

Research Paper

Temperature Dependent Damping in Additively Manufactured Polymer Structures

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Abstract. Temperature effects are predominantly ignored when computing the dynamic response of structures. Yet, in applications where large changes in temperature occur, the dynamic response can drastically change. This is particularly true for polymers. While the temperature effects on modulus and loss factor are often available for most polymers, this change is not addressed or corrected for. Meanwhile, the recent research on additively manufactured polymer metastructures has yet to consider the effects of temperature change on their ability to suppress vibrations. In order to fill this gap, the study presented in this paper focuses on the effects of temperature change on additively manufactured structures.

Keywords: Metastructures; Metamaterials; Temperature effects; Polymer structures; 3D printing; Additive manufacturing.

1. Introduction

Recent advancements in additive manufacturing (AM) of polymers have resulted in notable developments leading to useful and economic methods to create prototypes of structures with complex geometries [1–5]. Often, such prototypes are utilized to substantiate engineering applications such as vibration suppression using metastructures, also called mechanical metamaterials [6, 7]. Metastructures contain multiple, small, distributed vibration absorbers created from changes in geometry [8–12]. These structures have complex geometry and can be manufactured easily using AM. The behavior of the printed material is typically modeled with an elastic model with viscous damping, ignoring commonly known viscoelastic changes due to variations in temperature and frequency. The viscoelastic effects in polymers are important, particularly in vibration applications when temperatures change. Various materials display viscoelastic behavior including plastics, adhesives, and rubbers [13]. Although the effects of viscoelastic materials in vibration suppression has been studied extensively [13–16], the viscoelastic effects in AM materials have not been adequately studied. The viscoelastic behavior of a material can be characterized by evaluating the dynamic modulus, which varies with frequency and temperature. In recent years, researchers have improved the modeling and printing processes [10, 22] which enabled them to identify the effects of varying parameters on printed material properties [19]. Although significant attention has been given to the properties of printed materials, only few studies have included the dynamic modulus [20], and even fewer have utilized the dynamic modulus to model the dynamic behavior of the printed structures [21].

The study presented in this paper focuses on a AM material which is being utilized for vibration suppression [22–25] and known to show viscoelastic characteristics [26–28]. Previous work by the authors has characterized the viscoelastic behavior of AM material produced by Objet Connex 3D printer [29], which is used herein to explore the effects of these properties on the dynamic behavior of metastructures and on metastrucure design. The metastructures of interest to the authors are structures designed to lower the level of vibrations in the fundamental mode. In many applications, the fundamental mode has the largest contribution to the structural response, and thus being the mode of focus. Vibration absorbers are designed with their fundamental natural frequency close to the rest of the structure, allowing the absorber to absorb the energy at that frequency. While the largest contribution in structural response comes from a mode whose frequency is closer to the excitation frequency, the first mode typically dominates the structural response unless the excitation frequency is close to a higher mode [30]. For the specific study presented in this paper, the first mode is considered. Based on this, the study presented in this paper aims to accurately model the dynamic response of AM structures subjected to temperature change. This is accomplished by using complex modulus data and a Golla-Hughes-McTavish (GHM) model to examine the effects of temperature for two test cases. These concepts are extended to a metastructure concept, examining both the individual vibration absorbers and the entire metastructure.

1.1 Viscoelastic material properties

The theory of linear viscoelasticity for a one-dimensional system gives the following constitutive relationship:



$$\sigma(t) = G(t)\varepsilon(0) + \int_0^t G(t-\tau) \frac{d}{d\tau} \varepsilon(\tau) d\tau$$
(1)

where σ is the applied stress, ε is the resulting strain, and G(t) is the material relaxation modulus. Equation (1) assumes that the loading starts at zero time and the strain is continuous.

Transforming the constitutive relationship to the Laplace domain gives:

$$\tilde{\sigma}(\mathbf{s}) = \mathbf{s}\tilde{G}(\mathbf{s})\tilde{\varepsilon}(\mathbf{s}) \tag{2}$$

In eq. (2), the tilde represents the Laplace transform of the variable while the function $s\tilde{G}(s)$ is called the material dissipation function. Evaluating the material dissipation function along the imaginary axis ($s = j\omega$) for a viscoelastic material yields the dynamic modulus:

$$G^{*}(\omega) = j\omega \tilde{G}(j\omega) = G'(\omega) + jG''(\omega)$$
(3)

where G['] is called the dynamic (complex) modulus. The real and imaginary parts are referred to as the storage modulus, G', and the loss modulus, G", respectively. The loss factor is defined as the ratio of the loss modulus over the storage modulus:

$$\mu = \frac{G''}{G'} \tag{4}$$

It should be noted that the dynamic modulus values tend to be affected by temperature changes and frequencies [14].

1.2 Viscoelastic characterization of the Objet Connex printer

The Objet Connex 500 printer used in this study utilizes inkjet printing technology and it can print both stiff and rubber-like materials. Parts are formed by accumulating numerous small dots of resin that are cured to make a printed part that appears homogeneous. This procedure enables the printer to effectively mix two different base materials in different ratios to produce a gradient of materials with multiple levels of hardness [31]. The material jetting process is standard for this type of 3D printer and is detailed in Stavropoulos and Foteinopoulos [32]. In this study, the digital materials are formed by using two base materials, VeroWhitePlus and TangoPlus. VeroWhitePlus is a rigid opaque material and TangoPlus is a rubber-like transparent material [33]. Utilizing these two base materials, the printer is capable of printing ten different digital materials with varying stiffnesses [34]. Ge et al. [26] has completed a comprehensive study utilizing the Objet Connex 3D printer, and their accompanied publications present the details of the actual mechanisms used by the printer. Further previous work on materials printed by the Objet Connex printer showed viscoelastic behavior, which makes them temperature and frequency dependent [26–28].

The authors have already characterized the materials printed by the Objet Connex 3D printer utilizing a Dynamic Mechanical Analysis (DMA) machine (Q800 by Thermal Analysis) following the ASTM D5026 and ASTM D5418 standards [29, 35, 36]. The VeroWhitePlus, DM 8420, DM 8430 and TangoPlus materials were tested over various temperature levels and under different testing configurations [29]. The DMA measures the complex modulus by applying a sinusoidal excitation and measuring the resulting strain at various temperatures and frequencies. This is done in both a tensile and cantilever configuration. The experimental set-up for the tensile configuration is seen in Fig. 2. It needs to be emphasized that temperature-frequency equivalence is assumed and utilized to combine the effects of frequency and temperature into a single reduced frequency, f_r , which is calculated using:

$$f_r = f \cdot \alpha(T) \tag{5}$$

where *f* is the frequency at which the data was measured and α (T) is the shift factor. The correlation in eq. (5) is found by testing a specimen at various temperatures and frequencies. For temperature levels at which the data is measured, a corresponding shift factor is found graphically by analyzing the modulus with respect to reduced frequency graphs [13]. Using this information, the complex modulus versus reduced frequency data can be plotted for various reference temperatures. This work uses reference temperatures from 20° to 100° C in increments of 10° C. The data from the DM 8430 tensile sample for various reference temperatures is shown in Fig. 1. There is significant variation due to temperature changes in both the loss tangent and the storage modulus over the frequency range of interest (10 Hz to 2,000 Hz).



Fig. 1. (a) Storage modulus and (b) loss factor data for DM 8430 at various reference temperatures





Fig. 2. Experimental set-up of the tensile configuration in the dynamic mechanical analyzer [29]

1.3 Golla-Hughes-McTavish (GHM) model

In the literature, there are three common models utilized for vibration suppression to account for the frequency variation [15, 37]. These models are characterized by the equation used to represent the material dissipation functions presented in Table 1. The fractional derivative model developed by Bagley and Torvik uses a derivative of fractional order and is advantageous because five parameters are sufficient to represent the frequency-dependent behavior of many materials over a large frequency range [38, 39]. The anelastic displacement fields model were developed by Lesieutre *et al.* [40–42] and enhanced by de Lima *et al.* [37]. This model focuses on the elastic and the anelastic parts and represents the anelastic part by a first order differential equation which is formulated directly in the time-domain. This paper uses the Golla-Hughes-McTavish (GHM) model developed by representing the dynamic modulus as a series of mini-oscillators [43, 44]. Additionally, these models allow the damping of the structure to be determined directly from material properties.

The GHM method was initially introduced by Golla and Hughes [43] in 1985 and enhanced by McTavish and Hughes [45, 46]. The method was primarily developed in an attempt to analyze damping in large flexible space-structures; however, it can be used for all viscoelastic materials [47, 48]. This internal-variable method uses auxiliary dissipation coordinates to model the frequency-dependent loss factor inherent in viscoelastic materials. These dissipation coordinates added degrees of freedom to the overall system. This method is valuable because it produces mass, stiffness and damping matrices in the familiar second-order time-domain format. The resulting matrices can be easily integrated into existing finite element theory or analytical models following from Newton's laws. The only additional information required is fitting the complex modulus data to the form used in the GHM method. With that, the resulting parameters are inherent of the materials, not of the system.

The fit is performed in a logarithmic sense with equal weighting given to the real and imaginary parts. The number of terms necessary to obtain a good fit of the GHM approximation depends on the material, the frequency range and the desired accuracy. In this paper, the frequency range of interest is 10 to 2,000 Hz. The shape of the curve at the temperature and frequency of interests dictates the number of GHM parameters used. The GHM parameters for the Objet Connex material VeroWhitePlus at various temperatures are shown in Table 2. The storage modulus and the loss modulus data for VeroWhitePlus at 70° C with a GHM curve fit using 4 parameters over the frequency range 0.1 Hz to 2,000 Hz are shown in Fig. 3. For the comprehensive dataset of the GHM parameters for all materials tested, the reader is referred to [49].

2. Dynamic response of structures made from viscoelastic materials

The GHM model is applied to a simple uniform bar and beam with rectangular cross-section to study the effects of temperature change on the dynamic response. The derivation of the GHM model and the specific forms used in this paper are presented in the Appendix section. During the DMA characterization, both tensile and cantilever testing modes were used and the resulting data is used to model a cantilevered beam to determine how testing configurations affect the dynamic response. An experimental validation was also performed.

Table 1. Material dissipation function approximations for selected viscoelastic model	Table	1. Material	dissipation	function	approximations	for selected	viscoelastic	models
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Model	Form of Material Dissipation Function
Fractional Derivatives (FD) [38]	$s\tilde{G}(s) = \frac{G_o + G_1 s^{\alpha}}{1 + b s^{\alpha}}$
Anelastic Displacement Fields (ADF) [37]	$s\tilde{G}(s) = G_{0}\left[1 + \sum_{i=1}^{n} \Delta_{i} \frac{s}{\omega^{2} + \Omega_{i}^{2}}\right]$
Golla-Hughes-McTavish (GHM) [45]	$s\tilde{G}(s) = G^{\infty}\left[1 + \sum_{i=1}^{n} \hat{\alpha}_{i} \frac{s^{2} + 2\hat{\zeta}_{i}\hat{\omega}_{i}s}{s^{2} + 2\hat{\zeta}_{i}\hat{\omega}_{i}s + \hat{\omega}_{i}^{2}}\right]$



Table 2. GHM parameters for the Objet Connex material VeroWhitePlus at various temperatures					
Temperature	\mathbf{G}^{∞} [Pa]	j	$\widehat{oldsymbol{lpha}}_j$ [-]	$\widehat{\boldsymbol{\omega}}_{j}$ [rad/s]	$\widehat{\boldsymbol{\varsigma}}_{j}$ [-]
	1.78×10^{4}	1	1.31×10^{4}	1.74 × 10 ⁶	2.14×10^{2}
20° C		2	1.44×10^{3}	1.56 × 10⁵	8.65×10^{2}
20 C		3	9.70×10^{3}	5.93 × 10⁵	5.06×10^{2}
		4	8.95×10^{4}	2.00×10^{3}	9.77×10^{2}
	1.46×10^{2}	1	1.36×10^{4}	4.59 × 10⁵	7.69×10^{2}
		2	3.34×10^{4}	4.59 × 10⁵	2.48×10^{2}
30° C		3	8.94×10^{3}	1.84×10^{6}	8.26×10^{3}
		4	8.99×10^{4}	2.72×10^{4}	1.32×10^{3}
		5	7.42×10^{3}	2.83 × 10 ⁶	4.88×10^{2}
	4.84×10^{3}	1	9.83 × 10 ³	8.73×10^{4}	1.16 × 10 ³
70%C		2	2.21×10^{4}	6.87×10^{6}	1.31×10^{3}
70 C		3	1.70×10^{4}	1.63×10^{6}	1.34×10^{3}
		4	6.99×10^{4}	2.48×10^{7}	9.33×10^{2}

2.1 Dynamic response of a simple viscoelastic solid bar and beam

The simple case of a uniform bar with a hollow rectangular cross-section was modeled using a GHM model and the material properties from VeroWhitePlus. The geometry and material properties of the bar are listed in Table 3. The bar was fixed at the base and it was discretized into ten finite elements, with two nodes for each element.

Frequency response functions (FRFs) for a solid bar are plotted using the GHM parameters obtained at various temperatures (Fig. 4). The FRFs reveal that changing the temperature of a viscoelastic bar can result in a deviation in the natural frequency and affect the amount of damping in each mode. The structural response of the same structure with an impulse excitation at the tip of the bar was studied and the settling time was calculated for various temperatures. The settling time, in this context, is simply the time required for a structural response to become steady. In other words, a lower settling time shows that the structural vibration dies out more quickly, which is preferred in vibration suppression. Excited by a unit impulse at the tip, the settling time for a solid bar and a beam are shown in Fig. 5 for various temperatures. In both Fig. 5a and 4b, initially, as the temperature increases, the settling time decreases. This is due to the increasing damping in the material. After a certain temperature (in the vicinity of 60° C), an increase in temperature causes an increase in the settling time.



Fig. 3. (a) Storage modulus and (b) loss modulus data for VeroWhitePlus at 70° C with a GHM curve fit using 4 parameters over the frequency range 0.1 Hz to 2,000 Hz



Fig. 4. FRFs for a solid bar at varying temperatures



Table 3. Geometry and material properties of bar and beam models				
Property	Value			
Density [kg/m³]	1168			
Bar cross-sectional area [m ²]	1.131 × 10 ⁻³			
Bar length [cm]	45			
Beam cross-sectional width [mm]	12.71			
Beam cross-sectional height [mm]	2.96			
Beam length [cm]	20.32			
Beam area moment of inertia [m4]	2.747 × 10 ⁻¹¹			

2.2 Effects of testing configuration on dynamic response

During the characterization of the viscoelastic material properties of the AM materials, both cantilevered and tensile testing configurations were used. The effects of the testing configurations on the GHM model results during material characterization are investigated in this section. For the cantilevered configuration, the specimen is clamped at both ends; while one end of the beam is fixed and the other end can move cyclically. For the tensile testing configuration, the tensile clamp puts the specimen in tension while one end staying fixed and the other end can move cyclically [29]. A comparison of the testing configuration was conducted for DM 8430, which is slightly softer than the VeroWhitePlus material. GHM parameters were fit to the data for both testing configurations and used to model a solid cantilever beam. The natural frequency and damping of the first two modes were obtained for temperatures ranging from 50° to 70° C. As mentioned earlier, the presented work here is focusing on the first mode. Meanwhile, the second mode was obtained to observe the accuracy of the model. The natural frequency and damping values are presented in Fig. 6 revealing that the tensile test configuration leads to a higher natural frequency than the cantilever test configuration. This difference is more significant in the second mode than the first mode (Fig. 6a). Furthermore, as shown in Fig. 6b, the tensile test configuration results in lower damping values than the cantilever test configuration. The FRFs for these two testing configurations at a temperature of 50° C are shown in Fig. 7. The variation of the FRFs in Fig. 7 reveals that the testing configuration used to obtain the material properties can make a considerable difference in the results obtained by the GHM model. It must be emphasized here that the AM specimens are not homogeneous and the orientation of the specimen results in variance. As such, when tested in different configurations the layers of the AM materials interact with each other differently.



Fig. 5. Settling time of a solid (a) bar and (b) beam subjected to a unit impulse at the tip, at varying temperatures



Fig. 6. Variations in the (a) natural frequency and (b) damping factor of a cantilevered beam using GHM parameters obtained from cantilever and tensile testing configurations



2.3 Experimental verification of material characterization

The behavioral trends observed so far were experimentally validated using a cantilever beam. The beam with dimensions $8 \times 12 \times 120$ mm was printed using the Objet Connex 3D printer and it was clamped at one end to introduce fixed end condition. Then, the beam was placed in a thermal chamber, and tested at 20°, 30° and 40° C with a soak time of 30 minutes. The excitation force was introduced to the tip through a magnetic transducer (a magnetic disc was attached to the tip of the beam). The structural response was measured via a laser Doppler vibrometer mounted to the outside of the chamber. The excitation signal for the magnetic transducer was generated by the National Instruments data acquisition system via a voltage amplifier. The acquisition system also collected the signals from the laser vibrometer. Ten sine sweeps from 0 to 1500 Hz were conducted and the resulting FRFs were averaged. Then, the averaged FRF was curve fitted to obtain the natural frequencies and damping factors for the first two modes. The experimental procedure followed the ASTM E756 standard for a uniform beam [50]. During the tests at temperatures higher than 40° C, the beam became compliant resulting in the tip of the beam getting stuck to the magnetic transducer.

Comparison of the natural frequencies and damping factors of the GHM model to the experimental results of the VeroWhitePlus cantilevered beam are shown in Fig. 8 for the associated temperatures. The natural frequency results show good agreement for the first mode while the second mode shows deviations (Fig. 8a). The deviations could be attributed to several factors. The deviation could be because of the testing configuration. It is important to note that the experiment beam was in a cantilever test configuration, but the material parameters were obtained using a tensile test configuration. On another note, the tensile configuration produced higher natural frequencies than the cantilevered configuration, which was also more evident in the second mode. Additionally, the frequency range of the curve fit could be increased and more GHM parameters could be used to improve the accuracy of the model. This would also increase the degrees of freedom of the model.



Fig. 7. Variations in the FRF of a cantilever beam at 50° C using GHM parameters obtained from cantilever and tensile testing configurations



Fig. 8. Comparison of the GHM model and experimental results for a VeroWhitePlus cantilevered beam. The figures show the natural frequencies for the (a) first mode and (b) second mode and the damping factors for the (c) first mode and (d) second mode of the beam



Considering the results of the previous section, it would be expected that the GHM model would predict damping factors lower than that of the experimental results, but this is not the case in Fig. 8b. This shows that the GHM mode is not able to accurately predict the amount of damping in the beam for this specific case. This could be attributed to the mismatch in the dimensions of the material characterization testing specimen and the beam used for these experimental results. Furthermore, the testing clamp used to obtain the dynamic modulus could have introduced additional damping into the structure that was not produced from the viscoelastic effects of the AM material or because more terms are needed in the GHM approximation. The beam used for the experimental results used a non-contact excitation method along with a non-contact transducer which introduces relatively less damping into the system. Even though the GHM model is not able to predict the correct amount of damping in the structure, it can capture the trends due to temperature change which allows the exploration of the effects of temperature change using a GHM model.

3. Applications to metastructures

Viscoelastic effects result in a deviation in the natural frequency of the structure when compared to structures of identical geometry constructed of elastic material. The frequency response of metastructures is designed by modifying the geometric as well as the material properties of the distributed vibration absorbers. Meanwhile, the changes in temperature cause additional deviation of the natural frequency which must be accounted for. The previous sections show that when using the Objet Connex 3D printer, the GHM model can predict the first natural frequency of a simple beam and a simple bar by accounting for the viscoelastic properties of the material. Additionally, the GHM model captured the changes in damping due to temperature changes. In this section, the authors will examine how the viscoelastic properties affect a metastructure manufactured with Object Connex 3D printer. The metastructure being discussed here was studied in previous work [34, 35] by the authors and is designed to suppress vibrations along the axial direction of the metastructure bar. The metastructure is composed of the host structure and the vibration absorber system. The host structure has a hollow square cross-section as show in Fig 8 and is the part of the structure in which lower vibration response is desired. The vibration absorbers are basically cantilevered beams attached to the inner surfaces of the hollow cross-section with a tip mass arranged such that the bending motion of the cantilevered beams absorbs vibrations along the axial direction of the bar as shown in Fig 9. The entire metastructure is 3D printed from a single material. With that, the design of the metastructure is characterized by the number of vibration absorbers, w_i . The mass ratio, μ_i and the natural frequency of the individual vibration absorbers, w_i . The mass ratio is defined as:

$$\mu = \frac{\sum_{i}^{N} m_{i}}{\sum_{i}^{N} m_{i} + m_{\text{host}}}$$
(6)

where m_i is the mass of the ith absorber and m_{host} is the mass of the host structure.

The modeling techniques for a single absorber are presented in the next section and then incorporated into a full metastructure model. The metastructure model is compared to a baseline structure that has the same weight as the metastructure. It is important to note that this constant mass constraint shows that any increase in performance of the metastructure is due to the addition of the vibration absorbers and not due to any additional mass. The baseline structure is a simple bar with the same hollow square cross-section as the host structure, depicted in Fig. 9. The metastructure models use the VeroWhitePlus material properties.

3.1 Viscoelastic modeling of a single vibration absorber

Since the vibration absorbers are created from AM material, the GHM model must be utilized to capture the viscoelastic effects of the absorber. A single vibration absorber is composed of a beam with a tip mass. The dimensions of the vibration absorber are defined in Fig. 11a while the equivalent properties used in the GHM model are shown in Fig. 11b. These equivalent properties are calculated using the equations shown in Table 4.

Both elastic and viscoelastic finite element models of the vibration absorber are developed and used to compare the differences between these models. The vibration absorber is discretized into elements along the length of the beam and standard beam finite elements are used for all elements. The effects of the tip mass must also be accounted for. Since the tip mass is relatively large, both the rotational and translational motion of the tip mass must be taken in to consideration. With that, the kinetic energy expression for the tip mass is:

$$T = \frac{1}{2}m_{\rm M}\left[\dot{w}(\ell_{\rm B}) + e\dot{w}'(\ell_{\rm B})\right]^2 + \frac{1}{2}I\dot{w}'(\ell_{\rm B})^2 \tag{7}$$



Fig. 9. The host structure with hollow square cross-section



Fig. 10. The metastructure with vibration absorbers

Table 4. Effective properties of the vibration absorber				
Property	Equation			
Eccentricity of tip mass	$e = \frac{\ell_{M}}{2}$			
Mass of tip mass	$\mathbf{m}_{_{M}}=\rho\ell_{_{M}}\mathbf{w}_{_{M}}\mathbf{h}_{_{M}}$			
Mass moment of inertia of tip mass	$I = \frac{1}{12} m_{_{M}}(h_{_{M}}^{2} + \ell_{_{M}}^{2})$			

where w(x) is the vertical displacement of the beam; the dot represents the partial derivative with respect to time, and the prime represents the partial derivative with respect to x.

Equation (7) can be rewritten in matrix form as:

$$T = \frac{1}{2} \begin{bmatrix} \dot{w}(\ell_{\rm B}) & \dot{w}'(\ell_{\rm B}) \end{bmatrix} \begin{bmatrix} m_{\rm M} & m_{\rm M}e \\ m_{\rm M}e & m_{\rm M}e^2 + I \end{bmatrix} \begin{vmatrix} \dot{w}(\ell_{\rm B}) \\ \dot{w}'(\ell_{\rm B}) \end{vmatrix}$$
(8)

The mass terms are then added to the global mass matrix at the degrees of freedom associated with the displacement and rotation of the tip of the beam. Since the tip mass only affects the kinetic energy of vibration absorber, the stiffness matrix stays the same for all elements in the vibration absorber. The global mass and stiffness matrices are used for the elastic finite element model and in the derivation of the viscoelastic finite elements as detailed in the Appendix.

Using the methods described, a GHM model and two different elastic models were created for the vibration absorber. For the elastic models, a finite element model and a Rayleigh-Ritz model were used. The resulting FRFs for these models are shown in Fig. 12. The viscoelastic model predicts a relatively lower natural frequency with relatively higher damping. A mesh convergence study was conducted to determine that two elements are sufficient to accurately predict the natural frequency of the absorber using the GHM model.

3.2 Temperature effects of a single vibration absorber

Based on the model discussed in the previous section, FRFs for a single vibration absorber made from VeroWhitePlus at various temperatures are plotted in Fig. 13. It should be emphasized that the absorber was designed to have a natural frequency of 600 Hz at room temperature (20° C). Fig. 13 reveals that as the temperatures increases, the natural frequency of the vibration absorber decreases and the level of damping increases which is consistent with previous results. In an attempt to ensure this trend is valid for a variety of geometries, absorbers with room temperature natural frequencies ranging from 200 to 1,000 Hz were modeled at temperatures from 20° to 100° C. In Fig. 14, the effects of temperature change on the natural frequency and the damping values of vibration absorbers with various geometry are presented. Based on the results of Fig. 14, it can be stated that regardless of the design frequency of the vibration absorber, the trends from a temperature change standpoint are similar.



Fig. 11. Schematics of the vibration absorber consisting of a cantilever beam with a tip mass where (a) shows the dimensions of the vibration absorber and (b) shows the effective properties used for modeling



Fig. 12. Elastic and viscoelastic comparison of the FRF for a single vibration absorber





Fig. 13. FRFs for a single vibration absorber made from VeroWhitePlus at various temperatures



Fig. 14. Effects of temperature change on the (a) natural frequency and (b) damping values of vibration absorbers with various geometry



Fig. 15. (a) FRF and (b) impulse response of the a metastructure bar at 20° C with vertical lines representing the setting time of the corresponding structures

3.3 Viscoelastic modeling of a metastructure

This section presents the results for the GHM model of an AM metastructure. The metastructure has a mass ratio, $\mu = 0.23$ as defined in eq. (6) and 12 absorbers throughout the length of the bar. The vibration absorbers have geometries such that the room temperature natural frequency of the absorbers vary linearly from 500 to 980 Hz with the higher frequency absorbers located close to the base. Previous work has shown that vibration absorbers with linearly varying natural frequencies provide better suppression than vibration absorbers all tuned to the same natural frequency [8]. The mass of each vibration absorber is constrained to have a constant value of 12 g. The results from this model run at 20° C are shown in Fig. 15. The FRF in Fig. 15a shows that the linearly varying nature of the vibration absorbers leads to a more broad-band absorption near the fundamental natural frequency. The response of the top of the metastructure subjected to a unit impulse also at the tip is shown in Fig. 15b. The settling time of each structure is marked by a vertical line on Fig. 15b. This plot shows that the metastructure design significantly decreases the setting time of the structure.

3.4 Temperature effects of a metastructure

After the viscoelastic modeling, the temperature of the metastructure is changed to determine if similar performance improvements are observed at off-design temperatures. The temperature is changed for both the baseline structure and the metastructure and the results at 40° C are shown in Fig. 16. Fig 16a is presenting the FRF of the normalized tip displacements for the metastructure and the baseline structure while Fig. 16b is showing the impulse responses for both structures at 40° C. In Fig. 16b, the vertical lines are representing the settling time of the corresponding structure.

Similar trends are observed in the FRFs at higher temperatures, showing that as the natural frequency of the host structure changes with temperature, the frequencies of the vibration absorbers change at a similar rate. On another note, it is observed that increasing the temperature results in higher damping in the structure leading to smoother FRFs. At temperatures ranging from 20° to 70° C. The effects of the temperature change on the settling time of the baseline structure and the metastructure are calculated and plotted in Fig. 17 for temperatures ranging from 20° to 70° C. Initially, as the temperature increases, the settling times for both the baseline structure and the metastructure decrease. At 50° C the settling time of the metastructure begins to increase, whereas the settling time of the baseline structure does not start to increase until 60° C. At the higher temperatures the baseline structure outperforms the metastructure. This is due to the difference in static response between the two structures, which can be seen in the slight vertical shift in the FRF of the metastructure compared to the baseline structure. Since the baseline structure and the metastructure are constrained to have the same mass, the host structure part of the metastructure must have a slightly smaller wall thickness than that of the baseline structure. The smaller thickness causes a decrease in the stiffness of the structure and an increase in the static response. At higher temperatures, the static response of the structure dominates the response leading to higher settling times. At lower temperatures, the static response has a less-significant role in the response. These results show that the metastructure design can provide significant performance benefits for a range of temperatures even if the metastructure is designed for a fixed temperature. At higher temperatures, the metastructure no longer outperforms the baseline structure which shows that operating temperatures of the structure must be analyzed and studied verywell when determining if a metastructure design should be utilized.



Fig. 16. (a) FRF and (b) impulse response of the metastructure bar at 40° C with vertical lines representing the settling time of the corresponding structure



Fig. 17. Effects of temperature change on the settling time of a metastructure compare to the baseline structure with equal mass



Similar trends are observed regardless of the design of the metastructure thus are not presented here. The specific material used has the greatest impact on the performance of the metastructure at various temperatures. As the temperature changes, the increasing loss factor of the material is causing the changes in the amount of suppression. Once the material reaches a specific temperature, the damping due to the viscosity of the material is so large that the vibration suppression caused by the geometry of the metastructure is negligible. This temperature can be determined from the material characterization; specifically, the temperature at which the loss factor of the material reaches its peak for the frequency range of interest.

4. Conclusions

In this paper, the authors presented the complete process required to accurately utilize the GHM model; starting from the material characterization of the viscoelastic material properties to the experimental verification of the model. The results of the research presented in this paper indicate that the GHM model can accurately predict the fundamental natural frequency of a structure printed by the Objet Connex 3D printer. It is also revealed that the GHM model can successfully predict the changes in the natural frequency due to temperature changes.

It is also observed that the change in temperature can significantly alter both the storage modulus and the loss modulus of the material which leads to changes in the natural frequencies and the damping of the resulting structure. Based on this, the authors conclude that when experimentally characterizing the viscoelastic material properties of the AM material, the testing configuration of the specimen should be carefully considered. On another note, this work showed that the tensile testing configuration led to higher natural frequencies and lower damping values than the cantilevered configuration. In addition, the experimental verification results also had lower damping values than the modeled results.

Lastly, the developed GHM model was used to model the dynamics of a metastructure and study the effects of changing temperature. The results showed that the changes in natural frequency of the host structure and the vibration absorbers changed at a similar rate, resulting in similar trends in the dynamic response as the temperature increases. This was true up until a specific temperature where the baseline structure reached a relatively lower settling time which showed the significant reduction in the performance of the metastructure. At this temperature, the material achieves a high damping value thus the vibration suppression due to the geometry of the metastructure is insignificant compared to the suppression provided from viscosity of the material. Similar structures made of the same material would see this trend occur at a similar temperature.

Recommendations for future work on this project consist of the examining the effects of the various build parameters on the response of the metastructure and investigating the fatigue life of an AM metastructure. The AM process involves many different parameters. This work considered the effects of two different build orientations; the third build orientation could also be considered. Since the metastructures experience cyclic motion, the effects of fatigue must be considered. The literature on fatigue behavior of 3D printed material is growing [52] and should also be considered.

Author Contributions

K. Reichl contributed 90% of the work while O. Avci and D. Inman each contributed 5% of the work.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Nomenclature

- A Cross-sectional area, m²
- D Damping matrix, kg/s
- G Dynamic modulus, Pa
- G' Storage modulus, Pa
- G″ Loss modulus, Pa
- G^{∞} Equilibrium value of modulus, Pa
- $\mathbf{H}(\mathbf{x})$ Shape function
- I Area moment of inertia, m⁴
- J Polar moment of inertia, kg-m²
- **K** Stiffness matrix, N/m
- N Number of vibration absorbers
- f_r Reduced frequency, rad/s
- m Mass, kg
- n_q , n_z Number of physical and dissipation coordinates
- n_{MOT} Number of GHM curve fitting terms
- **q** Finite element physical coordinates, m
- t Thickness, m
- x_1, x_2 Coordinates of the finite element node, m

- w Width of cross-section, m
- w(x) Transverse displacement of beam, m
 - **z** Finite element dissipation coordinates

Greek Letters

â

- GHM curve fitting parameter
- $\alpha(T)$ Shift factor
 - $\hat{\zeta}$ GHM curve fitting parameter
- η Loss factor
- μ Mass ratio of metastructure
- $\hat{\omega}$ GHM curve fitting parameter, 1/s

Subscripts or Superscripts

- b Beam
- e Elastic
- i The ith vibration absorber
- *j* The *j* th GHM curve fitting term
- v Viscoelastic
- M Tip mass



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Appendix - GHM model details

The GHM method approximates the material modulus using the equations in Table 1. This approximation is comprised of terms called mini-oscillator terms which come from the single degree of freedom physical realization. The number of terms utilized depends on the test data, target frequency-range and desired accuracy. The following conditions, $\hat{\alpha}_j > 0$, $\hat{\omega}_j > 0$ and

 $\hat{\zeta}_i > 0$ ensure that the representation dissipates energy [43].

The viscoelastic finite element matrices are developed for a material dissipation function with a single term ($n_{MOT} = 1$) then are generalized for any number of terms. A typical finite element formulation, has the following equation of motion:

$$\mathbf{M}^{e}\ddot{\mathbf{q}}(t) + G\overline{\mathbf{K}}^{e}\mathbf{q}(t) = \mathbf{f}(t)$$
(9)

where \mathbf{M}^{e} and $\mathbf{K}^{e} = G\overline{\mathbf{K}}^{e}$ are the classical elastic finite element matrices and $\mathbf{f}(t)$ is the elemental nodal force vector corresponding the elemental degrees of freedom, $\mathbf{q}(t)$. This equation can be generalized by replacing the elastic constant with a viscoelastic representation resulting in:

$$\mathbf{M}^{e}\ddot{\mathbf{q}}(t) + G(t)\overline{\mathbf{K}}^{e}\mathbf{q}(0) + \int_{0}^{t} G(t-\tau)\overline{\mathbf{K}}^{e}\frac{d}{d\tau}\mathbf{q}(\tau)d\tau = \mathbf{f}(t)$$
(10)

where $\mathbf{q}(t)$ is restricted to be zero for $t \in (-\infty, 0)$. Transforming eq. (10) into the Laplace domain yields:

$$s^{2}\mathbf{M}^{e}\tilde{\mathbf{q}}(s) + s\tilde{G}(s)\overline{\mathbf{K}}^{e}\tilde{\mathbf{q}}(s) = \tilde{\mathbf{f}}(s)$$
(11)

A column of dissipation coordinates $\tilde{\hat{z}}$ are introduced such that

$$\tilde{\tilde{\mathbf{z}}}(s) = \frac{\omega^2}{s^2 + 2\hat{\zeta}\hat{\omega}s + \hat{\omega}^2}\tilde{\mathbf{q}}(s)$$
(12)

Using the approximation given in Table 1 and the relationship for eq. (12), the following equation of motion is equivalent to eq. (11).

$$\begin{bmatrix} \mathbf{M}^{e} & \mathbf{0} \\ \mathbf{0} & \frac{\hat{\alpha}}{\hat{\omega}^{2}} \mathbf{K}^{e} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \ddot{\mathbf{z}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{2\hat{\alpha}\hat{\zeta}}{\hat{\omega}} \mathbf{K}^{e} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{z}} \end{bmatrix} + \begin{bmatrix} (1+\hat{\alpha})\mathbf{K}^{e} & -\hat{\alpha}\mathbf{K}^{e} \\ -\hat{\alpha}\mathbf{K}^{e} & \hat{\alpha}\mathbf{K}^{e} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \mathbf{\hat{z}} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}$$
(13)

Next, a spectral decomposition on the stiffness matrix is utilized to simplify the equations. The elastic stiffness matrix possesses n_q non-negative eigenvalues. The zero eigenvalues represent rigid body modes which cannot dissipate energy thus they are neglected. Considering only the positive eigenvalues (λ_p) and their corresponding eigenvectors \mathbf{r}_p of $\mathbf{\bar{K}}^e$, the following matrices are constructed:

$$\overline{\mathbf{R}} = \operatorname{row}\{\mathbf{r}_{p}\}, \quad \overline{\mathbf{\Lambda}} = \operatorname{diag}\{\lambda_{p}\}, \quad \overline{\mathbf{R}}^{\mathsf{T}}\overline{\mathbf{R}} = 1$$
(14)

Leading to this spectral decomposition:

$$\mathbf{K}^{e} = \mathbf{G}^{\infty} \overline{\mathbf{R}} \overline{\mathbf{\Lambda}} \overline{\mathbf{R}}^{\mathrm{T}}$$
(15)

Taking eq. (13) and pre-multiplying the bottom row by $\overline{\mathbf{R}}^{T}$ and letting $\mathbf{z} = \overline{\mathbf{R}}^{T} \hat{\mathbf{z}}$ the equations of motion become:

$$\mathbf{M}^{\nu} \begin{bmatrix} \ddot{\mathbf{q}} \\ \ddot{\mathbf{z}} \end{bmatrix} + \mathbf{D}^{\nu} \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{z}} \end{bmatrix} + \mathbf{K}^{\nu} \begin{bmatrix} \mathbf{q} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}$$
(16)

where the viscoelastic mass, stiffness and damping matrices have the following form:

$$\mathbf{M}^{\nu} = \begin{bmatrix} \mathbf{M}^{e} & \mathbf{0} \\ \mathbf{0} & \frac{\hat{\alpha}}{\hat{\omega}^{2}} \mathbf{\Lambda} \end{bmatrix}, \quad \mathbf{D}^{\nu} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{2\hat{\alpha}\hat{\zeta}}{\hat{\omega}} \mathbf{\Lambda} \end{bmatrix}, \quad \mathbf{K}^{\nu} = \begin{bmatrix} \mathbf{K}^{e}(1+\hat{\alpha}) & \hat{\alpha}\mathbf{R} \\ \hat{\alpha}\mathbf{R}^{\mathrm{T}} & \hat{\alpha}\mathbf{\Lambda} \end{bmatrix}$$
(17)

where $\Lambda = G^{\infty}\overline{\Lambda}$ and $\mathbf{R} = \overline{\mathbf{R}}\Lambda$. Since the spectral decomposition determines the number dissipation coordinates that will be augmented onto the total system, the eigenvalue problem must be completed before the total degrees of freedom are known. The degrees of freedom depend on the number of non-negative eigenvalues of the stiffness matrix in addition to the size of the stiffness matrix and the number of terms in the complex modulus approximation. Generalizing this to a GHM approximation with *n* terms, the viscoelastic mass, damping and stiffness matrices have the following form:

$$\mathbf{M}^{v} = \begin{bmatrix} \mathbf{M}^{e} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \frac{\hat{\alpha}_{1}}{\hat{\omega}_{1}^{2}} \mathbf{\Lambda} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \frac{\hat{\alpha}_{n}}{\hat{\omega}_{n}^{2}} \mathbf{\Lambda} \end{bmatrix}$$
(18)



$$\mathbf{D}^{\nu} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \frac{2\hat{\alpha}_{1}\hat{\zeta}_{1}}{\hat{\omega}_{1}} \mathbf{\Lambda} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \frac{2\hat{\alpha}_{n}\hat{\zeta}_{n}}{\hat{\omega}_{n}} \mathbf{\Lambda} \end{bmatrix}$$
(19)
$$\mathbf{K}^{\nu} = \begin{bmatrix} \left(1 + \sum_{i=1}^{n} \hat{\alpha}_{i}\right) \mathbf{K}^{e} & -\hat{\alpha}_{1} \mathbf{R} & \cdots & -\hat{\alpha}_{n} \mathbf{R} \\ -\hat{\alpha}_{1} \mathbf{R}^{T} & \hat{\alpha}_{1} \mathbf{\Lambda} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ -\hat{\alpha}_{n} \mathbf{R}^{T} & \mathbf{0} & \cdots & \hat{\alpha}_{n} \mathbf{\Lambda} \end{bmatrix}$$
(20)

The viscoelastic matrices are square matrices with a dimension $n_q + n_z$ where n_q is the number of physical coordinates and n_z is the number of dissipation coordinates dictated by the number of terms included in the approximation from Table 1. To arrive at the global system of equations, the element matrices must be assembled into the global matrices. The degrees of freedom associated with the physical degrees of freedom are assembled using traditional finite element methods and the dissipation coordinate are simply augmented to the system since the dissipation degrees of freedom for neighboring elements do not interact with each other.

For a bar undergoing uniaxial motion, the elastic mass and stiffness elastic finite element matrices are given as [53]:

$$\mathbf{M}^{e}_{\text{bar}} = \frac{\rho A \ell}{6} \begin{bmatrix} 2 & 1\\ 1 & 2 \end{bmatrix}$$
(21)

$$\overline{\mathbf{K}}^{e}_{\text{bar}} = \frac{\mathbf{A}}{\ell} \begin{bmatrix} 1 & -1\\ -1 & 1 \end{bmatrix}$$
(22)

where ρ is the density, A is the cross-sectional area and ℓ is the length. By performing an eigenvalue analysis on the stiffness matrix, one degree of freedom can be eliminated since it corresponds to a zero eigenvalue. The resulting $\bar{\mathbf{R}}_{\text{bar}}$ and $\bar{\Lambda}_{\text{bar}}$ matrices are:

$$\overline{\mathbf{R}}_{\text{bar}}^{T} = \begin{bmatrix} -\sqrt{2} / 2 & \sqrt{2} / 2 \end{bmatrix}$$
(23)

$$\bar{\Lambda}_{\rm bar} = \frac{2A}{\ell} \tag{24}$$

Using eqs. (18) - (20), the resulting viscoelastic finite elements matrices are:

$$\mathbf{M}^{v}_{bar} = \begin{vmatrix} \frac{\rho A \ell}{3} & 0 & \cdots & 0 & \frac{\rho A \ell}{6} \\ 0 & \frac{2A}{\ell} \frac{\hat{\alpha}_{1}}{\hat{\alpha}_{2}^{2}} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{2A}{\ell} \frac{\hat{\alpha}_{n_{MOT}}}{\hat{\alpha}_{n_{MOT}}} & 0 \\ \frac{\rho A \ell}{6} & 0 & \cdots & 0 & \frac{\rho A \ell}{3} \end{vmatrix}$$

$$\mathbf{D}^{v}_{bar} = \begin{vmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & \frac{4A}{\ell} \frac{\hat{\alpha}_{1} \hat{\zeta}_{1}}{\hat{\alpha}_{1}} & \cdots & 0 & 0 \\ 0 & \frac{4A}{\ell} \frac{\hat{\alpha}_{n_{MOT}}}{\hat{\alpha}_{1}} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{4A}{\ell} \frac{\hat{\alpha}_{n_{MOT}}}{\hat{\alpha}_{n_{MOT}}} & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{G^{\infty} A}{\ell} \begin{vmatrix} 1 + \sum \hat{\alpha}_{j} & \sqrt{2} \hat{\alpha}_{1} & \cdots & \sqrt{2} \hat{\alpha}_{n_{MOT}} & -(1 + \sum \hat{\alpha}_{j}) \\ \sqrt{2} \hat{\alpha}_{1} & 2 \hat{\alpha}_{1} & \cdots & 0 & -\sqrt{2} \hat{\alpha}_{1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sqrt{2} \hat{\alpha}_{n_{MOT}} & 0 & \cdots & 2 \hat{\alpha}_{n_{MOT}} & -\sqrt{2} \hat{\alpha}_{n_{MOT}} \end{vmatrix}$$

$$(27)$$

where the degrees of freedom have been rearranged to facilitate the assembly process and to reduce the bandwidth of the final assembled matrices.

For a beam undergoing bending motion, the elastic mass and stiffness elastic finite element matrices are given as:

 \mathbf{K}^{ν}_{har}



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$$\mathbf{M}^{e}_{beam} = \frac{\rho A \ell}{420} \begin{vmatrix} 156 & 22\ell & 54 & -13\ell \\ 22\ell & 4\ell^{2} & 13\ell & -3\ell^{2} \\ 54 & 13\ell & 156 & -22\ell \\ -13\ell & -3\ell^{2} & -22\ell & 4\ell^{2} \end{vmatrix} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix}$$
(28)

$$\mathbf{K}^{e}_{beam} = \frac{G^{\propto}I}{\ell^{3}} \begin{vmatrix} 12 & 6\ell & -12 & 6\ell \\ 6\ell & 4\ell^{2} & -6\ell & 2\ell^{2} \\ -12 & -6\ell & 12 & -6 \\ 6\ell & 2\ell^{2} & -6\ell & 4\ell^{2} \end{vmatrix} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix}$$
(29)

where ℓ is the length of the finite element, ρ is the density of the material and I is the area moment of inertia [53]. By performing an eigenvalue analysis on the stiffness matrix, two degrees of freedom can be eliminated since they correspond to eigenvalues of zero. The spectral decomposition results in:

$$\bar{\mathbf{R}}_{\text{beam}}^{T} = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{2}{\ell\sqrt{8/\ell^{2}+2}} & \frac{1}{\ell\sqrt{8/\ell^{2}+2}} & \frac{-2}{\ell\sqrt{8/\ell^{2}+2}} & \frac{1}{\ell\sqrt{8/\ell^{2}+2}} \end{bmatrix}$$
(30)

$$\bar{\Lambda}_{\text{beam}} = \begin{bmatrix} \frac{2I}{\ell} & 0\\ 0 & \frac{6I(\ell^2 + 4)}{\ell^3} \end{bmatrix}$$
(31)

$$\mathbf{R}_{\text{beam}}^{T} = \frac{G^{\sim}I}{\ell^{3}} \begin{bmatrix} 6\sqrt{2(\ell^{2}+4)} & 3\sqrt{2(\ell^{2}+4)} & -6\sqrt{2(\ell^{2}+4)} & 3\sqrt{2(\ell^{2}+4)} \\ 0 & -\sqrt{2}\ell^{2} & 0 & \sqrt{2}\ell^{2} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{1}^{T} \\ \mathbf{R}_{2}^{T} \end{bmatrix}$$
(32)

Using eqs. (18) - (20), the viscoelastic finite element matrices are:

$$\mathbf{M}^{v}_{beam} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{M}_{12} \\ \mathbf{0} & \frac{\hat{\alpha}_{1}}{\hat{\alpha}_{1}^{2}} \mathbf{\Lambda}_{beam} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \frac{\hat{\alpha}_{n_{MOT}}}{\hat{\omega}_{n_{MOT}}^{2}} \mathbf{\Lambda}_{beam} & \mathbf{0} \\ \mathbf{M}_{21} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{M}_{22} \end{bmatrix}$$
(33)
$$\mathbf{D}^{v}_{beam} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 2\frac{\hat{\alpha}_{1}\hat{\zeta}_{1}}{\hat{\omega}_{1}} \mathbf{\Lambda}_{beam} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & 2\frac{\hat{\alpha}_{n_{MOT}}\hat{\zeta}_{n_{MOT}}}{\hat{\omega}_{n_{MOT}}} \mathbf{\Lambda}_{beam} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}$$
$$\mathbf{K}^{v}_{beam} = \begin{bmatrix} \phi \mathbf{K}_{11} & \hat{\alpha}_{1}\mathbf{R}_{1} & \cdots & \hat{\alpha}_{n_{MOT}}\mathbf{R}_{1} & \phi \mathbf{K}_{12} \\ \hat{\alpha}_{1}\mathbf{R}_{1}^{\mathrm{T}} & \hat{\alpha}_{1}\mathbf{\Lambda}_{beam} & \cdots & \mathbf{0} & \hat{\alpha}_{1}\mathbf{R}_{2}^{\mathrm{T}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \hat{\alpha}_{n_{MOT}}\mathbf{R}_{1}^{\mathrm{T}} & \mathbf{0} & \cdots & \hat{\alpha}_{n_{MOT}}\mathbf{\Lambda}_{beam} & \hat{\alpha}_{n_{MOT}}\mathbf{R}_{2} \end{bmatrix}$$
(35)

where $\phi = 1 + \sum_{j=1}^{n_{MOT}} \hat{\alpha}_j$ and the degrees of freedom have been rearranged to facilitate global matrix assembly and reduce the bandwidth of the stiffness matrix reducing the computational time required for matrix inversion.

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