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Research Paper

Multiobjective Geometric Analysis of Stiffened Plates under Bending through Constructal Design Method

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Abstract. Constructal design, finite element method and exhaustive search are applied to analyze different arrangements of steel plates with rectangular or trapezoidal stiffeners. As performance parameters, the maximum deflection and maximum von Mises stress are considered. A non-stiffened plate adopted as reference is studied together with 25 plates with rectangular stiffeners and 25 plates with trapezoidal stiffeners. The results show that trapezoidal stiffeners are more effective in minimizing the maximum deflection in comparison with rectangular stiffeners. However, regarding the minimization of stress, the rectangular stiffeners normally present better performance. When both performance parameters are concomitantly considered, a slight advantage of 4.70% for rectangular geometry is identified.

Keywords: Trapezoidal Stiffeners, Rectangular Stiffeners, Numerical Simulation, Geometric Optimization, Plate Bending.

1. Introduction

Attempts to design structures, which can carry large loads and have a relatively small deadweight at the same time, led to thin-walled structures. Those structures have excellent load capacities, but are susceptible to large deflections, buckling effects, and vibrations. There are various ways to respond to such challenges. Solutions have been sought in application of novel materials, such as fibre-reinforced composites [1] or functionally graded materials [2]. Also, the use of smart materials based active components is one of successful solutions [3, 4]. However, all those solutions imply significant costs and in many cases costs do play an important role. In such cases, less expensive solutions are sought.

One of the simplest, rather inexpensive and quite common solutions is to insert stiffeners welded to the plated structures, on the plane of the plate, being normally arranged in the longitudinal and/or transverse directions, allowing a considerable increase of rigidity with a minimum amount of material. The stiffened plates, due to their good relationship between weight and load-bearing capacity, as well as relatively low cost, are widely used in engineering structures, such as bridges, ship hulls and aircraft fuselages [5, 6].

The mechanical behavior analysis of non-stiffened plates can be performed analytically using the classic Kirchhoff differential equation, applying for instance, Navier or Lévy solutions. However, regarding to stiffened plates, the existing analytical solutions, such as the theory of orthotropic plate [7, 8] and approximations by energy principles [9, 10], are restricted to specific geometries, loading and boundary conditions. Besides that, mathematical solutions are usually rather complex, and may be prone to inaccuracies. For these reasons, numerical solutions become an interesting option, being the target of several studies over the past decades.

O'Leary and Harari [11] developed a computational model applying the Finite Element Method (FEM), so that the constraints between the plate and stiffeners were established through Lagrangian multipliers. In order to simplify the proposed model a two-dimensional beam finite element was used. Also using the FEM and based on the Mindlin theory, Deb and Booton [12] presented numerical models of eccentrically stiffened plates subject to bending due transverse loading. It was proposed two linear finite element models, being argued the advantages of one over the other. In turn, Biswal and Ghosh [13], developed a model for the numerical analysis of laminated stiffened plates of composite material by FEM, considering the First-order Shear Deformable Theory (FSDT), in a way that eliminates the use of shear correction coefficients. It was employed a four-noded rectangular element with seven degrees of freedom per node and the stiffness of the stiffeners is considered at all four nodes of the plate element.

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Using the Boundary Elements Method (BEM), Paiva [14] presented a computational model for the analysis of stiffened plates (building slabs). Its formulation considers three nodal values in displacements for the nodes at the boundary of the plate assuming that only vertical forces were transmitted between the plate and stiffener, considering the stress varying linearly in this region. However, it was highlighted that this formulation can be easily improved, allowing other models of connecting these two elements (plate-stiffener), as vertical forces and bending moments. Peng et al. [15] analyzed stiffened plates subjected to bending with stiffeners in eccentric and concentric conditions using the Galerkin Method, based on FSDT, taking account a meshless system, where a set of nodal points was distributed along the plate and the stiffener. The compatibility between the displacement of the stiffener and the plate allowed the field of displacement of the stiffener to be expressed as a function of the average surface of the plate. This approach allows greater flexibility in the placement of the stiffeners, once any change in their positions does not require a mesh refinement.

In more recent studies, applying the BEM, Waidemam and Venturini [16] proposed a simplified model for stiffened plates under bending, based on applying an initial moment field adopted to locally correct the stiffness of the reinforcement regions. So that, only the stiffness of the stiffener was considered, allowing to reduce the degrees of freedom of the problem. Likewise, Fernandes [17] also using the BEM presented a model for the analysis of stiffened plates subject to bending, considering different materials. The composed structure was treated as a single body in which the equilibrium and compatibility conditions were automatically considered.

Currently, with the availability of high-performance computers, computational numerical modeling through commercial software based on FEM allows an accurate analysis, being possible to evaluate and re-evaluate the model as many times as necessary with practicality. Aiming to find more effective structural arrangements, recent studies involving plates using the Constructal Design Method (CDM) and computational modeling through FEM, have shown relevant results. Dealing with buckling in perforated plates it is possible to quote Helbig et al. [18, 19], Rocha et al. [20], Lorenzini et al. [21] and Da Silva et al. [22]. As well as dealing with buckling on stiffened plates the works of Lima et al. [23, 24]. Regarding stiffened plates under bending, one can quote the researches of De Queiroz et al. [25], Pinto et al. [26], Troina et al. [27] and Amaral et al. [28]. Lastly, the analysis of aircraft structure can be found in Mardanpour et al. [29] and Izadpanahi et al. [30].

In this sense, the present study had as objective to find, by means the Exhaustive Search (ES) technique, optimized plate arrangements with rectangular or trapezoidal stiffeners, capable of minimizing the effects of maximum deflection and of maximum von Mises stress, set up due the application of the CDM and numerically solved through the ANSYS® software, which is based on FEM. It is worth to mention that the CDM application in stiffened plates considering two types of stiffeners and with a multiobjective approach can be highlighted as the novelty of this study; since in the majority of the previous works which adopted the CDM to investigate stiffened plates, the main goal was only focused on the deflection analysis with one type of stiffener. In addition, the adopted methodology coupling the CDM and ES allows not only to identify the optimized geometry, but also to evaluate the influence of the geometric configuration variation over the mechanical behavior of the structure.

The schematic representation of the employed methodology to promote the geometric analysis of the present work can be seen in Fig. 1.

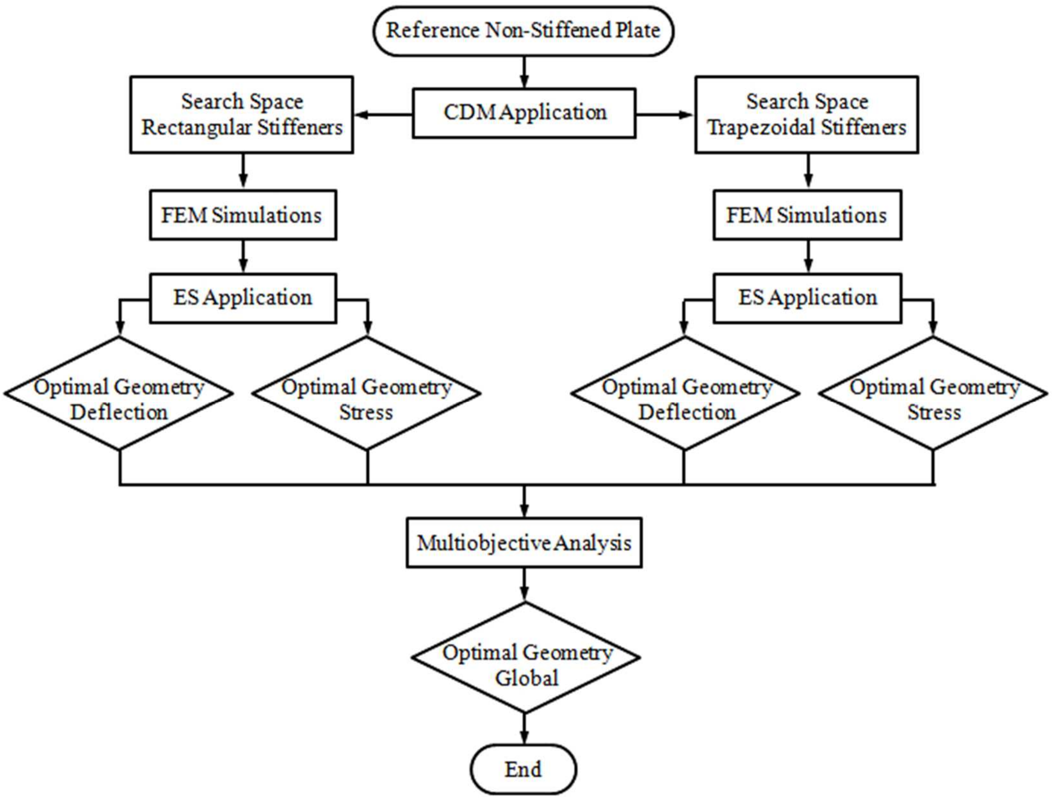

Fig. 1. Schematic representation of the employed methodology

2. Material and Methods

2.1 Computational Modeling

The computational modeling of the present study is based on the FEM, being developed in ANSYS® software. All performed numerical simulations considered a linear-elastic material behavior and a geometric linear structural analysis. For the discretization of the computational domains, the two-dimensional finite element SHELL281 was used in the triangular version. This element is suitable for modeling thin plates and has 6 nodes with 6 degrees of freedom per node: 3 translations in the x, y and z directions and 3 rotations around the x, y and z axes [31].

2.1.1. Computational Model Verification

The first computational model verification (see Fig. 2) is based on a case presented by Heinonen and Pajunen [32], where a numerical solution was obtained by FEM ANSYS® software. The stiffened plate was subjected to a uniform transverse loading of 100 kPa, having simply-supported boundary conditions in its 4 edges. The material has an elastic modulus of 0.2 GPa and a Poisson's ratio of 0.3.

Using the SHELL281 element in its triangular version, the stiffened plate of Fig. 2 was numerically simulated with a mesh formed by 189,630 finite elements, defined after the mesh convergence test presented in Fig. 3, in which it is also possible to view the result obtained by Heinonen and Pajunen [32] for the maximum von Mises stress of the plate.

It can be noted at Fig. 2 that the obtained von Mises stress result with the SHELL281 element of 198,62 MPa, showed a difference of 0.36% compared to the value of 199.35 MPa, obtained by Heinonen and Pajunen [32], verifying the proposed computational model.

After that, the second numerical model verification was carried out with a stiffened plate previously studied by Troina et al. [33]. This problem was numerically solved in [33] using the ANSYS® software by the three-dimensional finite element SOLID95 in the hexahedral version and was discretized with a mesh composed by 3,928 finite elements. The plate with rectangular stiffeners shown in Fig. 4 was subjected to a uniformly distributed load of 68.95 kPa with boundary conditions of simply-supported edges. The material has an elastic modulus of 206.8427 GPa and Poisson's ratio of 0.3.

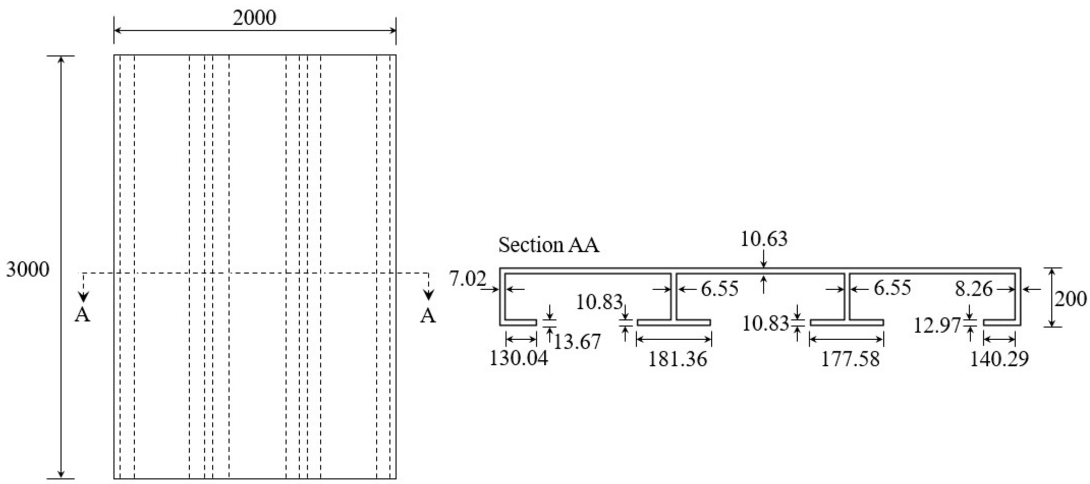

Fig. 2. Verification model 1 (unit: mm)

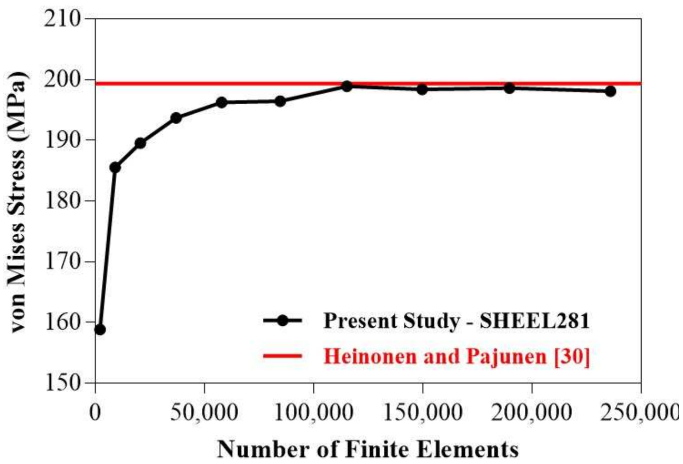

Fig. 3. Verification result: Model 1

The computational domain of Fig. 4 was discretized using the finite element SHELL281 in its triangular version with a converged mesh that totaled 30,400 elements, defined after the mesh convergence test presented in Fig. 4, in which the result obtained by Troina et al. [33] is also presented.

One can observe in Fig. 5 that the deflection result in the center of the plate U_z obtained with the present study was 0.281 mm, having a difference of 1.08% in comparison with the result of 0.278 mm, found by Troina et al. [33], verifying the proposed model.

2.1.2. Computational Model Validation

For the numerical model validation, it was adopted the experimental study presented by Carrijo et al. [34], as shown in Fig. 6. The square plate with eight rectangular stiffeners was subjected to a uniform transverse load of 0.96 kPa. As boundary conditions, only its four corners were considered simply-supported. The material has an elastic modulus of 2.5 GPa and a Poisson's ratio of 0.36.

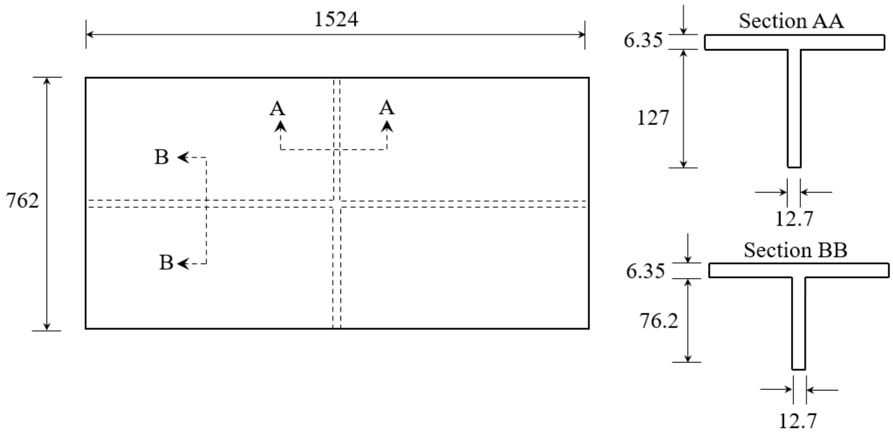

Fig. 4. Verification model 2 (unit: mm)

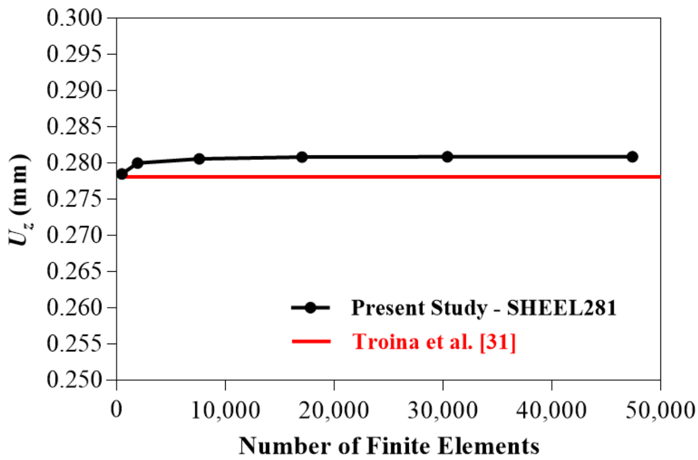

Fig. 5. Verification result: Model 2

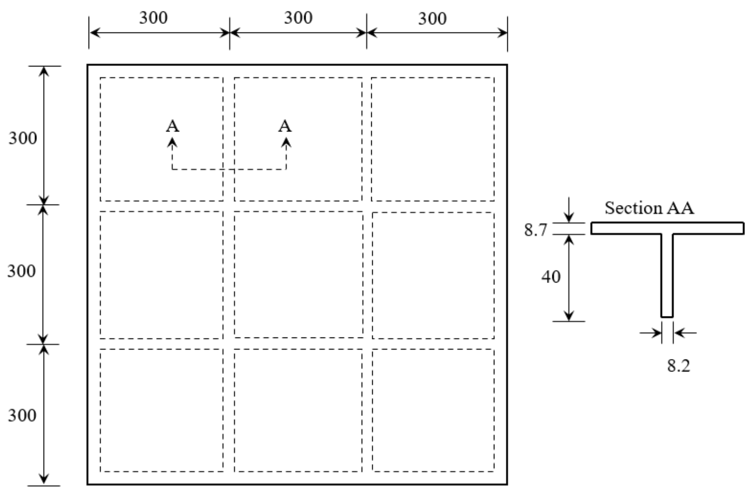

Fig. 6. Validation model (unit: mm)

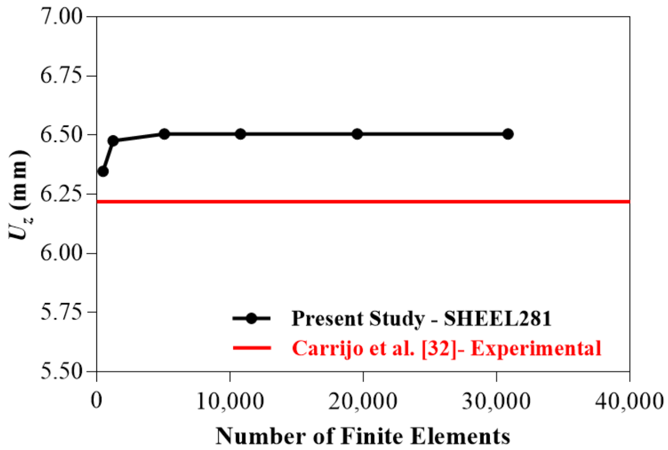

Fig. 7. Validation model result

Following the mesh convergence test shown in Fig. 6, the experiment was numerically solved here using the triangular finite element SHELL281 with a mesh of 5,070 computational cells. The experimental result obtained by Carrijo et al. [34] can also be viewed in Fig. 7.

From Fig. 7 it is possible to infer that the deflection U_z in the center of the stiffened plate obtained in this study (6.505 mm), showed a difference of 4.58% in comparison with the experimental model (6.220 mm), validating the developed computational model.

2.2 . Constructal Design Method (CDM)

Constructal Law is a physical principle that governs the broad geometric complexity of flow systems existing in nature. According to this principle the systems tend to be imperfect, so that their design must evolve in time (survive) in order to facilitate the flow that passes through them, distributing their imperfections in the best possible way [35, 36].

The Constructal Design Method (CDM) is the manner in which the Constructal Law is applied in the geometric analysis of flow systems, whether animated or inanimate. Based on restrictions, degrees of freedom and performance parameters the CDM guides the designer in the geometric evaluation of flow system. Once the restrictions are established, the degrees of freedom related to the geometric configuration of the system are modified in order to evaluate their influence on a predefined performance parameter seeking to achieve the best possible system performance [37-39]. Therefore, the CDM alone allows the development of a flow system geometric evaluation. However, if the CDM is applied in association with an optimization method, such as Heuristic Methods (HM) or Exhaustive Search (ES), a geometric optimization of the flow system is performed. In this last approach, the CDM has as function to define the search space in which the optimization method will identify the flow system geometric configuration that promotes the superior performance [38, 40]. Regarding the choice about the optimization method, the ES has the advantage of to allow a detailed analysis of the influence of the degrees of freedom variation over the performance parameter, while the HM defines the optimized geometry in a more direct way. On the other hand, when considering an elevated number of degrees of freedom the ES can be prohibitive due to its processing time, being in these cases recommended the use of HM.

The application of CDM, associated or not with an optimization method, is widespread in problems of heat transfer and/or fluid mechanics [41-46]. However, some studies showed its application feasibility in structural engineering problems in a corresponding way to the problems of these both areas, as in Bejan and Lorente [35], Lorente et al. [39] and Isoldi et al. [47].

Bejan and Lorente [35] explain that the CDM approach to structural problems is valid, since the flow in this situation is related to the flow of stresses in the solid component. Thus, the desired geometric configuration is one that presents a better distribution of the imperfections (regions submitted to the allowable stress of the material), which is only possible with a better distribution of the flow of stresses throughout over the structure.

In this study the CDM application consists of the geometric analysis of stiffened steel plates under bending, considering rectangular and trapezoidal stiffeners. For this, a non-stiffened plate was taken as a reference: with length a = 2000 mm, width b = 1000 mm and thickness t = 20 mm. Keeping the total material volume constant, as well as dimensions a and b, a steel volume portion was deducted entirety from its thickness and transformed into stiffeners through the volume fraction ϕ. The value of ϕ considered was 0.3, that is, 30% of the reference plate material volume transformed into stiffeners, based on the results obtained by Troina et al. [27]. Thus, the plate thickness t = 20 mm, became t_p = 14 mm.

In addition to the reference plate, a total of 50 stiffened plates were analyzed: 25 with rectangular stiffeners and 25 with trapezoidal stiffeners, concerning the maximum out-of-plane displacement (maximum deflection) and the maximum von Mises stress. Since two different stiffener geometries were considered, the mathematical definition of the volume fraction ϕ for rectangular and trapezoidal stiffeners, respectively, are given by:

$$\phi = \frac{V_s}{V_r} = \frac{n_{sx}(ah_st_s) + n_{sy}\left[(b - n_{sx}t_s)h_st_s\right]}{abt} \quad (1)$$

$$\phi = \frac{V_s}{V_r} = \frac{\left\{n_{sx}\left[\frac{(a+c)h_st_s}{2}\right] + n_{sy}\frac{(b+d)h_st_s}{2}\right\} - V_{int}}{abt} \quad (2)$$

where V_s is the material volume of the stiffeners; V_r is the material volume of the reference plate; and h_s and t_s are, respectively, the height and thickness of the stiffeners. It is also important to highlight that a 19 mm (commercial value ¾”) stiffener thickness was adopted. Moreover, to identify the plate arrangements the nomenclature P(n_{sx},n_{sy}) was adopted, being n_{sx} and n_{sy} the number of stiffeners in the x and y directions, respectively. Exclusively for plates with trapezoidal stiffeners, c and d are the length of the smaller bases of the trapezoidal stiffeners in the x and y directions, fixed at 1000 mm and 500 mm, respectively, as well as V_{int} is the stiffeners intersection volume. All of these parameters can be seen in Fig. 8 (a) and (b), while in Fig. 8 (c) all plate arrangements simulated in this study, i.e. the search space, are indicated.

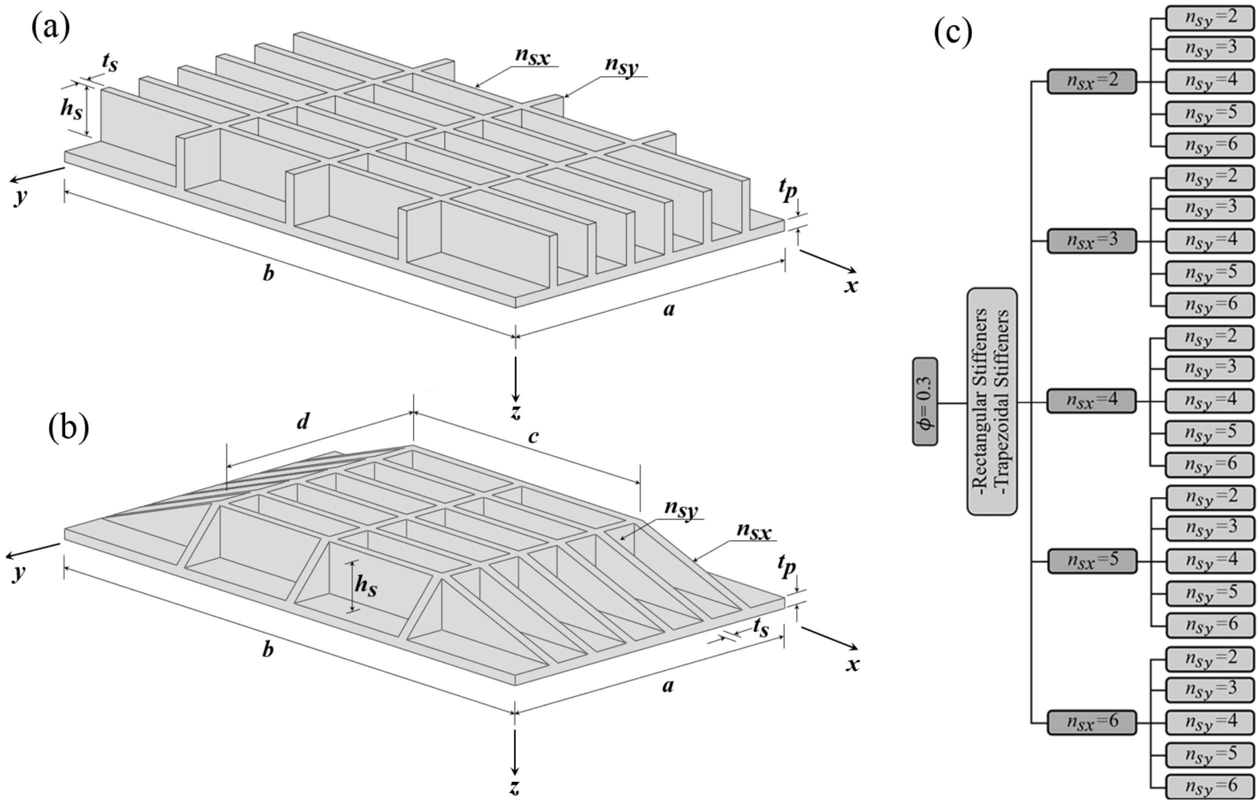

Fig. 8. Plate P(6,3): (a) Rectangular stiffeners; (b) Trapezoidal stiffeners; (c) Application of the CDM in the definition of the search space

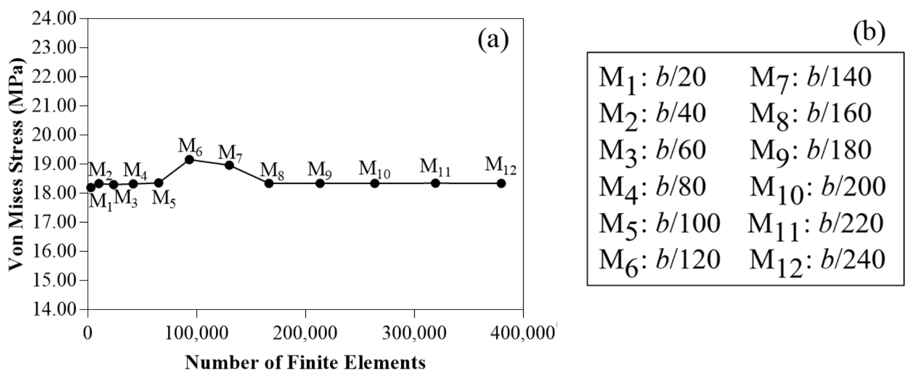

Fig. 9. Mesh convergence test: (a) von Mises stresses; (b) mesh refinement criteria.

As earlier mentioned, the performance parameters adopted in this study according to CDM are the maximum deflection and the maximum von Mises stress of the plates, which should be minimized. All plates were subjected to a uniform transverse loading of 10 kPa and boundary conditions of simply supported edges. The material adopted was the structural steel A-36 with an elastic modulus of 200 GPa and Poisson ratio of 0.3, which according to Mandal [48] has a relatively low cost and has mechanical properties that facilitate the assembly and welding process.

It is important to highlight that in the present work the CDM was used in association with the ES, allowing the definition of the optimized geometric configurations as well as showing the effect of the geometry variations over the performance parameters. This approach has been used in majority of similar studies, as in De Queiroz et al. [25], Pinto et al. [26], Troina et al. [27], Amaral et al. [28], and Pinto et al. [49]. However, regarding the other above mentioned possible approach, coupling CDM and HM, one can quote the recent work of Cunha et al. [50] that associated CDM and Genetic Algorithm (GA).

3. Results and Discussion

Initially, to define the appropriate size of the meshes used to discretize the studied computational domains, a mesh convergence test was performed. For this purpose, the plate with more complex geometry, P(6,6) with trapezoidal stiffeners was adopted. The size of the finite elements was reduced successively according to the criteria established by Troina et al. [27], based on the plate width b = 1000 mm. The results of convergence regarding the von Mises stress, as well as the mesh refinement criterion, can be seen in Fig. 9.

According to Fig. 9 (a), it was established that the M_9 mesh with finite element size of 5.55 mm, would be used to discretize all other computational domains (indicated in Fig. 8 (c)). It is worth to inform that the mesh convergence test was performed considering the von Mises stress as indicator, since it is well known that the stresses convergence is more complex than the deflections convergence. In other words, the converged mesh obtained by stresses analysis certainly promotes a converged solution for the deflections.

The results of the analyzed stiffened plates, for maximum deflection and maximum von Mises stress are presented, respectively, in Figs. 10 and 11. The bar graph format allows that the variations of deflections and stresses of the plates with rectangular and trapezoidal stiffeners be compared directly for each proposed arrangement.

As a first observation, it is noted that transforming in stiffeners a parcel of material deducted entirely from the thickness of a non-stiffened plate, an increase in the plate stiffness occurs, since all the results of stiffened plates showed values of maximum deflection lower than the value of the reference plate, which is U_{zMax} = 0.697 mm. However, regarding the stress analysis the same behavior was not observed. Most of the stiffened plate arrangements presented a maximum von Mises stress greater than the value of 13.313 MPa obtained by the non-stiffened reference plate.

Observing the Fig. 10, it is clear that the plates with trapezoidal stiffeners are more effective to minimizing maximum deflection, once in all analyzed arrangements the values of maximum deflection found for plates with trapezoidal stiffeners are lower than the values presented by the plates with rectangular stiffeners. It can also be observed that the smallest deflections tend to be for groups with less amounts of stiffeners (Groups A and B), which can be explained due premise of maintaining the total volume of material constant, imposed as a main restriction by the application of the CDM. Thus, smaller quantities of stiffeners imply stiffeners with greater height and consequently greater moment of inertia of the cross section, allowing a reduction of the maximum deflections.

Still regarding the Fig. 10, among all analyzed geometric configurations, the plate P(2,3) with trapezoidal stiffeners was the one that presented the lowest maximum deflection. Comparing the value of U_{zMax} = 0.0634 mm obtained by plate P(2,3) with trapezoidal stiffeners with the value of U_{zMax} = 0.0807 mm of plate P(2,3) with rectangular stiffeners, there is a reduction of 21.36% favorable to the plate with trapezoidal stiffeners. However, concerning percentage differences for maximum deflection, the highest value occurred for arrangement P(5,6), where the plate with trapezoidal stiffeners showed U_{zMax} = 0.1956 mm, that is, a value 33.86% lower than the value of U_{zMax} = 0.2958 mm reached by the plate with rectangular stiffeners.

As for Fig. 11, it can be seen that in terms of stress minimizing, trapezoidal plate arrangements become more advantageous mainly from Group C, with greatest expressiveness in comparison with plates with rectangular stiffeners in plates with more quantities of stiffeners (Group E). However, all geometric configurations of Group A and the arrangement P(3,2) of Group B are exceptions, since in these cases the plates with rectangular stiffeners presented lower von Mises stress than the respective plates with trapezoidal stiffeners. Among all analyzed arrangements the P(2,2) with rectangular stiffeners was the one that achieved the lowest stress value (12.119 MPa), representing a difference of 37.53% if compared to the value of 19.400 MPa found for the plate P(2,2) with trapezoidal stiffeners, being this stress percentage difference the largest registered among plates with rectangular and trapezoidal stiffeners.

It is worth noting that some plate arrangements with trapezoidal stiffeners achieved significant stress reductions in comparison with the arrangements with rectangular stiffeners, although they have not been the lowest values achieved among all analyzed cases. For example, the plate P(6,5) with trapezoidal stiffeners showed a von Mises stress of 19.536 MPa, which means a reduction of 34.64% compared to the value of 29.892 MPa presented by the plate P(6,5) with rectangular stiffeners.

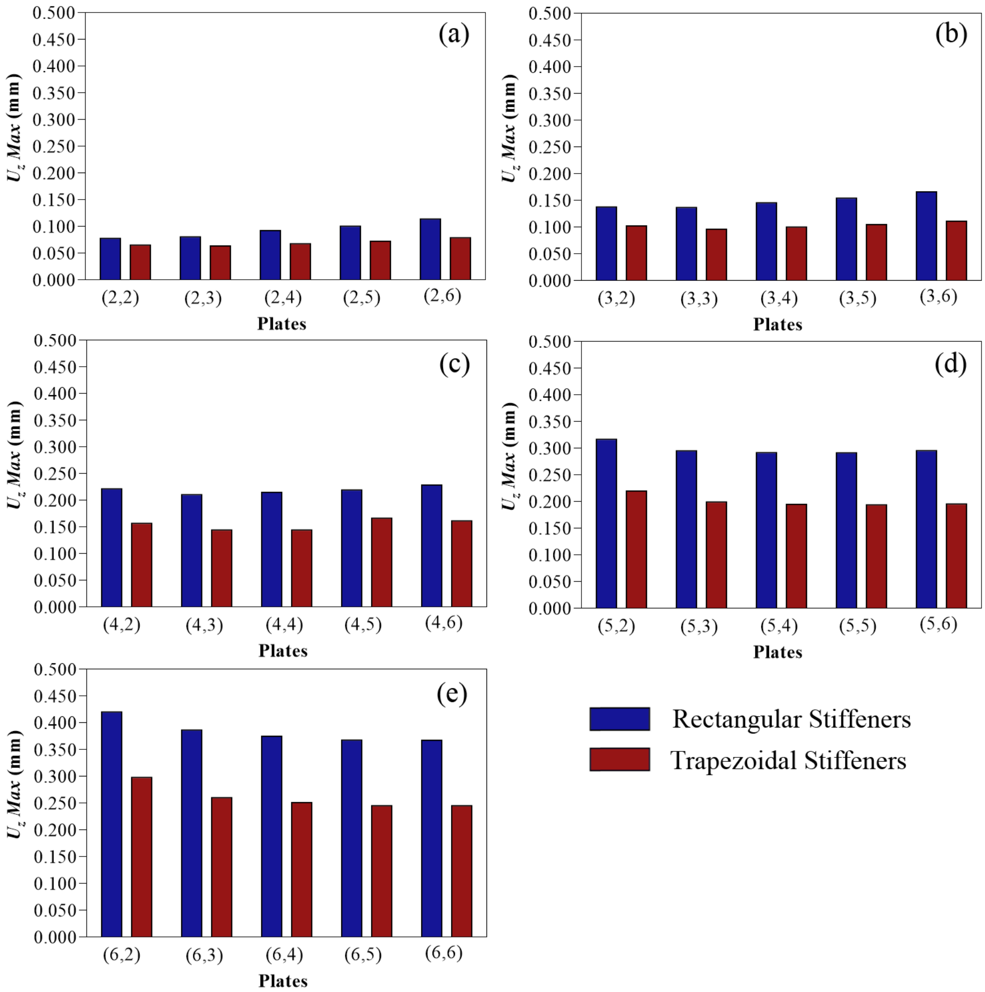

Fig. 10. Results of maximum deflection: (a) Group A; (b) Group B; (c) Group C; (d) Group D; (e) Group E

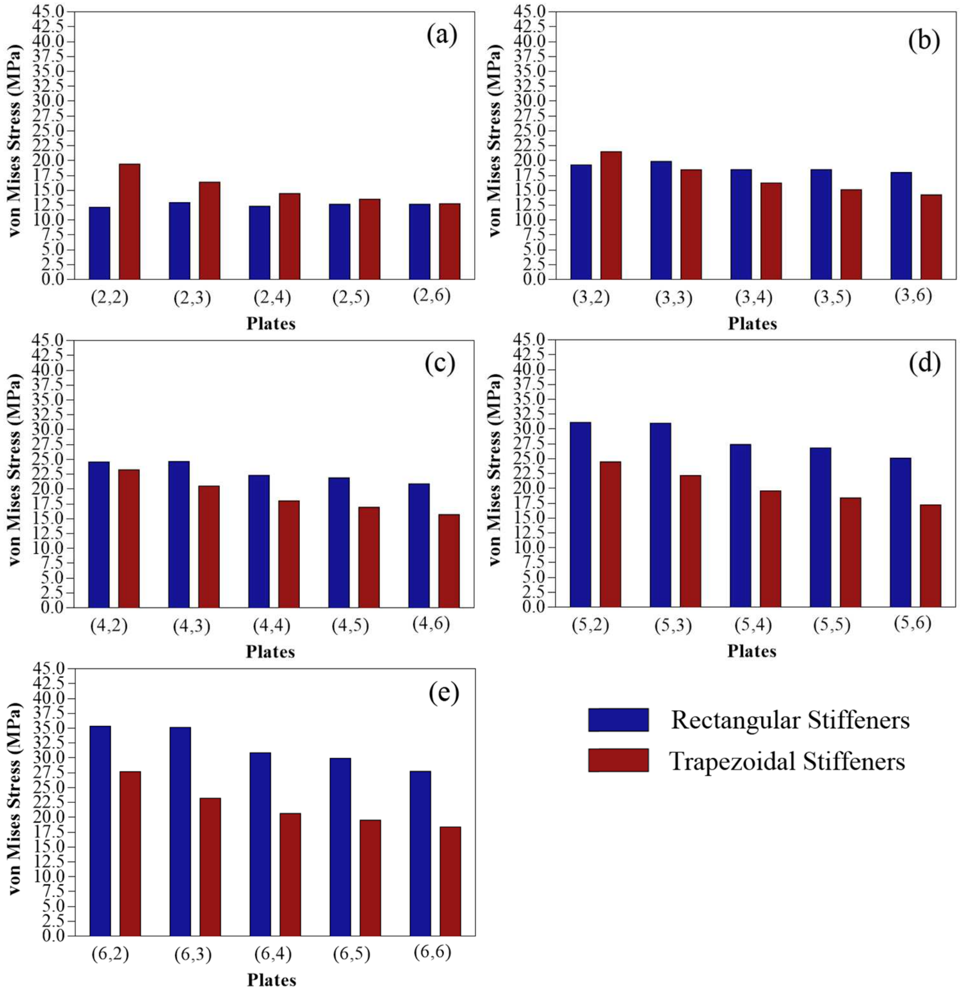

Fig. 11. Results of maximum von Mises stress: (a) Group A; (b) Group B; (c) Group C; (d) Group D; (e) Group E

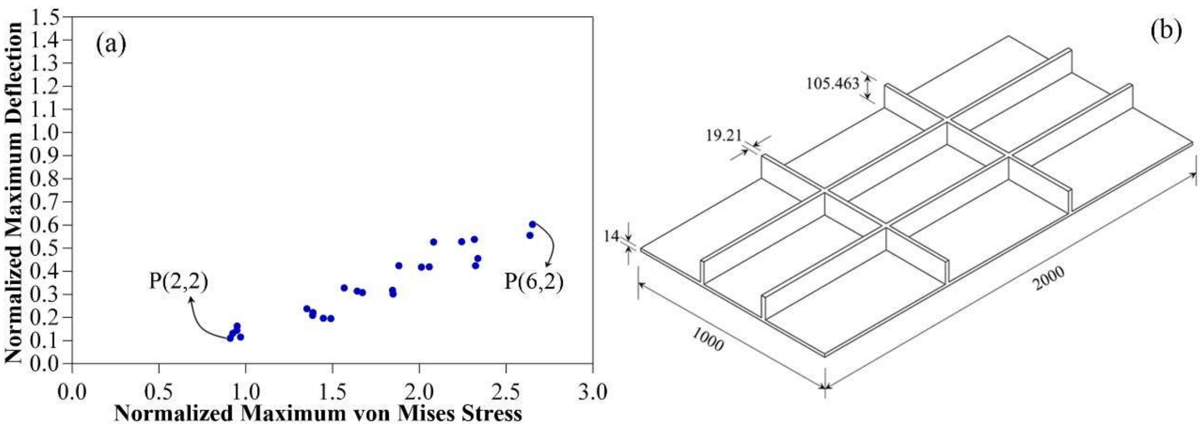

Fig. 12. Rectangular stiffeners: (a) Normalized Stress versus Normalized Deflection; (b) Optimized plate P(2,2), in mm

In sequence, it was carried out an analysis in order to find an optimized plate arrangement that was able to simultaneously minimize maximum deflection and maximum von Mises stress. For this purpose, these values for each plate with rectangular or trapezoidal stiffeners were normalized based on the non-stiffened reference plate, being the maximum deflection of 0.697 mm and the maximum von Mises stress of 13.313 MPa, as already presented. The normalized results were inserted in a scatter graph as shown in Figs. 12 (a) and 13 (a). Therefore, the optimized geometric configurations are those that came closest to the origin of the coordinate system of the scatter graphs.

From Figs. 12 (a) and 13 (a) the optimized plates are, respectively, the P(2,2) with rectangular stiffeners and the P(2,6) with trapezoidal stiffeners; while the worst geometry is the P(6,2) for both rectangular and trapezoidal stiffeners.

The optimized geometric configuration P(2,2) with rectangular stiffeners is depicted in Fig. 12 (b), reaching a maximum deflection of 0.0755 mm and maximum stress of 12.119 MPa. If compared the plates P(2,2) and P(6,2) (worst, see Fig. 12 (a)), the optimized geometry achieved an improvement of 66.28% concerning the rectangular stiffeners, for the concomitantly analysis of the both performance parameters.

In its turn, the Fig. 13 (b) shows the optimized arrangement P(2,6) with trapezoidal stiffeners, which presented a maximum deflection value of 0.0791 mm and a maximum stress of 12.720 MPa. This plate when compared with the worst P(6,2) (see Fig. 13 (a)), regarding the trapezoidal stiffeners, promotes an improvement in the mechanical behavior of 54.66% taking into account the minimization of maximum deflection and maximum von Mises stress considered in an associated way.

If a global comparison is carried out, the plate with rectangular stiffeners P(2,2) presented a maximum deflection value 4.55% lower than the plate P(2,6) with trapezoidal stiffeners. Dealing with stresses, the minimization was 4.72% in favor of the plate with rectangular stiffeners. However, analyzing simultaneously maximum deflection and maximum von Mises stress the plate with rectangular P(2,2) stiffeners achieved a performance 4.70% better than the plate P(2,6) with trapezoidal stiffeners.

Finally, from Figs. 12 and 13 the optimized and worst geometric configurations for each type of stiffener were adopted to show the distribution of deflections (Fig. 14) and the distribution of von Mises stresses (Fig. 15) occurred. Since these geometries were the extreme cases defined by the multiobjective analysis, it is possible to consider them as a representative sample among all studied stiffened plates.

One can observe that the optimized geometries can promote a better distribution of imperfections, being submitted an almost constant value both for deflection (Fig. 14 (a) and Fig. 14 (c)) and von Misses stress (Fig. 15 (a) and Fig. 15 (c)). In tits turn, the worst geometries have a considerably higher gradient both for deflections (Fig. 14 (b) and Fig. 14 (d)) and von Misses stress (Fig. 15 (b) and Fig. 15 (d)). These distinct mechanical behaviors are in agreement with the Constructal Principle of Optimal Distribution of Imperfection [35, 36].

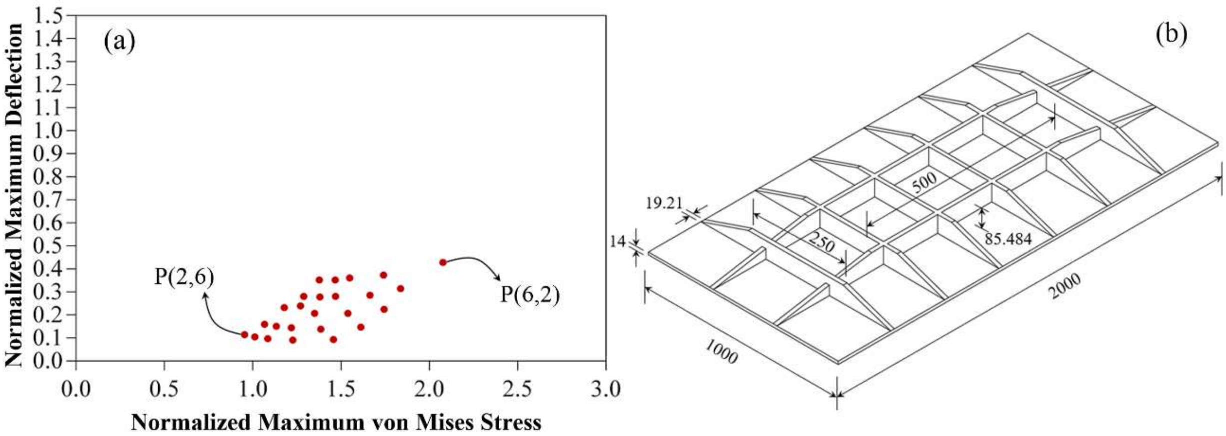

Fig. 13. Trapezoidal stiffeners: (a) Normalized Stress versus Normalized Deflection; (b) Optimized plate P(2,6), in mm

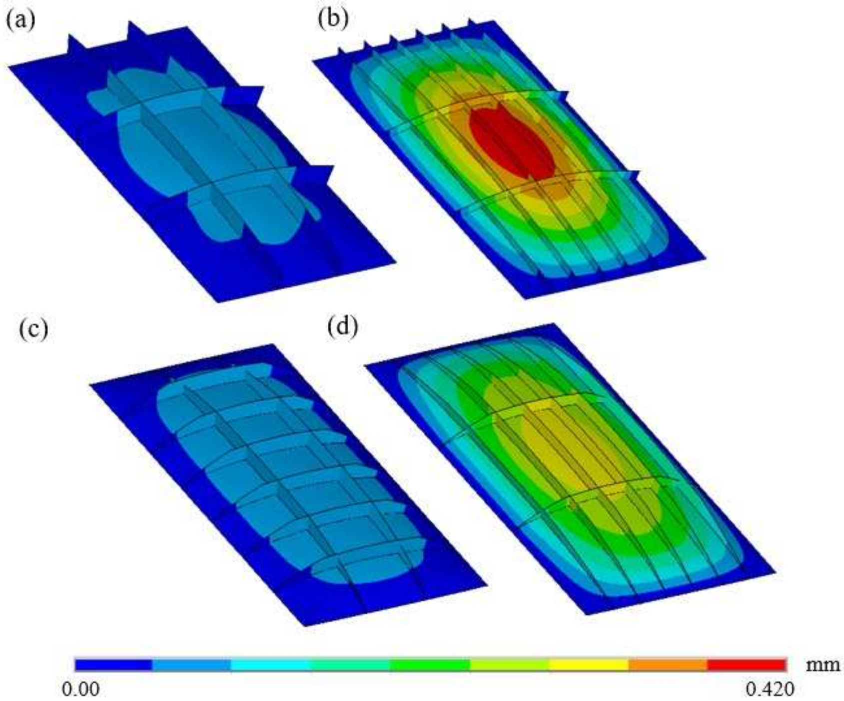

Fig. 14. Deflection distribution: (a) P(2,2) with Rectangular Stiffeners; (b) P(2,6) with Trapezoidal Stiffeners

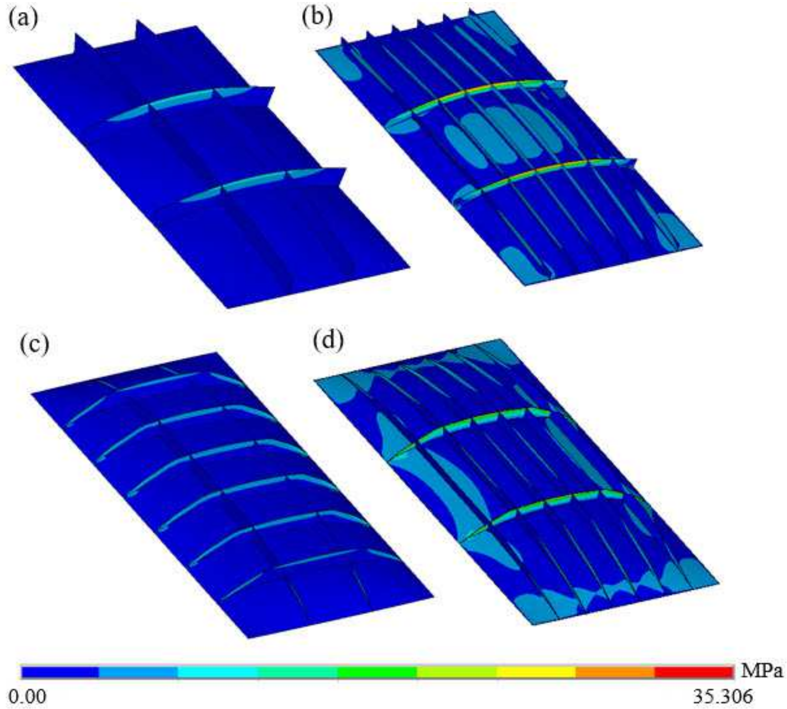

Fig. 15. von Mises stress distribution: (a) P(2,2) with Rectangular Stiffeners; (b) P(2,6) with Trapezoidal Stiffeners

4. Conclusion

Employing in associated way the Constructal Design, Finite Element Method, and Exhaustive Search, starting from a non-stiffened plate, several plate arrangements with rectangular or trapezoidal stiffeners were analyzed, aiming to identify the optimized geometric configurations capable of minimizing the maximum deflection and the maximum von Mises stress. First, it can be concluded that transforming a portion of steel from a non-stiffened plate (fully deducted from its thickness) into stiffeners, as expected, a considerable increase in the stiffness was promoted, since all the maximum deflection values found for the plates with rectangular or trapezoidal stiffeners are lower than the maximum deflection value obtained by the reference plate, achieving reductions of maximum deflection up to 90%. As for the minimization of the maximum von Mises stress, it was noted that the reference plate showed an advantage in comparison with most of the arrangements with rectangular and trapezoidal stiffeners, being a coherent result if considered the greatest structural complexity of the stiffened plates. In addition, between the two studied geometries of stiffeners (rectangular and trapezoidal), it can be inferred that trapezoidal stiffeners are more effective regarding the minimization of maximum deflection in all analyzed cases; while for the minimization of stresses, the trapezoidal geometry becomes more advantageous for larger quantities of stiffeners, i.e., groups C, D and E. Therefore, in concomitantly analysis of the both performance parameters, concerning the better and the worst cases among rectangular stiffeners, P(2,2) and P(6,2), respectively, the stiffened plate P(2,2) performed 66.28% better. In its turn, between the better and worst geometries considering trapezoidal stiffeners, P(2,6) and P(6,2), respectively, the plate P(2,6) reached an improvement of 54.66%. On the other hand, concerning the better result (optimized geometry) for each type of stiffeners, rectangular and trapezoidal, P(2,2) and P(2,6), respectively, there was a slight better performance of 4.70% in favor of the plate with rectangular stiffeners.

In future works, it would be interesting to investigate other ϕ values, as well as to consider different values for the stiffeners thicknesses t_s.

Author Contributions

Conceptualization: Vinícius Torres Pinto and Cristiano Fragassa. Methodology: Liércio André Isoldi and Cristiano Fragassa; Computational Model (development, verification and validation): Vinícius Torres Pinto and Elizaldo dos Santos; Formal Analysis: Luiz Alberto Oliveira Rocha and Liércio André Isoldi. Investigation: Vinícius Torres Pinto and Liércio André Isoldi; Resources: Luiz Alberto Oliveira Rocha, Elizaldo dos Santos, and Liércio André Isoldi. Data Curation: Cristiano Fragassa and Liércio André Isoldi. Writing (original draft preparation): Vinícius Torres Pinto and Luiz Alberto Oliveira Rocha. Writing (final version): Cristiano Fragassa and Liércio André Isoldi. Supervision: Luiz Alberto Oliveira Rocha and Elizaldo Domingues dos Santos. Project Administration: Liércio André Isoldi. All authors discussed the results, reviewed, and approved the final version of the manuscript.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Nomenclature

a	Length of reference plate and stiffened plates [mm]	n_{sx}	Number of stiffeners in the direction x
b	Width of reference plate and stiffened plates [mm]	n_{sy}	Number of stiffeners in the direction y
BEM	Boundary Elements Method	$P(n_{sx},n_{sy})$	Stiffened plates arrangements identification
c	Smaller bases length of the trapezoidal stiffeners in the direction x [mm]	t	Reference plate thickness [mm]
CDM	Constructal Design Method	t_p	Stiffened plates thickness [mm]
d	Smaller bases length of the trapezoidal stiffeners in the direction y [mm]	t_s	Stiffeners thickness [mm]
ES	Exhaustive Search	U_z	Central deflection [mm]
FEM	Finite Element Method	U_{zMax}	Maximum deflection [mm]
FSDT	First-order Shear Deformable Theory	V_r	Reference plate volume [mm^3]
GA	Genetic Algorithm	V_s	Stiffeners volume [mm^3]
HM	Heuristic Methods	V_{int}	Stiffeners intersection volume
h_s	Stiffeners height [mm]	ϕ	Ratio between stiffeners and reference plate volumes
		σVM_{Max}	Maximum von Mises Stress [MPa]

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