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Research Paper

# Predictive Control with Dynamic Hysteresis Reference Trajectory: Application to a Structural Base-Isolation Model

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**Abstract.** Over the last decades, in the field of control engineering, Model Predictive Control (MPC) has been successfully employed in many industrial processes. This due to, among other aspects, its capability to include constrains within the design control formulation and also its ability to perform on-line optimization. For instance, in the civil engineering field, different MPC approaches have been well developed to formulate active control algorithms able to reduce civil structural responses to earthquakes. Thus, in this paper, a customized version of a conventional Predictive Control (PC) strategy is proposed to mitigate the displacement on a base-isolated system with a nonlinear hysteresis behavior, that is excited by a seismic event. The proposal consists of including a dynamic hysteresis system into the control scheme to generate a reference trajectory that will softly drive the base-isolated structure to a rest status. The proposed control scheme is evaluated through numerical experiments, and then its performance is compared with respect to the conventional Predictive Control methodology. According to the numerical experiments, the approach here presented results more efficient than the conventional method due to the use of a suitable linear model of the structural system plus a new Driver Block with dynamic hysteresis within the Predictive Control scheme.

**Keywords**: Predictive Control, Dynamic reference trajectories, Hysteresis, Base-isolated structure.

## 1. Introduction

In the midst of the last decades, Model Predictive Control has considerably evolved and it has been applied to a wide range of engineering systems in the industrial field [1] [2] [3]. The usefulness of this control technique is now more remarkable due to the constant improvements in low-cost computer processors that are more efficient and faster as ever, allowing to solve complex control algorithms with less computational resources [4]. This control approach has mathematical aspects that makes it very attractive when choosing a control methodology, suitable to be applied in many industrial systems. Moreover, it is a simple understanding technique. On the overall view, the conventional MPC strategy requires three basic elements: 1) a linear mathematical model of the process to predict its future behavior, 2) an optimization strategy, and 3) the usage of a finite prediction horizon [5] [6]. Finally, this control methodology is established in a discrete-time formulation, which gives the possibility of computing the control signal and predicting the future evolution of the process output in real time [6].

The main advantages of this control technique are as follows: 1) it provides a good control performance to systems with complex dynamics (such as civil structures), or systems that are unstable or multivariable in nature [7], and 2) it provides an anticipated control action that may compensate external perturbations. On the other hand, this control methodology has the ability to handle both soft and hard constraints in the control design phase, which is particularly important in industrial applications where tight profit limits on the process operation are inevitably present [7]. Nevertheless, the MPC algorithms have some drawbacks, such as the complexity of designing and solving the optimization problem, mainly when the dynamic of the process changes with the time [2]. Moreover, the need of an appropriate process model may result in a notable expense of mathematical resource. Finally, another common disadvantage is that the stability cannot always be theoretically proved [5].

Nonetheless, even considering those disadvantages, in practice, Predictive Control techniques have demonstrated to be reasonable strategies that have been successfully applied in a large amount of industrial automatized processes. For instance, in the robotics area, MPC has been mostly employed along with vision-based techniques [8] [9] [10]. Meanwhile, in the health care field, this control strategy has been notably invoked [11], such as in the applications of clinical anesthesia or the management of diseases as diabetes [12] [13] [14]. Furthermore, in the automotive branch, MPC has also excelled. For instance, in autonomous operation for self-operated intelligent vehicles [15] [16]. On the other hand, in the power electronics field, the Predictive Control technique has been successfully used to control power converters and machine drives [17]. In the renewable power energy subject, especially in photovoltaic control applications, a Maximum Power Point Tracking is combined jointly to a Predictive Control for the DC-DC converter oriented to improve the efficiency production of the electrical energy provided by a photovoltaic

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panel [18] [19]. Finally, another application field where the MPC has highlighted is in the vibration structural control [20]. Specifically, different approaches have been granted to mitigate vibrations in civil structures subjected to external undesired ground perturbations [3] [21] [22] [23] [24].

In this paper, a Predictive Control strategy is proposed to mitigate vibrations on a hysteretic base-isolated system generated mainly by earthquakes. In the field of civil engineering and science, a lot of research contributions have been reported, most of them are related to detect earthquakes and reduce the negative impact that they cause on civil structures [25] [26]. In structural civil engineering, the hysteresis base-isolated mechanism is one of the most implemented and accepted seismic protection device against earthquakes [25]. In some cases, it is mathematically described by the hysteresis Bouc-Wen model [26]. Since the system is nonlinear, it is an important challenge for the control engineering community. Recently, most of the controllers in the state-of-art designed for this kind of systems are based on nonlinear control techniques, which in most cases, result too complex to follow. For instance, some of these are based on the well-known sliding mode control scheme, or on the adaptive control methodology [21] [27] [28] [29]. Hence, in the Predictive Control approach here presented, a simple and effective control strategy based on a modified version of the conventional PC scheme is developed. This strategy solves the control objective of mitigating the vibration of a hysteretic base-isolated model when it is subjected to a seismic perturbation. The main contribution is that a dynamic hysteresis model is employed to generate a reference trajectory to smoothly drive the structure response to its equilibrium status. This new trajectory is programmed within the Driver Block of the Predictive Control scheme. Hence, this block requires information from the current structure response, which is used, in real time implementation, as the initial condition to solve this reference model equation. According to the predictive control philosophy, this desired trajectory is recalculated at every sampling time over a finite-time prediction horizon. In addition, an optimal control problem is formulated to minimize an on-line linear quadratic index and by solving two Ricatti equations [30].

Finally, and according to the numerical simulations provided here, the proposed approach significantly improves the control performance when it is compared to the conventional Predictive Control technique. Hereinafter, Section 2 describes the modelling of the base-isolation structure for the control purposes. Section 3 presents the construction of the overall control scheme along with the proposed hysteretic system for the generation of the reference trajectory. In Section 4, the numerical simulations are implemented to evaluate the Model Predictive Control performance under two well-known seismic perturbations. Moreover, the control technique is validated for the case of saturation in the control signal, which is a very typical issue in the control area. Finally, Section 5 highlights a discussion of the results and gives some conclusions.

## 2. System Modelling and Problem Statement

As a case study, it is considered the typical civil engineering base-isolated scheme illustrated in Fig. 1. First, Fig. 1(a) represents one of the many applications that the base-isolated system may have. However, the structure over the base may be of any kind, such as bridges, sculptures, wind turbines, communication antennas, and so on. In this paper it is invoked the simplified version of the base-isolated device which considers an important mass connected to the base. This simplified configuration is shown in Fig. 1(b). Finally, Fig. 1(c) represents the physical schematic of the base-isolated device. Thus, the equation that represents the system to be controlled is given by [21]:

$$m\ddot{y}(t) + c\dot{y}(t) + k_1 y(t) + \phi(y)(t) = f(t) + u(t) \tag{1}$$

where $m$ is the mass of the base, $c$ is the viscous damping, $k_1$ is the stiffness coefficient, and $\phi$ is the hysteretic restoring force behavior of the isolator material. Additionally, $y(t)$ is the relative displacement of the base mass with respect to the position of the ground [30], $u(t)$ is the control law, and $f(t) = -ma(t)$ is the external perturbation force, where $a(t)$ is the earthquake ground acceleration. In the system (1), the restoring force $\phi$ is given by the hysteretic-isolator that is captured by the so-called Bouc-Wen model [29]. This model is extensively invoked to describe the hysteresis phenomenon in smart and civil engineering structures [26]. It is represented by a nonlinear first-order differential equation that relates the input displacement to the output restoring force in a rate independent hysteretic manner as follows:

$$\begin{aligned} \phi(y)(t) &= \alpha k_0 y(t) + (1-\alpha) D_i k_0 z(t) \\ \dot{z}(t) &= D_i^{-1}\left(A_i \dot{y}(t) - \beta\,|\dot{y}(t)|\,|z(t)|^{n-1} z(t) - \gamma \dot{y}(t)\,|z(t)|^{n}\right) \end{aligned} \tag{2}$$

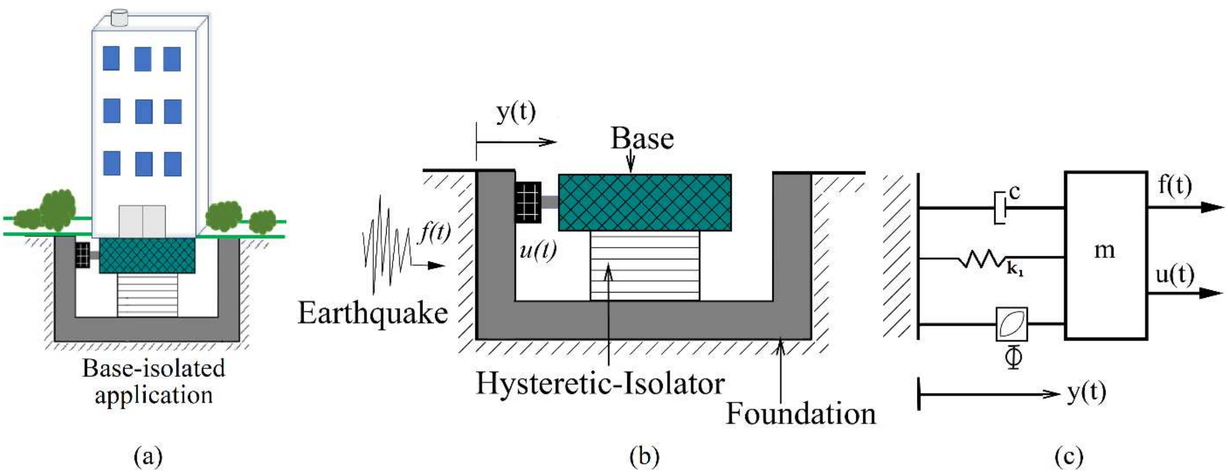


**Fig. 1.** (a) One base-isolated application. (b) Simplified configuration of the base-isolated system. (c) The mathematical model.

where $\alpha k_0 y(t)$ is the elastic component and $(1-\alpha)D_i k_0 z(t)$ is the hysteresis component [28] [30]. Additionally, $D_i > 0$ is the yield constant displacement, and $0 < \alpha < 1$ is the post-to pre-yielding stiffness ratio; $A_i, \beta$ and $\gamma$ are non-dimensional parameters that control the shape and size of the transition from the elastic response [31]. Additionally, it is assumed that $n > 1$, $k_0 > 0$ and $\beta + \gamma < 0$ [26] [32]. Furthermore, the hysteretic component involves a non-dimensional auxiliary internal variable $z(t)$ which is the solution of the non-linear differential equation (2) and this is a non-measurable variable. Finally, in this statement, it is assumed that the displacement $y(t)$ and velocity $\dot{y}(t)$ are measurable variables and the control input $u(t)$ is to be designed by accomplish the PC control objective. This is, to regulate the displacement around its rest position while maintaining the boundedness of the closed-loop state variables and reducing the vibration of the base-isolator.

In accordance with the control objective above mentioned, it is first required to adequately stablish a linear model that captures the main dynamical behavior of the actual hysteresis-isolation device. Hence, the proposed linear model to be employed includes an extra variable that works as a dissipative operator. The main objective of this extra dynamic variable is to incorporate the action of the Bouc-Wen restoring force within the linear model, and hence, within the PC structure. In this way, the controller will work in harmony with the hysteresis base-isolation. This is one of the key ideas of the proposed PC conceptualization. In other words, this was done with the intuition that the nonlinear part of the base-isolated system collaborates with the controller in the closed-loop system instead of seeing this non-linearity as a perturbation to fight. Thus, the proposed linear model is as follows:

$$\begin{aligned} &m\ddot{y}(t) + c\dot{y}(t) + k_1 y(t) + (1-\alpha)D_i k_0 z_i(t) = u(t) \\ &\dot{z}_i(t) = A_i \dot{y}(t) + \alpha k_0 y(t) \end{aligned} \tag{3}$$

Furthermore, a state-space model of the equation (3) can be built using the state variables $x_1 = y$, $x_2 = \dot{y}$ and $x_3 = z_i$, being the displacement, the velocity and a not-measurable auxiliar variable, respectively. Using these variables, the state model is yield as:

$$\begin{aligned} &\dot{x}_1(t) = x_2(t) \\ &\dot{x}_2(t) = \frac{-c}{m} x_2(t) - \frac{k_1}{m} x_1(t) - \frac{(1-\alpha)D_i k_0}{m} x_3(t) + \frac{u(t)}{m} \\ &\dot{x}_3(t) = A_i x_2(t) + \alpha k_0 x_1(t) \end{aligned} \tag{4}$$

It is worth to note that $z_i(t)$ is a fictitious variable. It means it does not have a physical interpretation and it is not measurable. However, for practical implementation purposes, it may be calculated trough the physical and measurable process variables: position and velocity. Finally, observe that the system in (3) gives an estimation of the internal variable of the Bouc-Wen state variable $z(t)$.

## 3. Predictive Control Formulation

In this section the Predictive Control formulation stated in [23] is recapitulated (see Fig. 2). The main idea of this control methodology is to use a model of the process to predict and optimize its future behavior along with a Driver Block for a self-generated reference signal, and a suited predictive control law. This optimization is made in the finite-time horizon by using a sample of the process output. Thus, a complete set of the predictive control signal in discrete-time domain is obtained, however, just the first element of this signal is employed to feed the process for control. This routine is then repeated at each sampling-time period [6].

In order to implement the Predictive Control strategy, the block diagram in Fig. 2 is followed. The Predictive Model is used to compute the predictive control signal produced at each sampling time. The resultant control signal should make the predicted output process follows a desired output trajectory as well as possible. The Driver Block is designed to self-generate this desired reference trajectory, which is calculated at every sample time by using information from the current output process (state-feedback) and the user set-point. Hereinafter, and for the position control objective, this set-point is assumed zero for a regulation control performance. Note that the process block contains the real nonlinear plant to control, which actually is the system represented by equations (1) and (2). Since Predictive Control is based on a discrete-time realization, the Predictive Model can be formulated by means of a discrete-time state notation as follows [23]:

$$\begin{aligned} &\hat{x}(k+j\,|\,k) = A\hat{x}(k+j-1\,|\,k) + B\hat{u}(k+j-1\,|\,k) \\ &\hat{y}(k+j\,|\,k) = C\hat{x}(k+j\,|\,k) \end{aligned} \tag{5}$$

where $\hat{x}(k+j\,|\,k)$ and $\hat{y}(k+j\,|\,k)$ are the state vector and the *virtual* process output predicted over a finite time-horizon $j = 0,1,\ldots,\lambda$ at the instant $k$, respectively, where $\lambda \in \Re^+$. On the other hand, $\hat{u}(k+j-1\,|\,k)$ is the predictive control sequence; and $A, B$ and $C$ are the discrete-time system matrix, the control matrix, and the output matrix, respectively. Additionally, for notation $\hat{x}(k\,|\,k) = x(k)$ and $\hat{u}(k\,|\,k) = u(k)$ where $x(k)$ is the output state vector at the sampling-time, and $u(k)$ is the control signal to be supplied to the process at each sampling-time. Nevertheless, in the optimization routine, the useful information will be $A, B, C$ and $\lambda$.

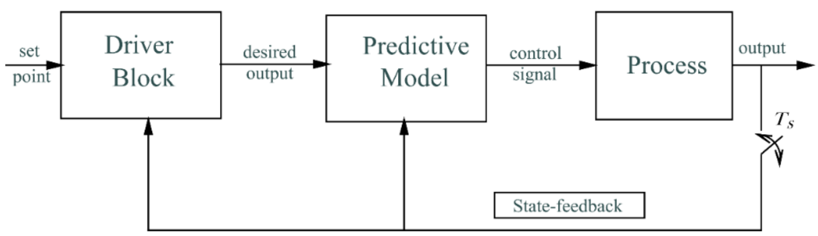


**Fig. 2.** Overall Predictive Control block diagram where $T_s$ is the sampling period.

Going on, and by following the formulation in equation (5), by invoking the well-known first Euler's approximation method, the system in (4) can be written in its discrete-time representation as follows:

$$\begin{aligned} x_1(k+1\,|\,k) &= x_1(k) + T_S x_2(k) \\ x_2(k+1\,|\,k) &= x_2(k) + T_S\left(-\frac{c}{m}x_2(k) - \frac{k_1}{m}x_1(k) - \frac{(1-\alpha)D_i k_0}{m}x_3(k) + \frac{u(k)}{m}\right) \\ x_3(k+1\,|\,k) &= x_3(k) + T_S\left(A_i x_2(k) + \alpha k_0 x_1(k)\right) \end{aligned} \tag{6}$$

where $T_S$ is the sampling-time period, $j = 0,1,\ldots,\lambda$ is the sequence of the finite prediction horizon, and $k$ is the sampling instant at $t = kT_S$ with $k = 0,1,2,\ldots$ .

### 3.1 Performance Criterion and Predictive Control Law

According to the previous presentation, the Predictive Control strategy consists in finding a predictive control sequence that minimizes the linear quadratic cost function over a finite horizon stated as [23]:

$$J = \frac{1}{2}\sum_{j=0}^{\lambda}\left[\hat{y}(k+j\,|\,k) - y_r(k+j)\right]^T Q(j)\left[\hat{y}(k+j\,|\,k) - y_r(k+j)\right] + \frac{1}{2}\sum_{j=0}^{\lambda-1}\hat{u}(k+j\,|\,k)^T R(j)\hat{u}(k+j\,|\,k) \tag{7}$$

where Q and $R$ are the real positive semi-definite and symmetric weighting matrices. Additionally, $y_r(k)$ is the reference trajectory, which is generated by the Driver Block. Therefore, the above equation is the cost function that minimizes the energy applied to manipulate the actuator and the error between the output plant and the reference trajectory through its weight factors Q and R. Furthermore, by following the standard quadratic optimization procedure, the minimization of the linear quadratic function in (7) requires the solution of two Riccati equations [23]. Thus, the first Riccati equation in discrete-time domain is given by [30]:

$$P(k+j) = C^T Q(j) C + A^T P(k+j+1)\left[I + BR^{-1}(j)B^T P(k+j+1)\right]^{-1} A \tag{8}$$

$$P(k+\lambda) = C^T Q(\lambda) C \tag{9}$$

where $j = 0,1,\ldots,\lambda$ is the prediction horizon and $I$ is the identity matrix. Here, matrix $P(k+j)$ is positive-definite and symmetric. Then, $P$ matrix is calculated at every sampling-time $kT_S, k = 0,1,2,\ldots$ . The dimension of this matrix is imposed by the linear system matrices data $A, B$ and $C$ . Additionally, the second Riccati equation to be solved, that incorporates a vector $\eta(k)$ is [23]:

$$\eta(k+j) = C^T Q(j) y_r(k+j) + A^T \eta(k+j+1) - A^T P(k+j+1)\left[I + BR^{-1}(j)B^T P(k+j+1)\right]^{-1} BR^{-1}(j)B^T \eta(k+j+1) \tag{10}$$

$$\eta(k+\lambda) = C^T Q(\lambda) y_r(k+\lambda) \tag{11}$$

The dimension of the vector $\eta$ is governed by the linear system model in discrete-time domain $A, B$ and $C$ , and it is also processed at each sampling-time. Note that both Riccati equations are solved in backwards in discrete-time by appealing to the final conditions (9) and (11), respectively. Hereinafter, Q and R are considered scalars. Then, the matrix and vector sequences that minimize the index are obtained as follows [23]:

$$D(k) = R^{-1}(j)B^T\left(A^T\right)^{-1}\left[P(0) - Q(j)I\right] \tag{12}$$

$$u_0(k) = R^{-1}(j)B^T\left(A^T\right)^{-1}\left[\eta(0) - Q(j)y_r(0)\right] \tag{13}$$

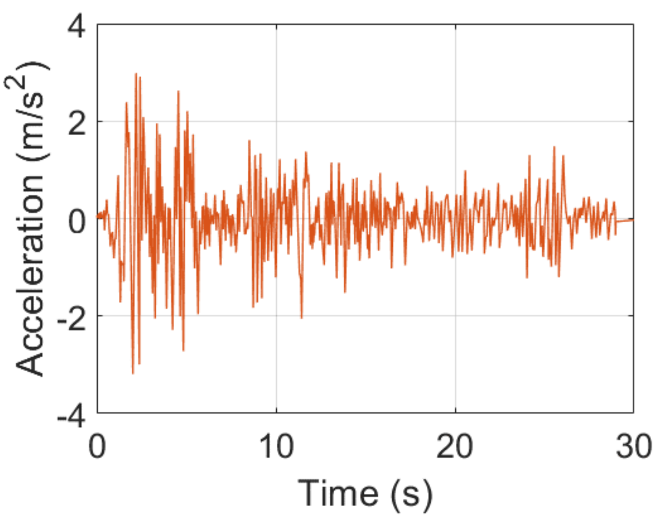


**Fig. 3.** Ground acceleration of El Centro earthquake.

Finally, and by taking into account that $\hat{x}(k|k) = x(k)$ , the following optimal predictive control law yields [23]:

$$u(k) = -D(k)x(k) + u_0(k) \tag{14}$$

Note that equation (14) requires the information of the state vector $x(k)$ at the instant $k$. On the other hand, it is worth mentioning that the control algorithm exposed above solves the optimization problem along with the Driver Block conception, since the desired trajectory is redefined at each sampling instant $k$. Obviously, this Driver Block has to generate the reference vector information $y_r(k+j)$, with $j = 0,1,\ldots,\lambda$ , before to calculate the Riccati equations.

### 3.2 The Driver Block

As was previously stated, the Driver Block builds a reference trajectory $y_r(k+j|k)$, with $j = 0,1,\ldots,\lambda$ that, at each sampling-time, is used to reach a predictive optimal control law $u(k)$. In this manner, the output process smoothly evolves towards the desired setpoint. In this subsection, it is proposed a novel manner to obtain the reference model within the Driver Block. This proposal consists of a hysteresis dynamic system that uses information from the output state vector to solve itself. Hence, the proposed hysteresis dynamic system leads the process output to the desired setpoint in a smoothly bearing and as fast as possible. Moreover, since the reference model is now a discrete-time system with hysteresis, it is intended to work to improve the closed-loop system performance. In accordance with the conventional Predictive Control strategy, the proposal follows the idea of using the current process output, in this case the position variable, as the initial condition. Thus, the hysteresis model reference is stated as follows:

$$\begin{gathered} y_r(j+1|k) = y_r(j|k) + \alpha_{hm}\left[-y_r(j|k) - x_S(j|k) + b_{hm}\,\mathrm{sgn}\left(y_{in}(j|k) + a_{hm}\,\mathrm{sgn}\left(y_{in}(j|k)\right)\right)\right] \\ x_S(j+1|k) = x_S(j|k) + c_{hm}\,\mathrm{sgn}\left(y_{in}(j|k)\right) \\ x_S(0|k) = 0 \end{gathered} \tag{15}$$

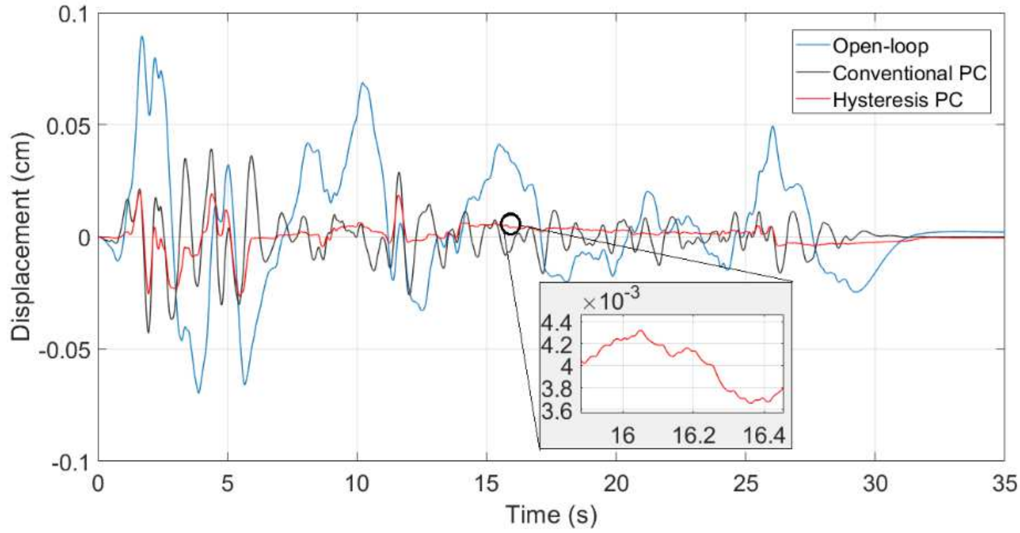


**Fig. 4.** Displacement response with El Centro earthquake.

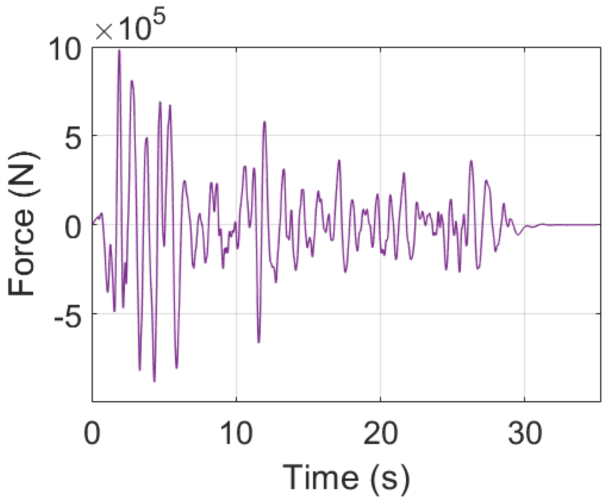


**Fig. 5.** Control signal for the case with the conventional Predictive Control strategy.

where $y_{in}$ is the input to the hysteresis system, $y_r$ is the main internal variable of the system and $x_S$ is a second internal variable that is proposed to work as an integral action to adjust possible offsets at the output of the process. As was mentioned before, the initial condition of the equation in (15) is the position of the base at the instant $k$, $y_r(0|k)=y(k)$. Finally, the input of the system is the output velocity, $y_{in}(0|k)=\dot{y}(k)$. Additionally, $b_{hm}$ represents the hysteretic output gain, $a_{hm}$ set the horizon width of the hysteretic-loop, $c_{hm}$ is the gain of the integral part, and $\alpha_{hm}$ is the constant-transition rate. For more details see [33]. Note that the solution of the equation in (15), $y_r(j)$, is now the dynamic reference trajectory to solve the optimization problem in (7).

## 4. Numerical Simulation Results

In this section, the numerical results are presented to validate the control algorithm and to compare these results with a Predictive Control methodology that employs a conventional reference model. This one-order reference model is given by [3]:

$$\begin{aligned} &y_r(j+1|k) = -0.8y_r(j|k) \\ &j = 0,1,\ldots,\lambda \end{aligned} \tag{16}$$

Furthermore, the system response is analyzed under three scenarios: 1) open-loop (no control signal), 2) the Driver Block with the conventional equation (16), and 3) the Driver Block with hysteresis as was stated in eq. (15). Moreover, to validate the control technique, it is tested with saturation in the control signal, since it is a typical issue in controlled systems. Additionally, two realistic earthquake accelerations are used to validate the control performance.

In the numerical simulations, the following true values for the Bouc-Wen model are set [26] [29] [32]: $m=6\times10^5$ Kg, $k_0=61224.49$ N/m, $c=0.1067\times10^7$ Ns/m, $\alpha=0.6$, $D_i=0.0245$ m, $A_i=1$, $\beta=0.5$, $\gamma=0.5$, $n=3$ and zero initial conditions. On the other hand, the values of the hysteresis equation (15) in the Driver Block are: $\alpha_{hm}=0.08$, $a_{hm}=-1$ and $b_{hm}=0.001$ and $c_{hm}=0.005$. The sampling period time is $T_S=0.01$ seconds and the prediction horizon is $\lambda=6$. Finally, the selected weighting values to solve the two Riccati equations are proposed as scalars $R=0.1\times10^{-4}$, $Q=0.1\times10^{12}$.

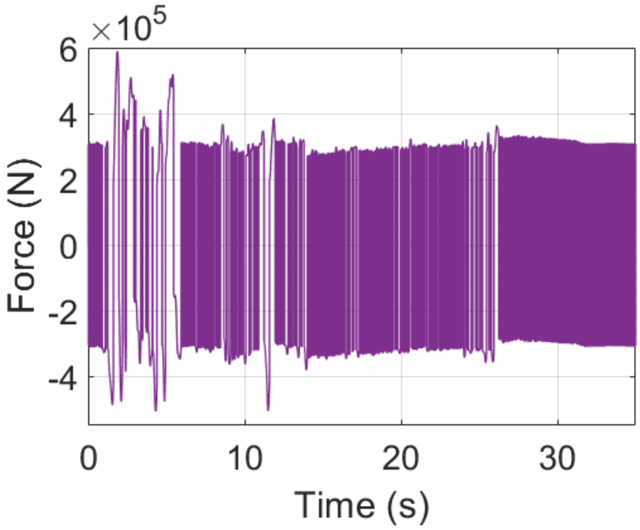


**Fig. 6.** Control signal for the case with the hysteresis Predictive Control strategy.

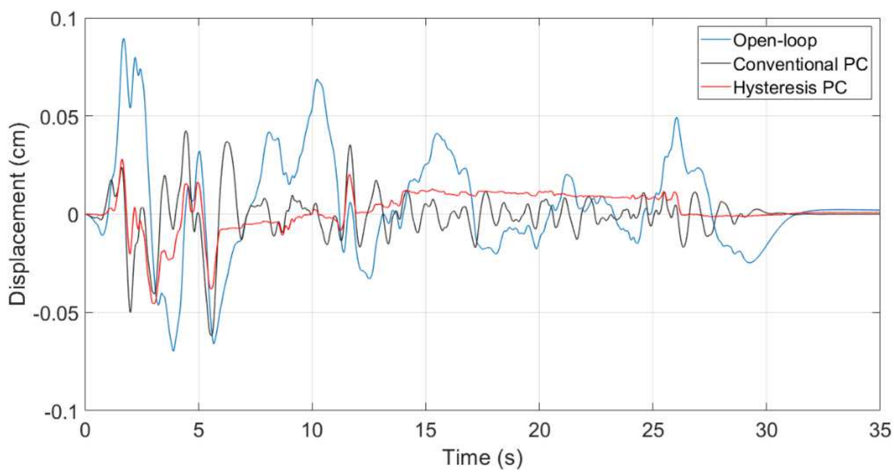


**Fig. 7.** Displacement of the base-isolated system with saturation in the control signal.

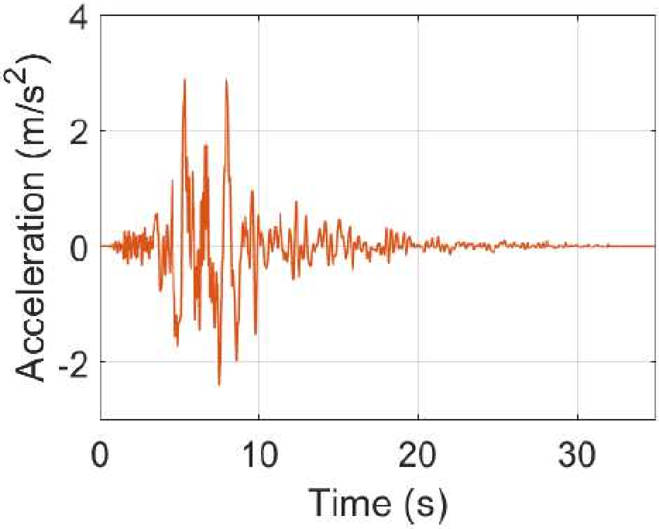


**Fig. 8.** Ground acceleration of the Kobe earthquake.

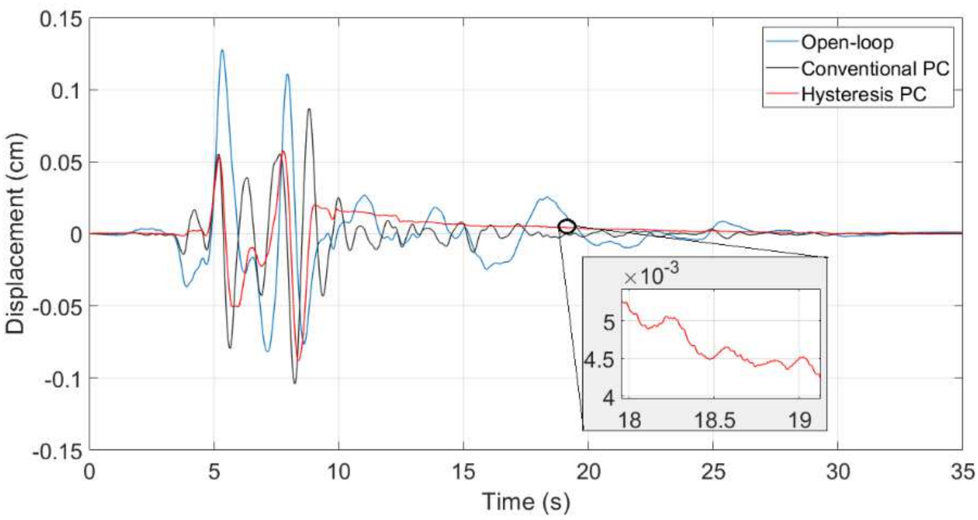


**Fig. 9.** Displacement response with Kobe earthquake.

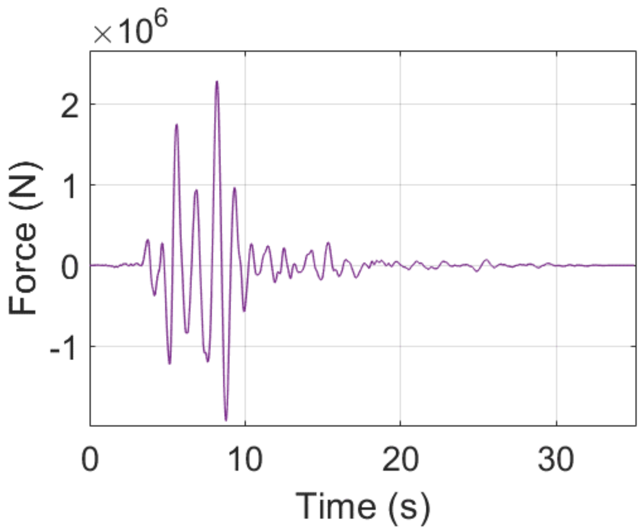


**Fig. 10.** Control signal for the case with the conventional Predictive Control strategy.

The first simulation was carried out by employing the acceleration of the El Centro earthquake as the seismic perturbation shown in Fig. 3 [21]. Then, Fig. 4 shows the displacement behavior of the base-isolated system under the three different scenarios. From here, it is notable that the PC with hysteresis has a better performance since there is an important reduction in the displacement of the base-isolated in comparison to the case with the simple model reference and the open-loop case. Besides, the vibration in the base also was significantly mitigated and the base was more softly guided to the rest position, thus fulfilling the control objective previously stated. In Fig. 4, it is also depicted a zoom version of a piece of the signal obtained with the proposed control technique. Here it is observable that the period of the small amplitude signal is about 0.6 seconds. On the other hand, the control signal obtained with the conventional PC strategy is depicted in Fig. 5, and the control signal with the hysteresis PC is exposed in Fig. 6. Additionally, a simulation with saturation in the control signal is carried out to validate the proposed technique under real scenarios typically presented in engineering systems. To emulate the saturation effect, a *Saturation Block* was inserted in the control signal, the value of it is from $-3\times10^5$ N to $3\times10^5$ N. The results of the control scheme with saturation are presented in Fig. 7 where also it is notable the improvement of the control performance when the hysteresis reference trajectory is invoked.

In order to ensure the performance of the proposed control technique, a second numerical simulation is implemented using the acceleration of the Kobe earthquake as ground perturbation [34]. The graphic that represents the Kobe acceleration is depicted in Fig. 8. Then, the results are exposed in Fig. 9, where similar conclusions from the previous simulation can be drawn. That is, the response of the base-isolated displacement is notable reduced when using the hysteretic reference model in comparison to the other two cases. Additionally, the corresponding control signals for this last numerical simulation are also presented. The control law obtained with the conventional predictive control is shown in Fig. 10. On the other hand, the control signal provided by the controller with hysteresis is shown in Fig. 11. Finally, the results with saturation in the control signal are presented in Fig. 12, where also it is notable the improvement of the technique with hysteresis in comparison with the conventional PC.

From Fig. 6 and Fig. 11 it is observable that the resultant control signal is a kind of modulated train of pulses which is a common behavior of the hysteretical systems. Additionally, this signal has a similarity to the typical control commuting signals in sliding mode control techniques, which has been demonstrated that an adequate high frequency designed signal indeed improves the control performance [29]. Hence, and by noticing all results above presented, it is clear that the proposal works better than the conventional Predictive Control scheme.

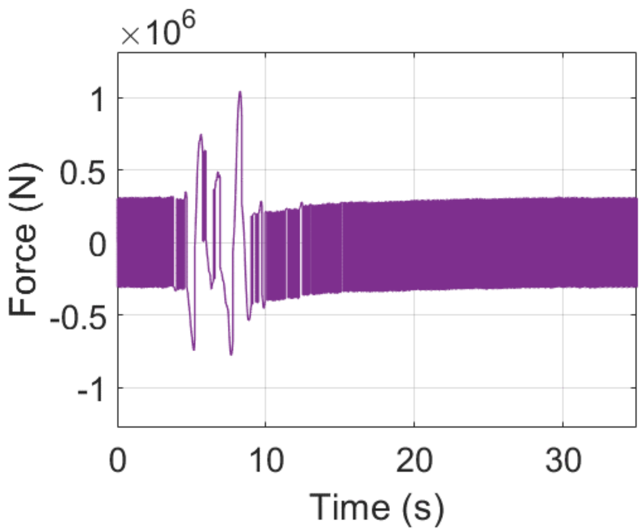


**Fig. 11.** Control signal for the case with the hysteresis Predictive Control strategy.

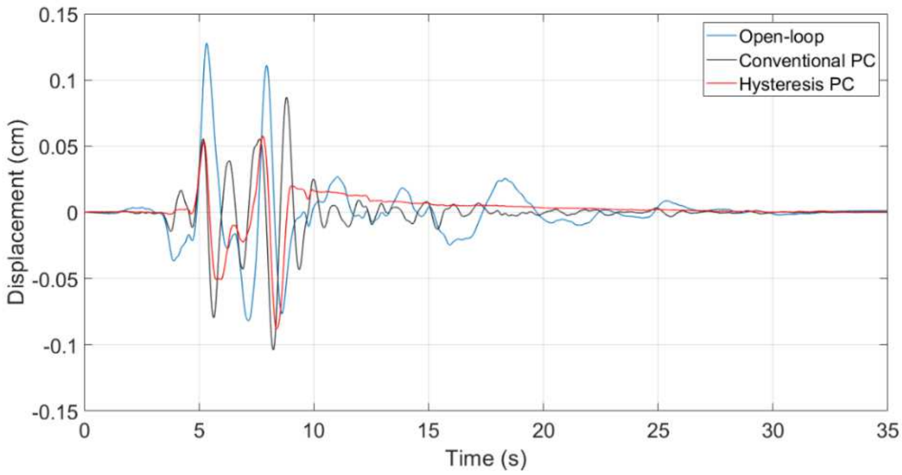


**Fig. 12.** Displacement of the base-isolated system with saturation in the control signal.

## 5. Conclusion

A recent proposed version of a MPC methodology has been presented. From it, it has been demonstrated that this approach results effective to accomplish the control objective of mitigating the vibration on a hysteretical structural base-isolated system. The hysteresis system employed as reference trajectory generator successfully outcomes to lead the base structural position to a rest status when it is subjected to a seismic perturbation. This is because, the reference trajectory with dynamic hysteresis provides a smooth behavior to guide the movement of the system around the desired value and it strings along with the hysteretic nature of the base-isolated structure. Moreover, this strategy works better than the conventional methodology and no extra effort is required to accomplish this. Note that one of the principal keys in the MPC approach is that it provides a suitable linear model of the base-isolation by adding an estimation information of the restoring force of this device. Moreover, the proposal made here it is simple to make since it does not require much more computational expense than the conventional strategy needs, and thanks to the current developments in the digital microprocessors field, it can be easily applied. Finally, as a near future work it is expected to study the controller performance and to adequate it, if would be necessary, when the base-isolated system is fixed to a super-structure, which is a relevant topic in the civil engineering field.

## Author Contributions

J. Rodellar proposed the basis of the control scheme and provided the mathematical model of the application as well as the adequate model and parameters of the Bouc-Wen system. Also, J. Rodellar suggested the numerical experiments to validate the proposal. Furthermore, J. Rodellar and L. Acho proposed the control scheme with hysteresis within the Driver Block. On the other hand, L. Acho examined and analyzed the theorical validation of the proposal. Meanwhile, N.I.P. de León Puig replanted the Driver Block strategy by adequately redesign the hysteresis model to be employed as the reference trajectory. Moreover, N.I.P. de L eón Puig programmed and tuned the controller parameters, and conducted the numerical simulations to validate the proposal. The manuscript was written through the contribution of all authors. Finally, all authors discussed the results, reviewed and approved the final version of the manuscript.

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## Conflict of Interest

The authors declare no conflict of interest. The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

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