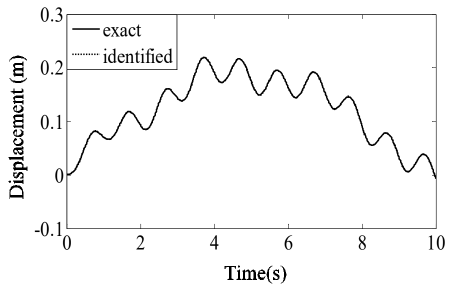
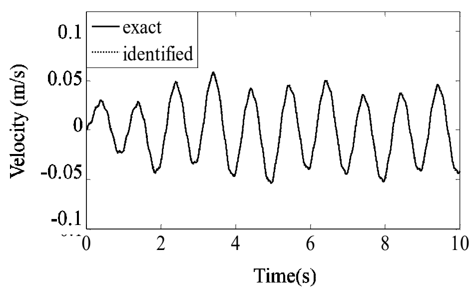


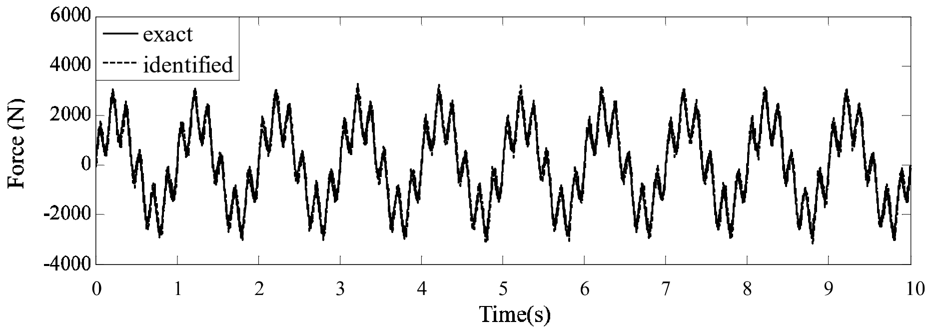
Fig. 3. A cantilever beam under unknown inputs



(a) Comparison of identified displacement of the14th node



(b) Comparison of identified velocity of the14th node



(c) Comparison of identified unknown inputs

Fig. 4. Comparisons of identified results with data fusion

4. Conclusions

In this paper, a generalized identification of joint structural state and unknown inputs is investigated based on modal Kalman filter with unknown inputs. The proposed method overcomes the limitation of many previous KF-UI methods that need to observe the acceleration responses at the unknown excitation or to assume the evolution of external excitation. The proposed MKF-UI method is based on the aggregation of classic KF in modal space and the recent KF-UI developed by the authors. The employment of modal transformation and modal truncation can reduce the scale of the structural state, and because the structural modal coordinates are independent of the position of external excitations, the accelerations at the excitation positions are not required. KF-UI developed by authors is adopted to realize concurrent estimation of structural modal coordinates and modal forces. Moreover, the proposed MKF-UI does not require the assumption of unknown external excitation. Then, the estimated modal forces can be returned to the unknown inputs. Therefore, compared with the existing KF-UI, the MKF-UI proposed in this paper offers an effective method to identify structural states and unknown inputs in more general cases.

Author Contributions

L. Liu developed the mathematical modeling and examined the theory validation; J. Zhu conducted the numerical simulation and analyzed the simulated results; Y. Lei planned the scheme and initiated the project. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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ORCID iD

Lijun Liu https://orcid.org/0000-0003-4385-8472
Jiajia Zhu https://orcid.org/0000-0002-7376-9078
Ying Lei https://orcid.org/0000-0002-7209-0629

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