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Research Paper

Nonstandard Dynamically Consistent Numerical Methods for MSEIR Model

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Abstract. In this paper, two numerical methods for solving the MSEIR model are presented. In constructing these methods, the non-standard finite difference strategy is used. The new methods preserve the qualitative properties of the solution, such as positivity, conservation law, and boundedness. Numerical results are presented to express the efficiency of the new methods.

Keywords: Positivity; Boundedness; Nonstandard finite difference.

1. Introduction

Finite difference methods are widely used to solve ODEs and PDEs problems [1-3], but in the construction of numerical methods using standard finite difference approximations, the necessary qualitative properties such as the positivity of the solution may not be transferred to the numerical solution. To solve this problem, Mickens [4, 5] introduced new methods called NSFD methods. The advantage of these methods are that in addition to maintaining the usual properties such as stability, consistency, and convergence, they produce solutions that maintain the qualitative properties of the exact solution [6-9].

This class of schemes and their formulations are determined around two issues: first, how they formulate the discretization of derivatives, and second, what are the appropriate ways to approximate nonlinear terms. The forward finite difference approximation for the first-order derivative is one of the most common methods for discretization. In standard mode, derivative dy/dx is approximated by $(y(x+h)-y(x))/h$, in which, h indicates the step-size. While, in the methods presented by Mickens, this term is approximated by $(y(x+h)-y(x))/\phi(h)$, where $\phi(h)$ is an increasing continuous function of h, which satisfies the following condition:

$$\phi(h)=h+O(h^2),\quad 0<\phi(h)<1,\quad h\to 0. \tag{1}$$

Note, when $h\to 0$ we must obtain the first derivative whatever the $\phi(h)$ taken. In fact, it must be:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\lim_{h\to 0}\frac{y(x+\phi_1(h))-y(x)}{\phi_2(h)}, \tag{2}$$

where $\phi_1(h)$ and $\phi_2(h)$ are continuous functions of the step-size h verifying (1). The finite difference method is called non-standard, when it satisfies one of the following conditions:

• The function in the denominator of the approximation of the discrete derivative must generally be expressed by a function such as ϕ of the step length, provided that (2) is verified. Using this rule, we can introduce a complex analytical function of h in the denominator, which holds in the following condition:

$$\phi(h)=h+O(h^2),\quad 0<\phi(h)<1,\quad h\to 0, \tag{3}$$

several functions $\phi(h)$ that satisfy in (3) are:

$$h,\quad \sin(h)\quad \text{or}\quad \frac{1-e^{-\lambda h}}{\lambda},\quad \lambda>0.$$

• The nonlinear expressions in the differential equation must be approximated non-locally, for example, the following can be mentioned (see [10-30])

$$y\approx 2y_n-y_{n+1},$$
$$y^2\approx 2(y_n)^2-y_{n+1}y_n,$$
$$y^3\approx y_{n+1}(y_n)^2,$$

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$$y^3 \approx y_n^2\left(\frac{y_{n+1}+y_{n-1}}{2}\right).$$

Note, that there are no general rules for choosing a denominator function or non-local expressions, although to see some techniques, you can refer to [13-28, 31-39].

In this paper, we develop the NSFD methods to obtain numerical solutions to the MSEIR epidemic model, where we apply the "Conservation Law" in the numerical schemes. New numerical methods retain their positive property. The rest of the paper is organized as follows: In Section 2, we introduce the mathematical model for the MSEIR epidemic model. In Section 3, we propose the new methods. In Section 4, boundedness, and the conservation law of methods are investigated. We investigate the positivity property in Section 5. In Section 6, we compared the obtained results from the new methods with the results of the classical methods. Finally, we end the paper with some conclusions in Section 7.

2. Mathematical Model

The mathematical model of MSEIR [2] consists of the following system of differential equations:

$$\begin{aligned} M'(t) &= b(N-S)-(\delta+d)M, \\ S'(t) &= bS+\delta M-\beta SI/N-dS, \\ E'(t) &= \beta SI/N-(\varepsilon+d)E, \\ I'(t) &= \varepsilon E-(\gamma+d)I, \\ R'(t) &= \gamma I-dR, \\ N'(t) &= (b-d)N, \end{aligned} \tag{4}$$

where

- M: the individuals with passive immunity, protected by maternal antibodies;
- S: the susceptible class, those individuals who can incur the disease but are not yet exposed to the disease;
- E: the individuals exposed to the disease but not yet infectious;
- I: the individuals infected by the disease and transmitting the disease to others;
- R: the recovered, with permanent immunity,

the parameters d, q, β, ε, δ and γ are positive numbers. By dividing each of the variables by N, we normalize the population, i.e. : $m = M/N$, $s = S/N$, $e = E/N$, $i = I/N$ and $r = R/N$. This reduces to the following equivalent system for the MSEIR:

$$\begin{aligned} m'(t) &= (d+q)(e+i+r)-\delta m, \\ s'(t) &= \delta M-\beta si, \\ i'(t) &= \beta si-(\varepsilon+d+q)e, \\ i'(t) &= \varepsilon e-(\gamma+d+q)i, \\ r'(t) &= \gamma i-(d+q)r \end{aligned} \tag{5}$$

The basic reproduction number of the system (5) is given by:

$$\mathcal{R}_0 = \frac{\varepsilon\beta}{(\varepsilon+d+q)(\gamma+d+q)},$$

The system (5) has a disease-free equilibrium $DFE = (0,1,0,0,0)^T$ and there exists also an endemic equilibrium $EE = (m^*, s^*, e^*, i^*, r^*)$ where:

$$\begin{aligned} m^* &= \frac{b+d}{\delta+d+q}(1-\frac{1}{\mathcal{R}_0}), \\ s^* &= \frac{1}{\mathcal{R}_0}, \\ e^* &= \frac{\delta(d+q)}{(\delta+d+q)(\varepsilon+d+q)}(1-\frac{1}{\mathcal{R}_0}), \\ i^* &= \frac{\varepsilon\delta(d+q)}{(\delta+d+q)(\varepsilon+d+q)(\gamma+d+q)}(1-\frac{1}{\mathcal{R}_0}), \\ r^* &= \frac{\varepsilon\delta\gamma}{(\delta+d+q)(\varepsilon+d+q)(\gamma+d+q)}(1-\frac{1}{\mathcal{R}_0}). \end{aligned}$$

The dynamics of the MESIR moel is shown in Fig. 1.

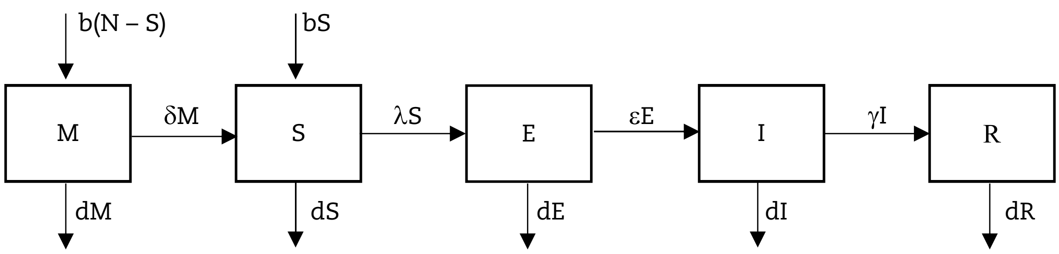


Fig. 1. Determine dynamics of the MESIR Model

3. Construction of New Methods

In this section, we propose two NSDF methods for the system (5) the following form:

3.1 Method 1

We propose our first new nonstandard (PESN) method as:

$$\begin{aligned}
\frac{m^{n+1}-m^n}{\varphi(h)} &= (d+q)(e^n+i^n+r^n)-\delta m^{n+1},\\
\frac{s^{n+1}-s^n}{\varphi(h)} &= \delta m^{n+1}-\beta s^{n+1}i^n,\\
\frac{e^{n+1}-e^n}{\varphi(h)} &= \beta s^{n+1}i^n-(d+q)e^n-\frac{\varepsilon}{2}(e^{n+1}+e^n),\\
\frac{i^{n+1}-i^n}{\varphi(h)} &= \frac{\varepsilon}{2}(e^{n+1}+e^n)-(d+q)i^n-\frac{\gamma}{2}(i^{n+1}+i^n),\\
\frac{r^{n+1}-r^n}{\varphi(h)} &= \frac{\gamma}{2}(i^{n+1}+i^n)-(d+q)r^n,
\end{aligned} \tag{6}$$

by simplifying the above relationships, we have:

$$\begin{aligned}
m^{n+1} &= \frac{\varphi(h)(d+q)(e^n+i^n+r^n)+m^n}{1+\delta\varphi(h)},\\
s^{n+1} &= \frac{\delta\varphi(h)m^{n+1}+s^n}{1+\beta\varphi(h)i^n},\\
e^{n+1} &= \frac{\beta\varphi(h)s^{n+1}i^n+e^n(1-\varphi(h)\left(d+q+\frac{\varepsilon}{2}\right))}{1+\frac{\varepsilon}{2}\varphi(h)},\\
i^{n+1} &= \frac{\frac{\varepsilon}{2}(e^{n+1}+e^n)+i^n(1-\varphi(h)\left(d+q+\frac{\gamma}{2}\right))}{1+\frac{\gamma}{2}\varphi(h)},\\
r^{n+1} &= \frac{\gamma}{2}\varphi(h)(i^{n+1}+i^n)+r^n(1-\varphi(h)(d+q)).
\end{aligned} \tag{7}$$

3.2 Method 2

We propose our second new NSDF method for the system (5) in the following form:

$$\begin{aligned}
\frac{m^{n+1}-m^n}{\varphi(h)} &= (d+q)(e^n+i^n+r^n)-\frac{\delta}{2}(m^{n+1}+m^n),\\
\frac{s^{n+1}-s^n}{\varphi(h)} &= \frac{\delta}{2}(m^{n+1}+m^n)-\beta s^{n+1}i^n,\\
\frac{e^{n+1}-e^n}{\varphi(h)} &= \beta s^{n+1}i^n-(d+q)e^n-\frac{\varepsilon}{2}(e^{n+1}+e^n),\\
\frac{i^{n+1}-i^n}{\varphi(h)} &= \frac{\varepsilon}{2}(e^{n+1}+e^n)-(d+q)i^n-\frac{\gamma}{2}(i^{n+1}+i^n),\\
\frac{r^{n+1}-r^n}{\varphi(h)} &= \frac{\gamma}{2}(i^{n+1}+i^n)-(d+q)r^n,
\end{aligned} \tag{8}$$

by simplifying the above relationships, we have:

$$\begin{aligned}
m^{n+1} &= \frac{\varphi(\mathrm{h})(d+q)(e^n+i^n+r^n)+m^n(1-\frac{\delta\varphi}{2})}{1+\frac{\delta\varphi(h)}{2}},\\
s^{n+1} &= \frac{\frac{\delta\varphi(h)}{2}(m^{n+1}+m^n)+s^n}{1+\beta\varphi(h)i^n},\\
e^{n+1} &= \frac{\beta\varphi(h)s^{n+1}i^n+e^n(1-\varphi(h)\left(d+q+\frac{\varepsilon}{2}\right))}{1+\frac{\varepsilon}{2}\varphi(h)},\\
i^{n+1} &= \frac{\frac{\varepsilon}{2}(e^{n+1}+e^n)+i^n(1-\varphi(h)\left(d+q+\frac{\gamma}{2}\right))}{1+\frac{\gamma}{2}\varphi(h)},\\
r^{n+1} &= \frac{\gamma}{2}\varphi(h)(i^{n+1}+i^n)+r^n(1-\varphi(d+q)).
\end{aligned} \tag{9}$$

4. Boundedness and the Conservation Law

In this section, we investigate the boundedness and the conservation law.

Definition 4.1 *Using [5], consider a system modeled by n-first-order differential equations*

$$\frac{dx}{dt} = f(x), \tag{10}$$

where,

$$x(t)^T = (x_1(t), \dots, x_n(t))^T, \qquad f(x)^T = (f_1(x), \dots, f_2(x)), \tag{11}$$

let

$$M(t) = \sum_{i=1}^{n} x_i(t). \tag{12}$$

If $M(t)$ satisfies a scalar differential equation of the form

$$\frac{dM}{dt} = f(M), \tag{13}$$

where f is a function depending only on M, then Eq. (13) is a conservation law for the system given by Eqs. (10) and (11).

Proposition 4.2 *Methods (7) and (9) preserve the conservation law.*

Proof. By summing the equations in (6) we have:

$$\begin{aligned}\frac{d}{dt}(m+s+e+i+r) = (d+q)(e^n+i^n+r^n) - \delta m^{n+1} + \delta m^{n+1} - \beta s^{n+1} i^n \\ +\beta s^{n+1} i^n - (d+q)e^n - \frac{\varepsilon}{2}(e^{n+1}+e^n) \\ +\frac{\varepsilon}{2}(e^{n+1}+e^n) - (d+q)i^n - \frac{\gamma}{2}(i^{n+1}+i^n) \\ +\frac{\gamma}{2}(i^{n+1}+i^n) - (d+q)r^n = 0,\end{aligned}$$

and

$$m^{n+1} + s^{n+1} + e^{n+1} + i^{n+1} + r^{n+1} - m^n - s^n - e^n - i^n - r^n = 0,$$

where

$$m^n + s^n + e^n + i^n + r^n = 1,$$

we have

$$m^{n+1} + s^{n+1} + e^{n+1} + i^{n+1} + r^{n+1} = 1,$$

therefor the method (7) preserve the conservation law. Similarly, by summing the equations in (8) we have:

$$\begin{aligned}\frac{d}{dt}(m+s+e+i+r) = (d+q)(e^n+i^n+r^n) - \frac{\delta}{2}(m^{n+1}+m^n) + \frac{\delta}{2}(m^{n+1}+m^n) - \beta s^{n+1} i^n \\ +\beta s^{n+1} i^n - (d+q)e^n - \frac{\varepsilon}{2}(e^{n+1}+e^n) \\ +\frac{\varepsilon}{2}(e^{n+1}+e^n) - (d+q)i^n - \frac{\gamma}{2}(i^{n+1}+i^n) \\ +\frac{\gamma}{2}(i^{n+1}+i^n) - (d+q)r^n = 0,\end{aligned}$$

and

$$m^{n+1} + s^{n+1} + e^{n+1} + i^{n+1} + r^{n+1} - m^n - s^n - e^n - i^n - r^n = 0,$$

where

$$m^n + s^n + e^n + i^n + r^n = 1,$$

we have

$$m^{n+1} + s^{n+1} + e^{n+1} + i^{n+1} + r^{n+1} = 1,$$

therefor the method (9) preserves the conservation law. ∎

Definition 4.3 *Solutions of system*

$$x' = f(t,x), \quad t > 0 \tag{14}$$

are bounded if the solution $x(t, t_0, x_0)$ of the system satisfies in the following:

$$\| x(t,t_0,x_0) \| \leq C(\| x_0 \|, t_0), \forall t \geq t_0 \tag{15}$$

where $C: \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a constant dependent on t_0 and x_0.

Proposition 4.4 *Methods (7) and (9) are bounded.*

Proof. Since the proposed schemes are positive, and all variable are nonnegative real numbers and according to the conservation law, the sum of theme variables equal to one, it follows that the methods have the boundedness.

5. Positivity

In this section, we investigate the positivity property of the constructed methods in the previous section.

Definition 5.1 *The finite-difference method is called positive, if, for any value of the step size h, and $y_0 \in \mathbb{R}_+^n$ its solution remains positive, i.e. $y_k \in \mathbb{R}_+^n$ for all $k \in \boldsymbol{N}$.*

Theorem 5.2 *Sufficient condition for methods (7) and (9) to be positive is the following:*

$$0 \le \varphi(h) \le \frac{1}{d+q+\frac{\gamma}{2}}.$$

Proof. Assume that m^n, s^n, e^n, i^n, and r^n are nonnegative real numbers so for positivity of method (7) it is enough to show that:

$$1-\varphi(\mathrm{h})\left(d+q+\frac{\varepsilon}{2}\right) \ge 0, \quad \Leftrightarrow \quad \varphi(\mathrm{h}) \le \frac{1}{\left(d+q+\frac{\varepsilon}{2}\right)}, \tag{16}$$

$$1-\varphi(h)\left(d+q+\frac{\gamma}{2}\right) \ge 0, \quad \Leftrightarrow \quad \varphi(\mathrm{h}) \le \frac{1}{\left(d+q+\frac{\gamma}{2}\right)}, \tag{17}$$

$$1-\varphi(h)(d+q) \ge 0, \quad \Leftrightarrow \quad \varphi(\mathrm{h}) \le \frac{1}{(d+q)} \tag{18}$$

from (16)-(18) can be seen that

$$\varphi(h) \le \min\left\{\frac{1}{\left(d+q+\frac{\varepsilon}{2}\right)}, \frac{1}{\left(d+q+\frac{\gamma}{2}\right)}, \frac{1}{(d+q)}\right\} \tag{19}$$

which leads to

$$0 \le \varphi(h) \le \frac{1}{d+q+\frac{\gamma}{2}}. \tag{20}$$

Similarly, for the positivity of method (9), it is enough to show that

$$1-\frac{\delta\varphi(h)}{2} \ge 0, \quad \Leftrightarrow \quad \varphi(h) \le \frac{2}{\delta}, \tag{21}$$

$$1-\varphi(h)\left(d+q+\frac{\varepsilon}{2}\right) \ge 0, \quad \Leftrightarrow \quad \varphi(h) \le \frac{1}{\left(d+q+\frac{\varepsilon}{2}\right)}, \tag{22}$$

$$1-\varphi(h)(d+q+\frac{\gamma}{2}) \ge 0, \quad \Leftrightarrow \quad \varphi(h) \le \frac{1}{(d+q+\frac{\gamma}{2})}, \tag{23}$$

$$1-\varphi(\mathrm{h})(d+q) \ge 0,, \quad \Leftrightarrow \quad \varphi(h) \le \frac{1}{(d+q)}, \tag{24}$$

therefore from (21)-(24) we have

$$\varphi(h) \le \min\left\{\frac{2}{\delta}, \frac{1}{\left(d+q+\frac{\varepsilon}{2}\right)}, \frac{1}{\left(d+q+\frac{\gamma}{2}\right)}, \frac{1}{(d+q)}\right\} \tag{25}$$

which leads to

$$0 \le \varphi(h) \le \frac{1}{d+q+\frac{\gamma}{2}}. \tag{26}$$

Thus inequalities (20) and (26) gives:

$$0 \le \varphi(h) \le \frac{1}{d+q+\frac{\gamma}{2}}, \tag{27}$$

and this completes the proof. ∎

6. Numerical Results

In this section, we present some numerical simulations to confirm the advantages of the constructed NSFD methods. Consider model (5) with the parameters:

$$d = 1/(40*365), \beta = .14, \delta = 1/180, \varepsilon = 1/14, \gamma = 1/7, q = 0,$$

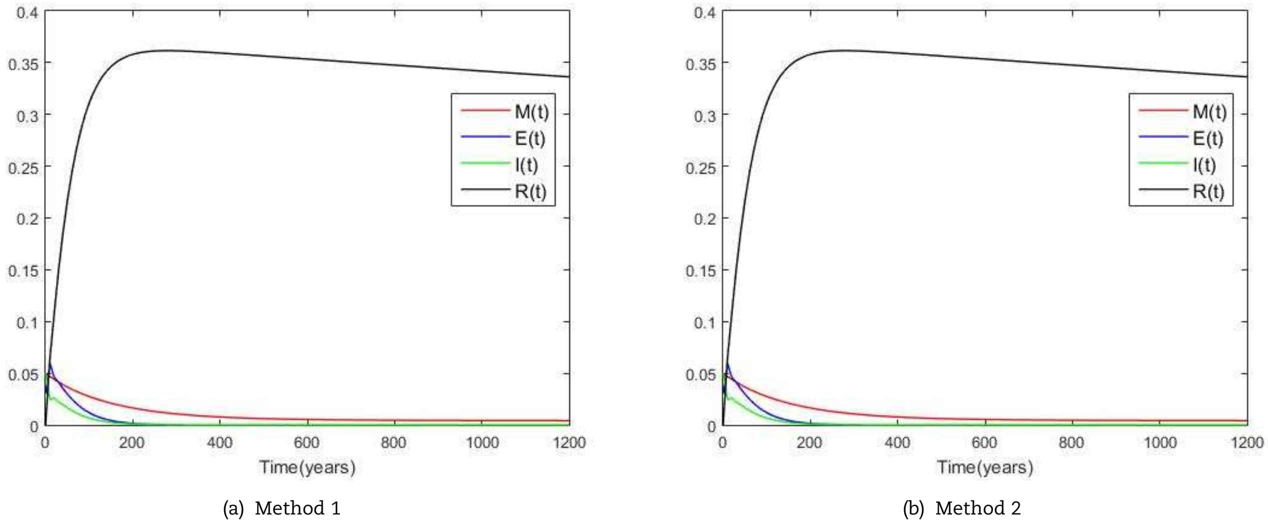


Fig. 2. Numerical results of the new methods with $h = 10, m(0) = 0.05, s(0) = 0.87, e(0) = 0.03, i(0) = 0.05, r(0) = 0$.

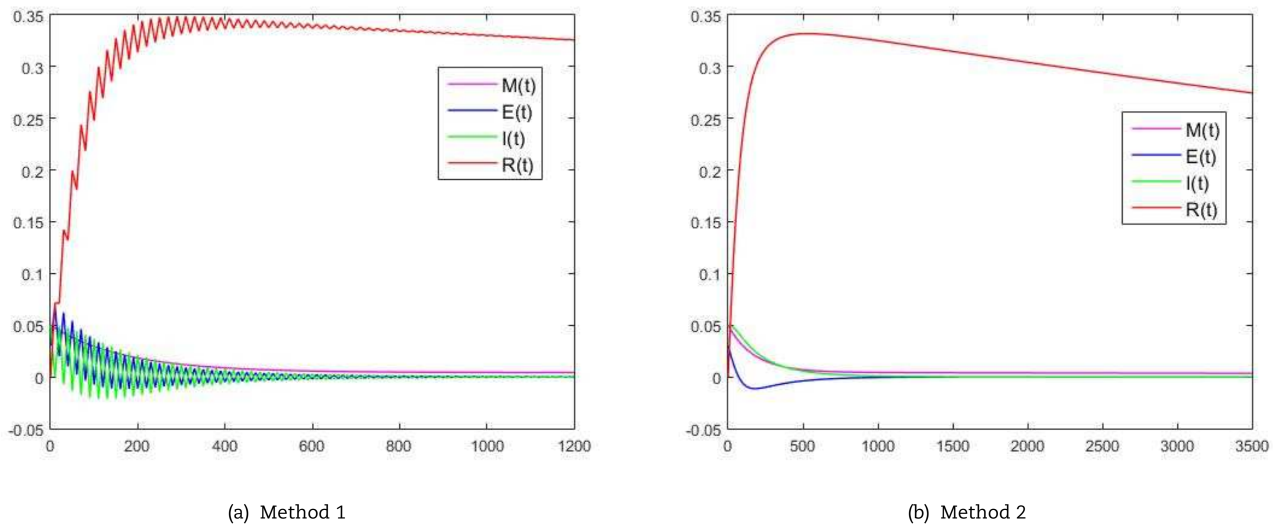


Fig. 3. Numerical results of the new methods with $h = 10, m(0) = 0.05, s(0) = 0.87, e(0) = 0.03, i(0) = 0.05, r(0) = 0$.

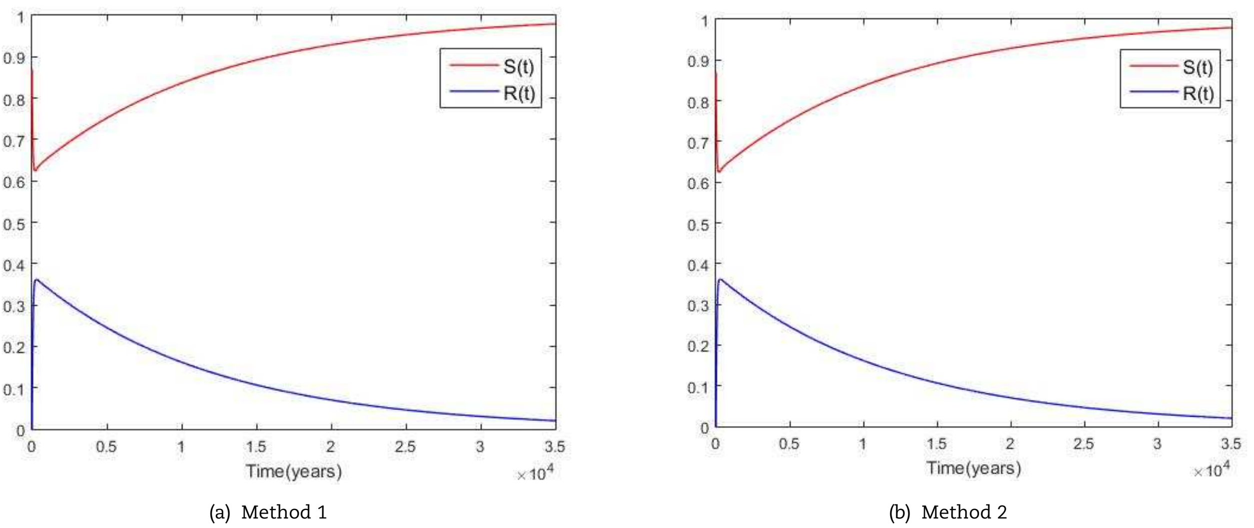
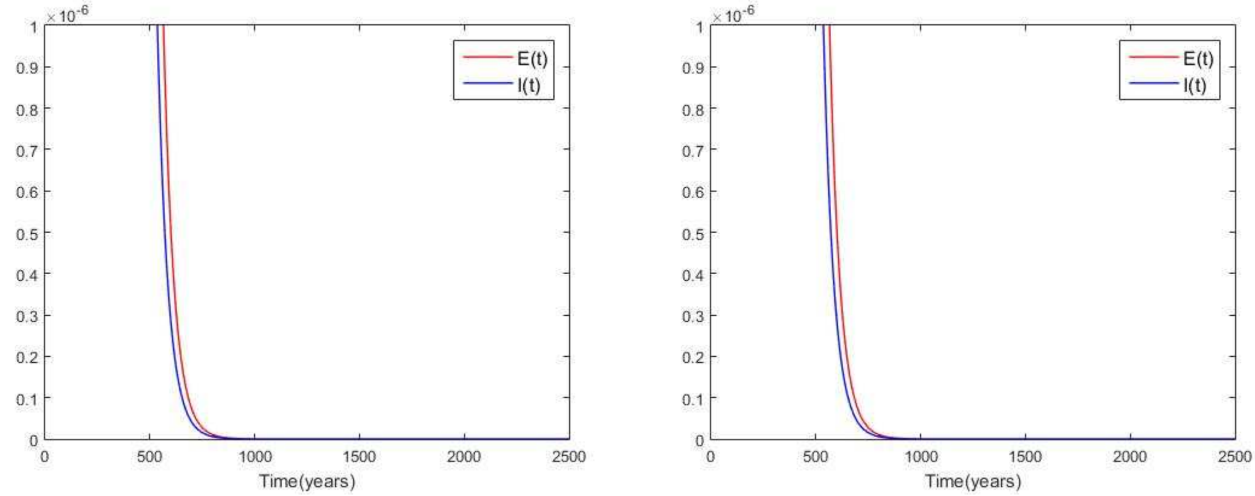
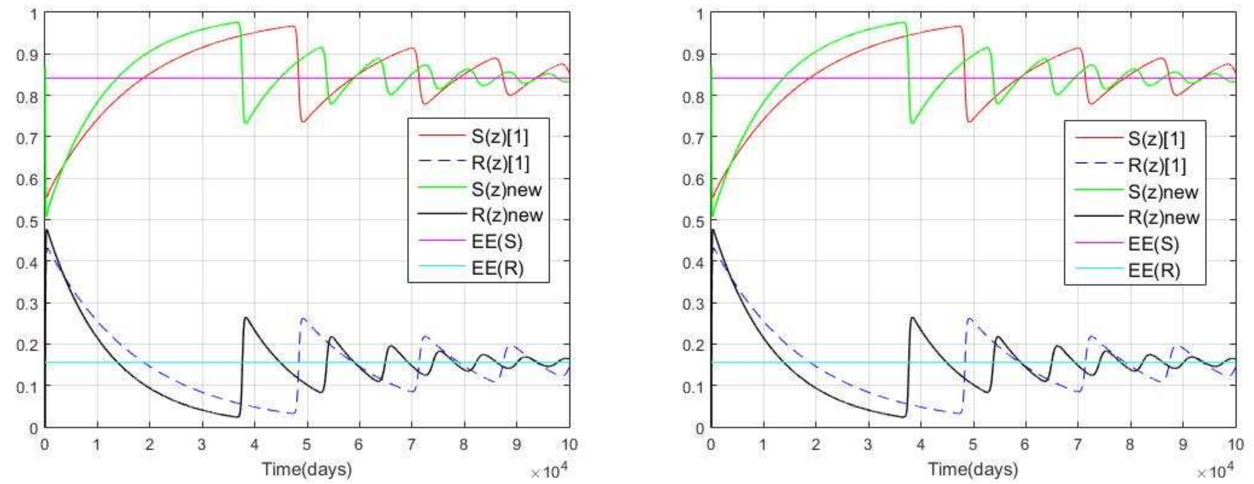


Fig. 4. Numerical results of the new methods with $h = 10, m(0) = 0.05, s(0) = 0.87, e(0) = 0.03, i(0) = 0.05, r(0) = 0$.



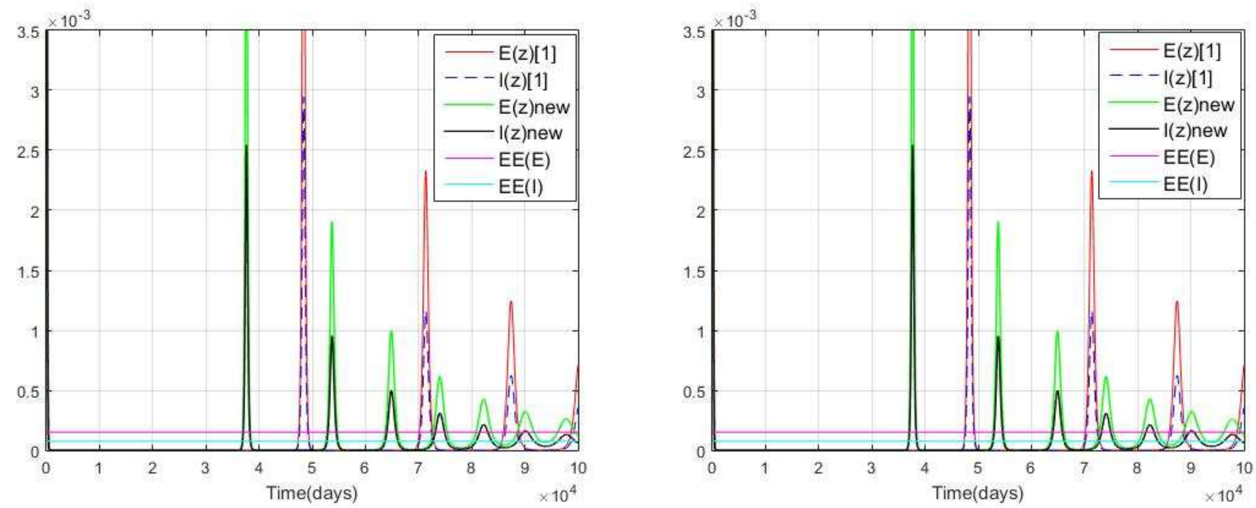
(a) Method 1 (b) Method 2

Fig. 5. Numerical results of the new methods with $h = 10, m(0) = 0.05, s(0) = 0.87, e(0) = 0.03, i(0) = 0.05, r(0) = 0$.



(a) Method 1 (b) Method 2

Fig. 6. Numerical results of the new methods with $h = 10, m(0) = 0.05, s(0) = 0.87, e(0) = 0.03, i(0) = 0.05, r(0) = 0$.



(a) Method 1 (b) Method 2

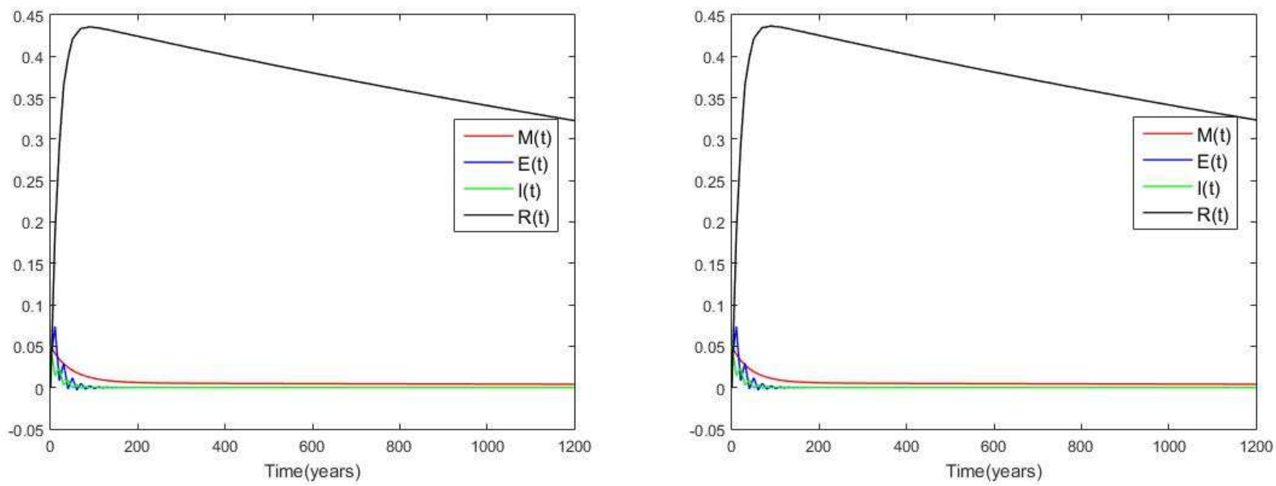
Fig. 7. Numerical results of the new methods with $h = 10, m(0) = 0.05, s(0) = 0.87, e(0) = 0.03, i(0) = 0.05, r(0) = 0$.

Table 1. Qualitative behavior with respect to E_0 of the methods considered on the problem (4) with $m(0) = 0.05, s(0) = 0.87, e(0) = 0.03, i(0) = 0.05, r(0) = 0$.

h	$Euler$	$RK2$	$Method1$	$Method2$
0.1	Convergence	Convergence	Convergence	Convergence
1	Convergence	Convergence	Convergence	Convergence
5	Convergence	Convergence	Convergence	Convergence
10	Divergence	Convergence	Convergence	Convergence
15	Divergence	Divergence	Convergence	Convergence
20	Divergence	Divergence	Convergence	Convergence
50	Divergence	Divergence	Convergence	Convergence
100	Divergence	Divergence	Convergence	Convergence

Table 2. The computational time used to obtain Figures 1-7.

Figure	(a)	(b)
Fig. 1	0.65s	0.66s
Fig. 2	3.62s	5.71s
Fig. 3	0.72s	0.73
Fig. 4	0.65s	0.68s
Fig. 5	1.8s	1.81s
Fig. 6	1.8s	1.82s
Fig. 7	0.64s	0.65s



(a) Method 1 (b) Method 2

Fig. 8. Numerical results of the new methods with $h = 10, m(0) = 0.05, s(0) = 0.87, e(0) = 0.03, i(0) = 0.05, r(0) = 0$.

and according to inequalities (19) and (27), we take:

$$Q = \min\left\{\frac{2}{\delta}, \frac{1}{\left(d+q+\frac{\varepsilon}{2}\right)}, \frac{1}{\left(d+q+\frac{\gamma}{2}\right)}, \frac{1}{(d+q)}\right\},$$

and

$$\varphi(h) = \frac{\tan Qh}{Q}.$$

The parameters used in these simulations have been taken from [40]. Figure 2 shows that the new methods retain the positivity property, while the Euler method and the second-order Runge-Kutta method (RK2) give negative values when using the MSEIR model, (see Figure 3). Figure 4 shows that the values of $R(t)$ and $S(t)$ are approaching 0 and 1, respectively. In Figure 5, we see that the values $E(t)$ and $I(t)$ approach zero and remain positive, which means that they do not go beyond the dynamic system. In Table 1, we see that the Euler and RK2 methods diverge with increasing the step-size, but the new schemes are convergent for all h. In Figures 6-7, we used the same parameters as above for δ, ε, γ, and q but we choose $\beta = 0.17$ so that $R_0 \approx 1.1883$. As can be seen for each h, nonstandard methods give good approximations and converge to the endemic equilibrium, and it is also clear that our schemes converge to the endemic equilibrium better than the method presented in [40]. If the sufficient condition in Theorem 5.2 is violated, then the numerical solutions may exhibit spurious oscillations and the new methods produce negative values, (see Figure 8). In Table 2, computational time used to obtain Figures 2-8 presented. It is worth pointing out that the required codes written in MATLAB. The feature of computer, that authors used to solve these four numerical methods, was RAM 8GB and CPU 2.40GHz.

To obtain numerical solutions, we do the following:

- Step 1 Select values: m^0, e^0, s^0, i^0, r^0, such that $m^0 + e^0 + s^0 + i^0 + r^0 = 1$.
- Step 2 For n = 0, 1, 2,…, do
- Step 3 Calculate m^{n+1}.
- Step 4 Using this value of m^{n+1}, i^n and s^n, calculate s^{n+1}.
- Step 5 Using this value of s^{n+1}, i^n and e^n, calculate e^{n+1}.
- Step 6 Using this value of e^{n+1}, e^n and i^n, calculate i^{n+1}.
- Step 7 Using this value of i^{n+1}, i^n and r^n, calculate r^{n+1}.

7. Conclusions and Discussion

In this paper, we introduced two nonstandard finite difference (NSFD) methods for the MSEIR epidemic model. The properties such as positivity and boundedness for the proposed methods have been shown. Also, these methods preserve the conservation law.

Author Contributions

All authors planned the scheme, developed the mathematical modeling and examined the theory validation. The manuscript was written through the contribution by all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript

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Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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