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Research Paper

Modeling of Flight of the Line Thrower Projectile

Alexander K. Belyaev[1], Vladimir A. Piskunov[2], Tatiana V. Zinovieva[3]

[1] Institute for Problems in Mechanical Engineering, Russian Academy of Sciences, Saint Petersburg, Russia, Email: vice.ipme@gmail.com

[2] Concern Avrora scientific and production association JSC, Saint Petersburg, Russia, Email: vladimirpiskunov17@mail.ru

[3] Institute for Problems in Mechanical Engineering, Russian Academy of Sciences, Saint Petersburg, Russia, Email: tatiana.zinovieva@gmail.com

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Corresponding author: T.V. Zinovieva (tatiana.zinovieva@gmail.com)

Abstract. The actual problem of increasing the flight range of line thrower projectile which is a container with a line (thin rope) inside. The line leaves the container during the flight, i.e. the projectile has a variable mass. Mathematical model of the projectile flight is constructed using the Lagrange equations of the second kind. The projectile is considered as a material particle, the line considered as an elastic thread with the tensile Cauchy strain. An approximation of the projectile flight trajectory is introduced in terms of three generalized coordinates. The dependence of the projectile's flight distance on the projectile departure angle is constructed for several values of the tensile rigidity of the line.

Keywords: Line thrower, Variable mass flight, Elastic thread, Lagrange equations.

1. Introduction

An important task for modern engineering practice is to increase the throwing distance of the projectile with a thin elastic rope (line). The line throwers are an example of these devices which allow ship to ship line deployment, for instance, to launch a line from a ship in distress to a rescue ship. The Ocean Engineering field is an extraordinary source of interesting dynamical problem. In this paper problem is selected to illustrate how extended Lagrange equation can help modeling tasks and interpretation. According to the International Convention for the Safety of Life at Sea, SOLAS-74, the safe distance between ships is 230 m in a storm. In such a situation, it is necessary to use the line thrower to guide navigation (Fig. 1).

Pneumatic line-thrower is similar to a pneumatic gun, the main unit of which is a quick-acting valve (Fig. 2).

Testing pneumatic line gauges is extremely expensive and time consuming. For this reason, mathematical modeling can be used in the design of line throwers. Mathematical modeling of the dynamics of such projectiles requires account for the variable mass of the projectile since the line leaves the container during the flight.

A fairly large number of works are devoted to the problems of motion of bodies with variable mass. However, even now the solution of these mechanical problems is of scientific and practical interest due to the relevance of the topic.

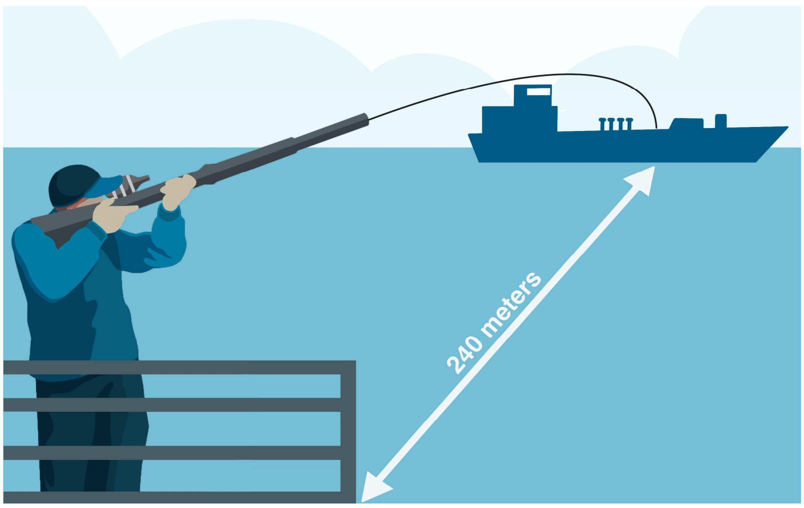

Fig. 1. Marine rescue.

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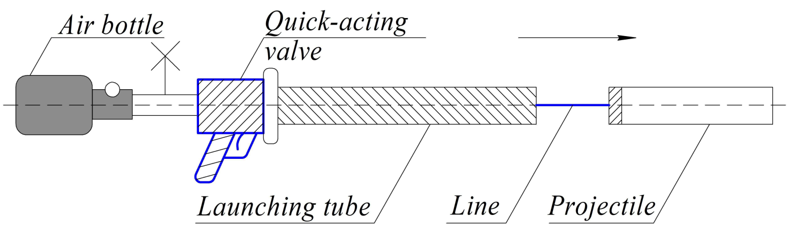

Fig. 2. Line thrower scheme.

The dynamics of mechanical systems with variable mass is considered in [1, 2]. Some problems of the stability of flying bodies are considered in [3, 4]. The classical Meshchersky equation for a body with variable mass is presented in [5, 6] by a variational statement which greatly expands the application possibilities. The study of dynamics of rotating bodies with increasing mass allows one to apply the approaches to rotors and drums, cf. [7]. The dynamics of body after separation of its part as well as the research results are explored and summarized in [8, 9]. The problem of dynamics of a falling chain is considered in [10, 11]. The Lagrange equations are used in [12, 13] for solving the Cambridge problem of the classical falling chain. The generalized canonical Hamilton equations are used to describe the dynamics of a body of variable mass in [14].

The present paper addresses the application of Lagrange equations to construct a mathematical model for the flight of a projectile with a thin line leaving it. The work continues the research begun by the authors [15]. In addition to analyzing the flight of line thrower projectile, the tensile strain arising in the line during flight was calculated. In this regard, we acknowledge the works by Fabio Casciati and Lucia Faravelli devoted to dynamics of the tensioned cables of cable-stayed bridges [16, 17] and the estimation of tension [18].

2. Mathematical Statement

The container is considered to be a material point connected with an elastic line. Model takes into account the tensile strain of the line. The part of a line stretched out of the projectile is considered as an elastic thread consisting of material particles with translational degrees of freedom. For marking each particle of the thread, the material (Lagrangian) coordinate s is introduced in the unstressed reference configuration. The thread motion is determined by the position vector $\boldsymbol{r}(s,t)$, whereas the inertial property of the thread is determined by the linear mass $\rho(s,t)$.

Let us now consider the scheme for calculating the flight of the projectile and the line in the vertical plane, see Fig. 3. The origin $s = 0$ of the curvilinear coordinate is taken at the projectile. The front end of the launcher is characterized by $s=\sigma(t)$, thus, this function determines the length of the line stretched out of the flying projectile. Let us introduce Cartesian coordinates x and y.

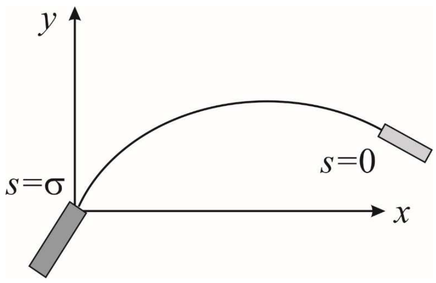

Fig. 3. Flight of the line thrower projectile in vertical plane.

Equations of flight of a body thrown with initial velocity v_0 at angle γ to the horizon describe a parabola

$$\begin{aligned} x &= v_0 t\cos\gamma, \\ y &= v_0 t\sin\gamma - gt^2/2 = \tan\gamma\, x - gx^2/2v_0^2\cos^2\gamma. \end{aligned} \tag{1}$$

This trajectory is taken an approximation to the actual configuration of the system consisting of the projectile and the line:

$$x(s,q_i) = \sigma - s, \quad y(s,q_i) = \alpha(\sigma - s) - \beta(\sigma - s)^2, \quad i = 1,2,3. \tag{2}$$

Three generalized coordinates $q_1 = \sigma, q_2 = \alpha, q_3 = \beta$ are introduced here [19]. They are the sought-for functions of time. This assumption implies that the deformed line has the shape of a parabola whose coefficients are time-dependent.

The adopted approximation (2) is best suited for describing flat shooting characterized by low initial angle of the projectile and the small curvature of the trajectory.

The expressions for the kinetic and potential energies are needed for Lagrange equations [20]. The kinetic energy of the projectile and the line is given by

$$K = \frac{1}{2}\left(\rho\int_0^{\sigma} \boldsymbol{v}^2 ds + m_p \boldsymbol{v}_p^2\right), m_p = m_0 + \rho\left(l - \sigma(t)\right), \tag{3}$$

where $\boldsymbol{v}$ and $\boldsymbol{v}_p = \boldsymbol{v}(0,t)$ denote velocity of the line particles and the projectile, respectively, and m_p stands for the projectile mass. The velocity vector is determined in terms of the position vector of the line particles

$$\boldsymbol{v} = \sum_i \frac{\partial \boldsymbol{r}}{\partial q_i}\dot{q}_i, \quad \boldsymbol{r} = \boldsymbol{r}(s,t), \quad 0 \le s \le \sigma(t). \tag{4}$$

The potential energy is an additive function, that is $\Pi = \Pi_g + \Pi_\varepsilon$. The first term is due to the gravity

$$\Pi_g(q_i) = g\left(\rho\int_0^\sigma y(s,q_i)ds + m_p y(0,q_i)\right) \tag{5}$$

The second one is associated to stretching the line

$$\Pi_\varepsilon(q_i) = \frac{B}{2}\int_0^\sigma \varepsilon^2(s,q_i)ds, \tag{6}$$

where B denotes the tensile stiffness and ε is the Cauchy tensile strain

$$\varepsilon = \frac{1}{2}\left(\left|\frac{d\boldsymbol{r}}{ds}\right|^2 - 1\right) \tag{7}$$

rather than the linear tensile strain $\varepsilon_{lin} = |d\boldsymbol{r}/ds| - 1$, cf. [16].

The Lagrange equations read as

$$\begin{aligned} &K(q_i,\dot{q}_i) = \frac{1}{2}\sum_{i,k} m_{ik}(q)\dot{q}_i\dot{q}_k \\ &\frac{\partial K}{\partial \dot{q}_i} = \sum_k m_{ik}\dot{q}_k, \quad \frac{\partial K}{\partial q_i} = \frac{1}{2}\sum_{n,k}\frac{\partial m_{nk}}{\partial q_i}\dot{q}_n\dot{q}_k \\ &\left(\frac{\partial K}{\partial \dot{q}_i}\right)^{\cdot} = \sum_k m_{ik}\ddot{q}_k + \sum_{n,k}\frac{\partial m_{ik}}{\partial q_n}\dot{q}_n\dot{q}_k - \frac{\partial K}{\partial q_i} + \frac{\partial \Pi}{\partial q_i} = Q_i, \qquad i = 1,2,3. \end{aligned} \tag{8}$$

The projectile is subject to quadratic drag from air resistance, the drag force is given by:

$$\boldsymbol{F} = -k|\dot{\boldsymbol{r}}|\dot{\boldsymbol{r}}, \qquad k = \frac{1}{2}CS\eta, \tag{9}$$

where C is the coefficient of frontal air resistance, S is area of the projectile mid-section and η denotes the density of air, cf. [21, 22].

The air drag of the projectile (9) yields the following generalized forces:

$$\begin{gathered} \delta A = \boldsymbol{F}\cdot\delta\boldsymbol{r}_p \equiv Q_1\delta\sigma + Q_2\delta\alpha + Q_3\delta\beta, \\ Q_1 = -k|\boldsymbol{v}_p|[v_{px} + v_{py}(\alpha - 2\sigma\beta)], \quad Q_2 = -k|\boldsymbol{v}_p|v_{py}\sigma, \quad Q_3 = k|\boldsymbol{v}_p|v_{py}\sigma^2, \end{gathered} \tag{10}$$

where the subscripts x and y denote the corresponding projections.

The derived equations need the initial conditions for $q_i, \dot{q}_i$. At the initial moment of time t = 0 the container with the line flies out from the origin of the coordinate frame with the speed v_0 at angle γ to the horizon

$$t = 0: \qquad x = 0,\ y = 0,\ \dot{x} = v_0\cos\gamma,\ \dot{y} = v_0\sin\gamma. \tag{11}$$

Taken the adopted approximation (2), these conditions are satisfied if $\sigma(0) = 0, \quad \dot{\sigma}(0) = v_0\cos\gamma, \alpha(0) = \tan\gamma$. Comparing the dependence y (x) of the trajectories of the free body (1) and the container (2), it is reasonable to assume that $\beta(0) = g/2v_0^2\cos^2\gamma$. The remaining initial conditions are taken as follows: $\dot{\alpha}(0) = 0, \dot{\beta}(0) = 0$. Numerical calculations showed that the nontrivial values of $\dot{\alpha}(0), \dot{\beta}(0)$ change only the initial part of the flight trajectory rather than the overall flight distance.

3. Numerical Calculation

The solution of the problem was found numerically by the Mathematica package using the “ExplicitRungeKutta” method with the “StiffnessSwitching” option [23]. Calculations have been performed with the parameters $m_0 = 0.5$ kg, $\rho l = 0.3$ kg, S = 20 cm^2, C = 0.8, $\eta = 1.2$ kg/m^3, initial velocity $v_0 = 84$ m/s. The flight range of the projectile versus the initial angle γ is plotted in Fig. 4 for several values of the tensile stiffness of the line.

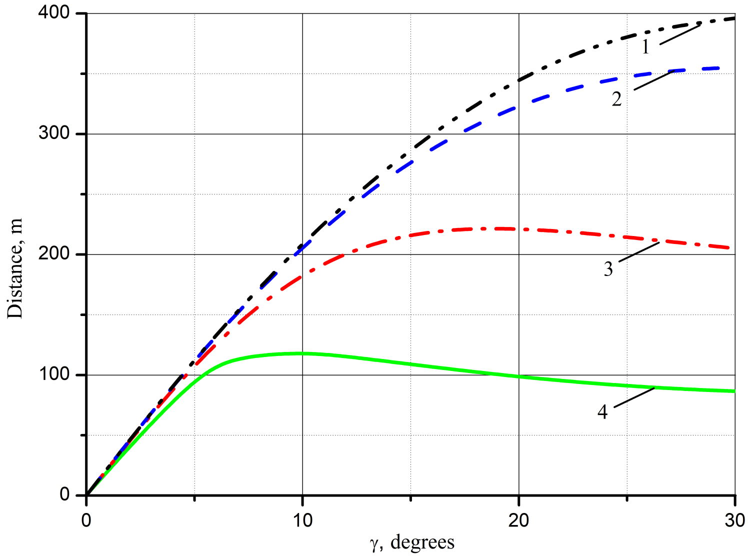

Fig. 4. The flight range vs. the initial angle for some values of the line rigidity $B = 0$ N (curve 1), $B = 100$ N (curve 2), $B = 10^3$ N (curve 3), $B = 10^4$ N (curve 4).

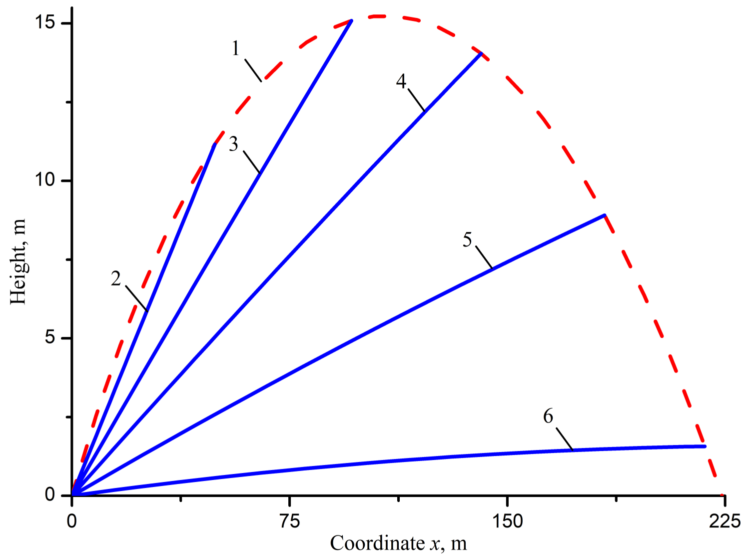

Fig. 5. The projectile trajectory (1) and the line position at some time instants of the flight: $t = 0.2T$ (curve 2), $t = 0.4T$ (curve 3), $t = 0.6T$ (curve 4), $t = 0.8T$ (curve 5), $t = 0.97T$ (curve 6), $T = 3.1$ sec.

Analyzing the graphs, we can draw the following conclusion: the lower the tensile stiffness of line, the greater the flight range of projectile. When designing a line thrower, one needs to find a compromise between the required line stiffness and the flight distance.

One sees that angle of the projectile departure ensuring the maximum range of flight increases as rigidity B decreases. The values of these optimal angles are summarized in Table 1 for some values of the line rigidity along with the corresponding values of the flight range.

The line may break during the flight, thus it is necessary to estimate the line strain. Inserting the adopted approximation (2) in eq. (7) we obtain the following expression for the tensile strain of the line

$$\varepsilon(x,t) = \frac{1}{2}\left[2\beta(t)x - \alpha(t)\right]^2. \tag{12}$$

Table 1. Optimal angle and flight range

Line rigidity B (N)	Optimal angle γ (degree)	Flight range (m)
0	30	396
100	30	357
10^3	18	224
10^4	9	119

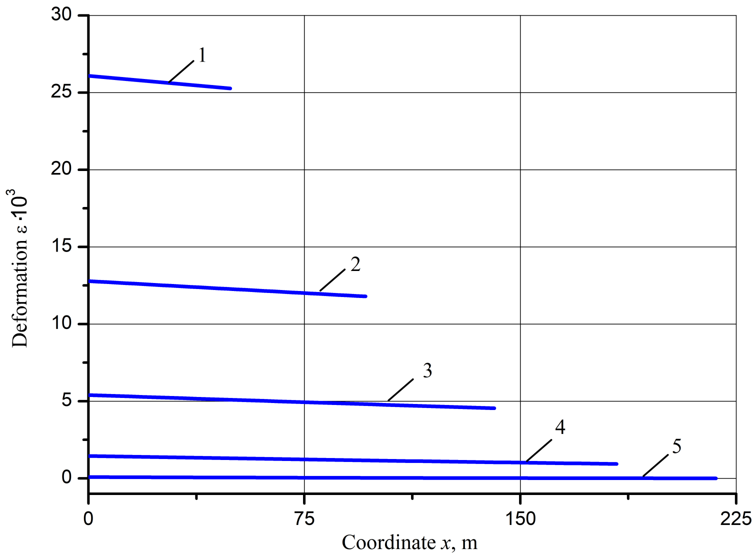

Fig. 6. Tensile strain of the line at some time instants of the flight:
t = 0.2T (curve 1), t = 0.4T (curve 2), t = 0.6T (curve 3), t = 0.8T (curve 4), t = 0.97T (curve 5), T = 3.1 sec.

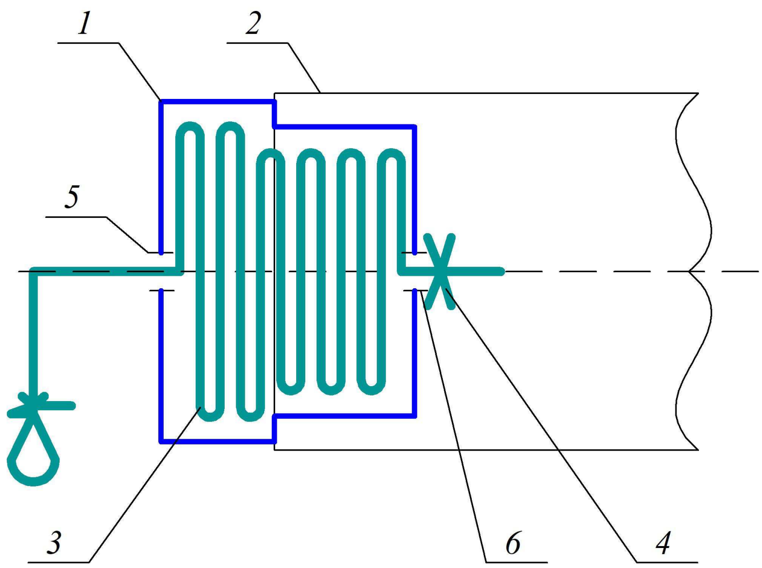

Fig. 7. Layout of a free line section in a throwing container:
1 – container cap, 2 – container, 3 – freely laid line, 4 – knot, 5 – outlet of the cap, 6 – inlet of the cap.

Let us obtain the strain values at different time instants for the line with rigidity $B = 10^3$ N at the departure angle $\gamma = 18^\circ$. In this case the overall flight time of the projectile is T = 3.1 seconds. Figure 5 displays the deformed state of the line at some time instants while Fig. 6 shows the tensile strain as functions of x coordinate for the same time instants.

One can see from the graphs that the tensile strain in the line is always the greatest at the launcher (the origin of coordinates) and reaches the maximum value at the moment of launch.

4. Model Refinement

At the time instant of the shot, significant pressure is exerted on the end of container in the line thrower barrel, and this can cause the line to break. To avoid this, a cap is placed into the projectile, which a line is laid into and drawn through the outlet (Fig. 7). The outlet diameter is less than the line diameter, this is, pulling the line causes some resistance.

Length L_* of the line in the cap should be sufficient to prevent exit of the container cap in the barrel as it causes jamming and breakage. During the flight, the knot on the line pulls the cap and then the remaining line is unwound without any resistance.

To calculate the container flight it is enough to make minimal changes to the developed model. We find the time instant t_1 of the cap breakaway from the condition that the length of the unwound part of the line is equal to the prescribed value $\sigma(t_1) = L_*$. In equations of Section 2 we insert use the line stiffness as the function of time:

$$B(t) = \begin{cases} B_1, & t \le t_1 \\ 0, & t > t_1 \end{cases}, \tag{13}$$

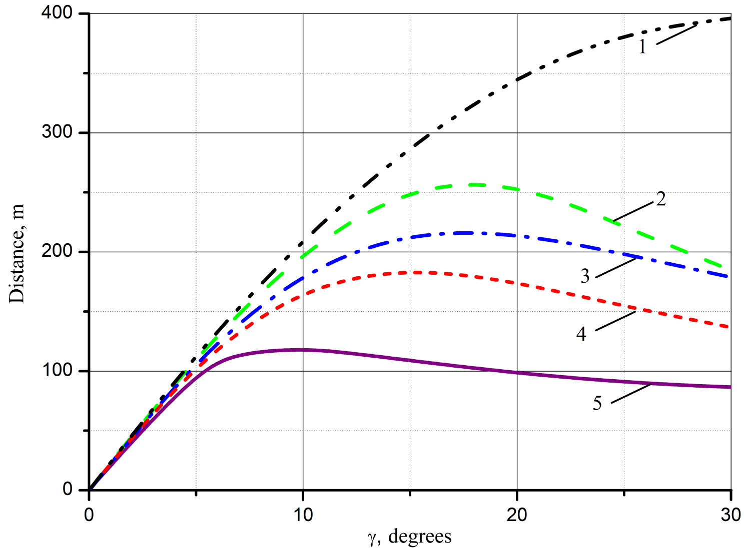

Fig. 8. The flight range vs. the initial angle for some values of the line length in bung $L.$ = 0 m (curve 1), $L.$ = 2 m (curve 2), $L.$ = 6 m (curve 3), $L.$ = 10 m (curve 4), $L.$ equals the total line length (curve 5).

where B_1 is the line stiffness before the cap breakaway.

The refined dependences of the flight range on shot angle of the line thrower are shown in Figure 8 for several values of the length $L.$ of the line laid in the cap. The line stiffness is B_1=10^4 N.

Obviously, all the curves lie between curve 1 which corresponds to a line with zero rigidity and curve 5 which corresponds to the case when the entire line is completely laid in a cap (this is curve 4 from Fig. 4).

Table 2 shows the calculated values of the optimal shot angles providing the maximum flight range for the model with a cap.

Our calculations confirmed the parameters which have been experimentally determined for the line thrower: the best technically possible value of length $L.$ is 2 meters, and the maximum flight range of 265 m is provided at the shot angle of 18 degrees.

Table 2. Optimal angle and flight range in model with a cap

Length $L.$ (m)	Optimal angle γ (degree)	Flight range (m)
2	**18**	**265**
6	17	220
10	15	187

5. Conclusion

Mathematical modeling of the flight of a line thrower projectile is carried out by means of the Lagrange equations of the second kind. The solution to the Cauchy problem is found numerically using Mathematica package. The dependence of range of the projectile flight on the angle of projectile departure is found. The tensile stiffness of the line was shown to affect significantly the flight range. The results were elaborated for the container with a cap. The resulting equations can be applied to the design and use of marine rescue equipment.

Author Contributions

Alexander K. Belyaev initiated the project and set a mathematical problem; Vladimir A. Piskunov developed mathematical modeling; Tatiana V. Zinovieva performed numerical calculations. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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References

[1] Belyaev, A.K., Irschik, H., *Dynamics of mechanical systems with variable mass*, Springer Wien Heidelberg New York Dordrecht London, 2014.
[2] Eke, F.O., Mao, T.C., On the dynamics of variable mass systems, *International Journal of Mechanical Engineering Education*, 30 (2), 2002, 123-137.
[3] Belyaev, A.K., Irschik, H., Krommer, M., *Mechanics and model-based control of advanced engineering systems*, Springer Wien New York, 2014.
[4] Pesce, C.P., Tannuri, E.A., Casetta, L., The Lagrange equation for systems with mass varying explicitly with position: some applications to offshore

engineering, *Journal of the Brazilian Society of Mechanical Science and Engineering*, 28, 2006, 496-504.
[5] Casetta, L., Pesce, C.P., The inverse problem of Lagrangian mechanics for Meshchersky's equation, *Acta Mechanica*, 225, 2014, 1607-1623.
[6] Casetta, L., Pesce, C.P., A brief note on the analytical solution of Meshchersky's equation within the inverse problem of Lagrangian mechanics, *Acta Mechanica*, 226(7), 2015, 2435-2439.
[7] Cveticanin, L., Kovacic, I., On the dynamics of bodies with continual mass variation, *Journal of Applied Mechanics*, 74, 2007, 810-815.
[8] Cveticanin, L., Djukic, Dj., Dynamic properties of a body with discontinual mass variation, *Nonlinear Dynamics*, 52, 2008, 249-261.
[9] Cveticanin, L., *Dynamics of Bodies with Time-Variable Mass*, Springer, 2015.
[10] Calkin, M.G., March R.H., The dynamics of a falling chain, *American Journal of Physics*, 57(2), 1989, 154-157.
[11] Calkin, M.G., March R.H., The dynamics of a falling chain: II, *American Journal of Physics*, 57(2), 1989, 157-159.
[12] Irschik, H., Holl, H.J., The equations of Lagrange written for a non-material volume, *Acta Mechanica*, 153, 2002, 231-248.
[13] Irschik, H., Holl, H.J., Mechanics of variable-mass systems – part 1: balance of mass and linear momentum, *Applied Mechanics Review*, 57(2), 2004, 145-160.
[14] Casetta, L., Pesce, C.P., On the generalized canonical equations of Hamilton for a time-dependent mass-particle, *Acta Mechanica*, 223(12), 2012, 2723-2726.
[15] Chulkin S.G., Zinovieva T.V., Piskunov V.A., Fomichov A.B., Influence of design parameters on the flight distance of shell of marine pneumatic life saving line-thrower (in Russian), *Marine Intellectual Technologies*, 1(43), 2019, 60-68.
[16] Faravelli, L., *Cable-Stayed Bridges: A Monitoring Challenge*, Dynamics and Control of Advanced Structures and Machines, 2017, 161-168.
[17] Faravelli, L., Ubertini, F., Nonlinear State Observation for Cable Dynamics, *Journal of Vibration and Control*, 15(7), 2009, 1049-1077.
[18] Casciati, F., Casciati, S., Elia, L., and Faravelli, L., *Tension estimates in cable stayed bridges*, International Conference on Performance-based and Life-cycle Structural Engineering, Brisbane, QLD, Australia, 9-11 December 2015, Brisbane, QLD, Australia: School of Civil Engineering, The University of Queensland, 2015.
[19] Lurie, A.I., *Analytical mechanics*, Springer-Verlag, Berlin-Heidelberg, 2002.
[20] Eliseev, V.V., *Mechanics of Deformable Solids (in Russian)*, St. Petersburg, Polytechnic University Press, 2003.
[21] Loitsyanskii, L.G., *Mechanics of liquids and gases*, Pergamon Press, 1966.
[22] Aleshkevich, V.A., Dedenko L.G., Karavaev V.A., *General physics course. Mechanics (in Russian)*, Fismathlit, 2011.
[23] Borwein, J.M., Skerrit, M.B., *An introduction to modern mathematical computing: with Mathematica*, Vol. XVI, Springer, 2012.

ORCID iD

Alexander K. Belyaev https://orcid.org/0000-0002-5934-8138
Vladimir A. Piskunov https://orcid.org/0000-0002-0834-4671
Tatiana V. Zinovieva https://orcid.org/0000-0002-7953-6849

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