J. Appl. Comput. Mech., 8(2) (2022) 545-556
DOI: 10.22055/JACM.2020.32499.2028

ISSN: 2383-4536
jacm.scu.ac.ir

Shahid Chamran University of Ahvaz

Journal of Applied and Computational Mechanics

Research Paper

Chaos Control in Gear Transmission System using GPC and SMC Controllers

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Received February 02 2020; Revised April 26 2020; Accepted for publication April 26 2020.
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Abstract. Chaos is a phenomenon that occurs in some non-linear systems. Therefore, the output of the system will be heavily dependent on the initial conditions. Since the main characteristic of the chaos is an abnormal behavior of the system output, it should be considered in designing control systems. In this paper, controlling chaos phenomenon in a time-variant non-linear gear transmission system is investigated. To do this, a non-linear model for the system is introduced considering the effective parameters of the system, and then it is shown that chaos appears in the system by plotting phase plane of state-space variables. It should be noted that there is a great difference between random and chaotic behavior. In random cases, the model or input contains uncertainty, and therefore, the system behavior and output are not predictable. However, in chaotic behavior, there is only a brief uncertainty in the system model, input or initial conditions, and designing controller based on output prediction could be achieved. Therefore, model predictive control (MPC) algorithms are used to control the chaos, using the output prediction concept. In many cases, perturbation term also can be considered as uncertainty, and therefore, a robust controller family can be used for eliminating chaos. Both generalized predictive controller (GPC) and sliding mode controller (SMC) are used for chaos control here. The simulation results show the efficiency of the proposed algorithms.

Keywords: Gear transmission system, Chaos, Model predictive control, GPC, Sliding mode, SMC, Nonlinear.

1. Introduction

In stable linear systems, the low difference in initial conditions can only lead to small changes in the output, while non-linear systems can exhibit chaos phenomenon, which makes the output of the system susceptible to the initial conditions.

The main characteristic of chaos is its output irregularity that must separate from stochastic processes. In stochastic processes, the system model or input contains uncertainty, and therefore the output changes cannot accurately predicted, and only statistical measurements are used. However, in chaotic systems, there is only a brief uncertainty in the system model, input, or initial conditions [1].

Chaos occurs in many physical systems by non-linear dynamics. For example, the atmosphere clearly shows chaotic behavior, which is why long-term weather forecasting is not possible. Some famous mechanical and electrical systems show chaotic vibration in their behavior. These systems include interlocking elastic structures, mechanical systems with lagging or backlash, wheel-rail dynamics in railway systems and, etc.

Todays, in the field of control science, development of methods for controlling chaotic systems has considered due to irregular behavior, high speed and high sensitivity to the initial conditions, which is often complicated and difficult to implement. For example, in [2], an adaptive method has been used for controlling chaotic disturbances in a two degrees of freedom robot. In [3] sliding mode control with non-linear input is used to control chaos and uncertainty. In [4], the difference synchronization and chaos control of chaotic systems with non-linear exponential terms have been studied by using the feedback control method. In [5], the transition from chaotic behavior to periodic behavior in a 2nd order discrete system has been analyzed. In [6], the dynamic behavior of a rotational machine with centrifugal governor which is subjected to two different forms of external disturbance is studied and The Lyapanov direct method is applied to obtain conditions of stability of the equilibrium points of the system. In [7], A coupled lateral-torsional non-linear model of gear-rotor-bearing transmission system is developed after comprehensive considering the non-linear features associated with time varying meshing stiffness, backlash, transmission error, friction force, input/output load, gravity and gear eccentricity. The results show that the friction force could enlarge the vibration amplitude and affect the low frequency components seriously. In [8], the problem of finite time stabilization for vehicle suspension systems presents with hard constraints based on terminal sliding mode (TSM) control.

The gear transmission system is also a chaotic system which by effects of nonlinearity ,time varying properties, lagging, gear transmission errors, friction between the levels of the teeth and so forth, shows some kind of chaotic behavior. This system has many applications in various industries such as aerospace, robotics and industrial machinery, whose chaotic vibrations directly

Published online: July 29 2020

affect the performance of other related subsystems and may cause damage to the whole system [9-11]. Hence, the researchers have discussed analyzing the behavior of this system as well as the control of it. In [12], the dynamic model of the gear transmission system is obtained by considering the lagging, time varying hardness, external input and the static transmission error and the global bifurcation and transition to chaotic behavior of the system are studied. The torsional vibration model used in this work is a simplification of the model discussed in [13], which is sufficient to replicate the chaotic responses. There are also investigations of chaos in gears in consideration of the multi-DOF coupling, involving torsional and lateral displacement DOFs [14], or additional torsional DOFs by including the shafts [15]. There are also multi-DOF gear models where by considering the elasticity of each tooth, allows to present much more complex dynamical behaviors [16].

The feedback control method is used to stabilize system state variables and hence chaos control in [17]. An analytical approach concerning the elimination of chaos in a gear system by applying the external control excitation using Melinkov's method is presented in [18]. In [19] a systematic analysis of the dynamic behavior of a gear-bearing system with non-linear suspension, non-linear oil film force, and non-linear gear mesh force is performed. Phase plane, Poincaré map and bifurcation diagram of the gear system are also discussed using numerical methods. In [20], the existence of chaos in a semi direct gear drive system is shown using time series analysis, Poincaré map and phase plane, and then the disturbance control is investigated using a proportional integral (PI) method. Then the bending and torsion coupling non-linear dynamic model of the first stage reduction gear of SDGDS is established. In [21], a back stepping based adaptive output feedback control scheme is presented for a gear transmission servo system with an uncertain sandwiched dead zone nonlinearity and uncertain motor parameters. In [22], the incremental harmonic balance method is extended to analyze the non-linear dynamics of a spur gear pair.

Nowadays, by increasing computing power of computers, the use of digital controllers for controlling analog systems is increasing rapidly, among which the use of model predictive control (MPC) algorithms has become widespread in many applications. The main idea of MPC algorithms is minimizing the difference between output and reference, taking into account the possible constraints of the problem on the input, output or state variables. In this method, the future output of the system is predicted based on its model, and then by defining the cost function and applying the constraints, we seek to obtain optimal control inputs for optimizing the cost function. The advantages of MPC against other methods are explicit use of model, current state optimization based on future state, ability of applying constraints to the optimization problem, simple tuning of controller, ability of predicting future events and adopting appropriate control actions. In this paper, generalized predictive control algorithm (GPC) used for chaos control in gear transmission system. One of the benefits of this method is the ability to implement for unstable and Nonminimum phase plants.

This algorithm was established by Clarke et al [23] in 1978 and quickly became a popular method in the industry and academic researches and by implementing for many industrial applications, provided a satisfactory performance and degree of robustness [24]. In [25], a GPC based method for parafoil path tracking in wind environment is designed. GPC also has been used in cement production to control the cement rotary kiln calcining zone temperature [26]. In [27], GPC is used to control the path in a biaxial motion system against the traditional control structure to overcome the redundancy of the system. In [28], GPC is used for load tracking in micro grid DC system of renewable energy sources. In [29] GPC algorithm is used based on reduced model of PWR nuclear reactor for load tracking. In [30] MPC also is used in general state space form for load tracking in PWR nuclear reactors.

Sliding mode control (SMC), also known as variable structure control, is an important robust control approach and has attractive features to keep systems insensitive to uncertainties on the sliding surface. SMC is widely used as a powerful method to tackle uncertain non-linear systems [31]. In [32] an observer based adaptive sliding mode with an integral sliding surface is used for non-linear Markovian jump systems with partly unknown transition probabilities and the stochastic stability of the closed-loop system is guaranteed. In [33] a new sliding mode control design methodology for fuzzy singularly perturbed systems is introduced and its applicability and superiority are verified by a controller design for an electric circuit system. Trajectory tracking of an under actuated autonomous underwater vehicle (AUV) is done by using sliding mode controller in [34].

In this paper, the dynamic model of the gear transmission system described in section 2. GPC and SMC controllers will discussed in section 3 and 4 respectively, and the simulation results presented in section 5.

2. Chaotic Behavior of Gear Transmission System

In this section, a general model for the gear transmission system is first introduced and its dynamic equations will be presented [14]. Then, by analyzing the phase plane of the proposed model, the chaotic behavior of this system is investigated.

2.1 Extracting the model

In this paper, the gear transmission system modeled as a pair of rigid disk connected by a spring damper set along the line of action [18], as shown in Fig. 1.

The radius of the gears a and b are considered to be r_a and r_b respectively. I_a and I_b are also represent the moment of inertia of two gears, k_m is spring stiffness, T_a and T_b are torques applied to the gears and c_m is the damping factor that is equivalent to the gear pair engaged. The backlash function f_h, is used to represent gear clearances, and the displacement function $e(t)$, is also applied at the gear mesh interface to represent static transmission error. By considering θ_p and θ_g as the torsional displacements of pinion and gear, whole system can be modeled by equations (1) and (2).

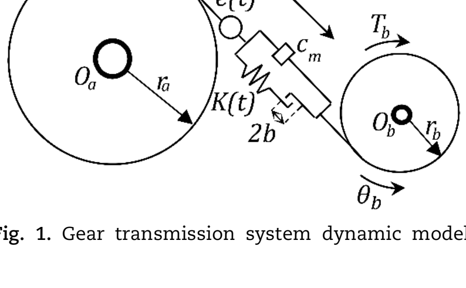

Fig. 1. Gear transmission system dynamic model [18]

$$I_a\frac{d^2\theta_a}{dt^2}+c_m\left(r_a\frac{d\theta_a}{dt}-r_b\frac{d\theta_b}{dt}-\frac{de(t)}{dt}\right)r_a+r_ak_mf_h(r_a\theta_a-r_b\theta_b-e(t))=T_a \tag{1}$$

$$I_a\frac{d^2\theta_b}{dt^2}-c_m\left(r_a\frac{d\theta_a}{dt}-r_b\frac{d\theta_b}{dt}-\frac{de(t)}{dt}\right)r_b-r_ak_mf_h(r_a\theta_a-r_b\theta_b-e(t))=-T_b \tag{2}$$

Both gears are required to have backlash designed for better lubricating and less interacting. Installation and erosion errors are also factors in creating this backlash. Therefore, backlash function f_h is considered to be piecewise linear function (3) to present both gears backlash [18].

$$f_h=\begin{cases}r_a\theta_a-r_b\theta_b-(1-\alpha)b,\ b<(r_a\theta_a-r_b-e)\\ \alpha(r_a\theta_a-r_b\theta_b-e),\ \ -b\le(r_a\theta_a-r_b\theta_b-e)\le b\\ r_a\theta_a-r_b\theta_b-e+(1-\alpha)b,\ b<-(r_a\theta_a-r_b\theta_b-e)\end{cases} \tag{3}$$

By defining the new variable $\tilde{x}=r_a\theta_a-r_b\theta_b-e(t)$, which is the difference between the dynamic and static transmission error, the vibrating equations of the gear system are simplified as follows.

$$m\frac{d^2\tilde{x}}{dt^2}+c_m\frac{d\tilde{x}}{dt}+k_mf_h(\tilde{x})=\hat{F}_m+\hat{F}_e(t) \tag{4}$$

where

$$f_h(\tilde{x})=\begin{cases}\tilde{x}-(1-\alpha)b,\ b<\tilde{x}\\ \alpha\tilde{x},\ \ -b\le\tilde{x}\le b\\ \tilde{x}+(1-\alpha)b,\ b<-\tilde{x}\end{cases},\ m=\frac{I_aI_b}{I_ar_a^2+I_br_b^2},\ \hat{F}_e(t)=-m\frac{d^2e}{dt^2},\ \hat{F}_m(t)=m(\frac{T_ar_a}{I_a}+\frac{T_br_b}{I_b}) \tag{5}$$

Here, m is the equivalent mass representing the total inertia of the gear pair, $\hat{F}_m(t)$ is the average force transmitted through the gear pair, and the internal excitation term $\hat{F}_e(t)$ arises from the static transmission error. Static transmission error f_h considered as one of the most effective parameters in the vibration of the system that is one of the most important source of vibration in gears due to manufacturing and deformation errors of the teeth. Since the mean angular velocities of the gears are constant, the static transmission error can approximated as a periodic function that its fundamental frequency is the meshing frequency [18]. So, the static transmission error that considered as harmonic with

$$e(t)=e(t+2\pi/\omega_e)=e\cos(\omega_et+\varphi_e) \tag{6}$$

is entered to the equations [21]. The dimensionless form of the system can be obtained by the defining parameters

$$x=\frac{\tilde{x}}{b},\ \omega=\sqrt{\frac{k_m}{m}},\ \tau=\omega_nt,\ \Omega_e=\frac{\omega_e}{\omega_n},\ \tilde{\mu}=\frac{c}{2m\omega_n},\ \tilde{F}_e=\frac{e}{b},\ \tilde{F}_m=\frac{\hat{F}_m}{bk_m} \tag{7}$$

as follows

$$\frac{d^2x}{d\tau^2}+2\tilde{\mu}\frac{dx}{d\tau}+f_h=\tilde{F}_m+\tilde{F}_e\Omega_e^2\cos(\Omega_e\tau+\varphi_e) \tag{8}$$

where

$$f_h(x)=\begin{cases}x-(1-\alpha)b,\ 1<x\\ \alpha x,\ \ -1\le x\le 1\\ x+(1-\alpha)b,\ 1<-x\end{cases} \tag{9}$$

By approximating backlash function f_h with a third order function as $f_h=-0.1667x+0.1667x^3$, and considering $\alpha=0$, the equation (8) is rewritten as follows.

$$\frac{d^2x}{d\tau^2}+2\tilde{\mu}\frac{dx}{d\tau}+(-0.1667x+0.1667x^3)=\tilde{F}_m+\tilde{F}_e\Omega_e^2\cos(\Omega_e\tau+\varphi_e) \tag{10}$$

Further, by obtaining the state space equations of the system, and changing bifurcation parameter, we will investigate the chaotic behavior in the gear transmission system.

2.2 Chaotic behavior of system

To investigate the vibrating equations of the system, we first obtain state space equations by defining new parameters $\tilde{F}_e=\varepsilon f_e$, $\tilde{F}_m=\varepsilon f_m$ and $\tilde{\mu}=\varepsilon\mu$, where ε is a small parameter which takes the value in the interval $0\le\varepsilon\le1$. The perturbed system is as follows.

$$\begin{cases}\dot{x}=y\\ \dot{y}=-2\varepsilon\mu y+(0.1667x-0.1667x^3)+\varepsilon(f_m+f_e\Omega_e^2\cos(\Omega_e\tau+\varphi_e))+u(\tau)\end{cases} \tag{11}$$

where $u(\tau)$ is added to the equations as control input and its main idea in designing is to use actuators to create a secondary excitation torque on the gear a, which control actuation is usually applied by hydraulic or magnetic operators to control the system's performance. Equation (11) is state space equations of the system that will use for designing controller and simulation in this article.

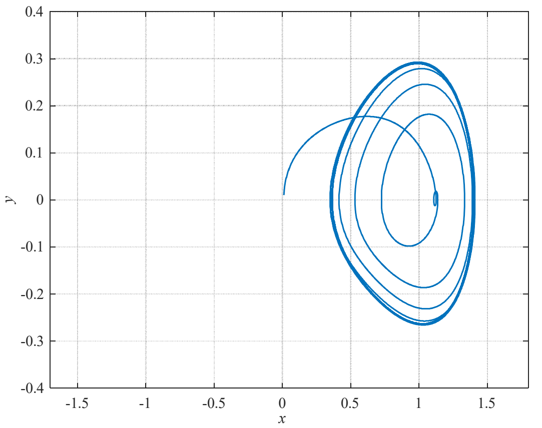

Fig. 2. Phase plane with $f_e = 20$

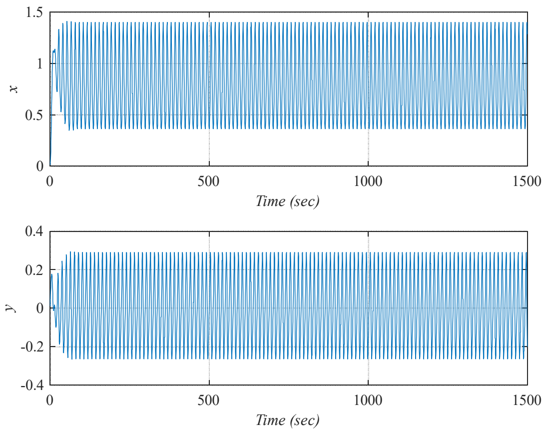

Fig. 3. Time history of state space variables with $f_e = 20$

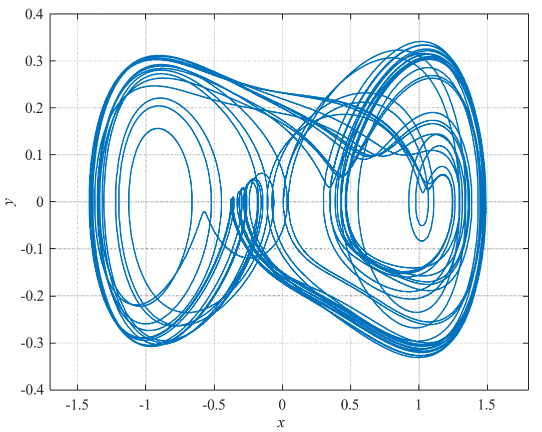

Fig. 4. Phase plane with $f_e = 30$

By considering constant values of the model as $\varepsilon = 0.01$, $f_m = 1$, $\Omega_e = 0.5$ and $\mu = 9$, and changing bifurcation parameter f_e in the interval $[0,40]$, the presence of chaos will be investigated by plotting phase plane of the state space variables. Unperturbed system with $\varepsilon = 0$, has two center fixed points in $(\pm 1,0)$ and an unstable saddle point in $(0,0)$. f_e is bifurcation parameter of the system, that for $f_e > 26$ bifurcation occurs and system will shows chaotic behavior. Now, taking into account the initial values of the state variables as $(0.01,0.01)$, the phase plane and time history of state variables will be plotted by Euler approximation with sampling time as 0.01 seconds. By considering $f_e = 20$, these diagrams are as follows.

In this case, the system does not show chaotic behavior. Let assume bifurcation parameter as $f_e = 30$ and plot the same diagrams.

As it is clear, for $f_e = 30$ the transition to chaos has occurred and the disorder in the behavior of the system is quite evident. In the next section, we will design the generalized predictive controller for controlling the chaos in the system for $f_e = 30$.

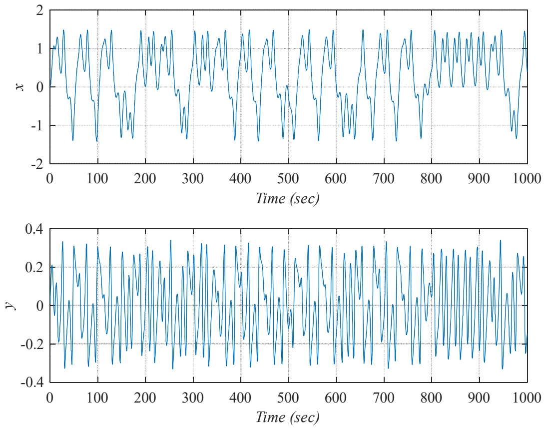

Fig. 5. Time history of state space variables with $f_e = 30$

3. Generalized Predictive Controller (GPC)

The purpose of this section is to design GPC Controller for chaos control in gear transmission system. The model used in this algorithm is the transfer function as follows.

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1) + \frac{C(z^{-1})e(t)}{\Delta(z^{-1})} \tag{12}$$

This model also is known as Controller Auto Regressive Moving Average or CARIMA model. In this model, the polynomials A and B are denominator and nominator of the transfer function respectively. An integrator is also considered as $1/\Delta(z^{-1})$ in the dynamic of the disturbance $e(t)$, and d indicates system delay. Polynomials A, B, C are defined as follows.

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{na} z^{-na} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{nb} z^{-nb} \\ C(z^{-1}) &= 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_{nc} z^{-nc} \end{aligned} \tag{13}$$

The first step for designing a MPC controller is to define a cost function for optimization that it can defined to track the reference signal $W(t)$ and minimize control input as follows.

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} \delta(j)\left[\hat{y}(t+j\,|\,t) - w(t+j)\right]^2 + \sum_{i=1}^{N_u} \lambda(j)\left[\Delta u(t+i-1)\right]^2 \tag{14}$$

Where N_1 and N_2 are the minimum and maximum costing horizons and N_u is the control horizon. In order to minimize the cost function in each sample, prediction of output over the prediction horizon using the model of the system is needed. To derive the system output relationship, we use equation (12) and Diophantine equation, which expressed as follows.

$$1 = E_j(z^{-1})A(z^{-1}) + z^{-j}F_j(z^{-1}) \tag{15}$$

where $E_j(z^{-1})$ and $z^{-j}F_j(z^{-1})$ are the quotient and remainder of the division $1/A$, and These polynomials could be calculated recursively. In other hand they can be obtained by dividing $1/\tilde{A}(z^{-1})$ until the remainder can be factorized as $z^{-j}F_j(z^{-1})$ and then the quotient of the division is the polynomial $E_j(z^{-1})$. By considering the definition of polynomial A, it can be concluded that.

$$\begin{aligned} &E_1 = 1 \\ &F_1 = z(1 - \tilde{A}(z^{-1})),\ \tilde{A}(z^{-1}) = \Delta A(z^{-1}) \end{aligned} \tag{16}$$

By having two initial points above, and using the recursive equation (17), the rest of the coefficients of these polynomials can also obtained by following equations.

$$\begin{aligned} &E_{j+1} = E_j + f_{j,0} z^{-j} \\ &F_{j+1,i} = f_{j,i+1} - f_{j,0}\tilde{a}_{i+1},\ i = 0,1,\ldots,na-1 \end{aligned} \tag{17}$$

Where $f_{j,i}$ is the coefficient z^{-i} in the j-th polynomial. Finally, after calculating the polynomials $E_j(z^{-1})$, $F_j(z^{-1})$ and using the Diophantine equation and considering perturbation term in the form of Gaussian noise with zero mean, the output prediction using the transfer function will be as follows.

$$\hat{y}(t+j\,|\,t) = G_j(z^{-1})\Delta u(t+j-d-1) + F_j(z^{-1})y(t) \tag{18}$$

Where $G_j(z^{-1}) = BE_j(z^{-1})$. With precision on the output prediction equation (18), it is clear that $F_j(z^{-1})y(t)$ depends on past signals, and the other term depends on the past and future signals. In this way, by dividing the output prediction into two parts of free response, which depends on the future control input and the forced response that depends on past signals we can rewritten equation (18) in the matrix form as follows.

$$Y = \Phi Y_{-} + \Pi U_{-} + \Omega U \tag{19}$$

Where Y is the vector of output predictions and Y_{-} is the system's past output. U and U_{-} are also future and past control input respectively. The matrices Ω and Φ also contain the coefficients of the polynomials $G_j(z^{-1})$ and $F_j(z^{-1})$ respectively, which are defined as follows.

$$\Phi = \begin{bmatrix} f_{d+1,0} & \cdots & f_{d+1,na} \\ \vdots & \ddots & \vdots \\ f_{d+N,0} & \cdots & f_{d+N,na} \end{bmatrix}, \ \Omega = \begin{bmatrix} g_{d+1,0} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ g_{d+N,N-1} & \cdots & g_{d+N,0} \end{bmatrix} \tag{20}$$

Having output prediction and defining reference signal as

$$W = [w(t+d+1) \ \dots \ w(t+d+N)]^T \tag{21}$$

we can optimize the cost function, which can expressed in the matrix form as follows.

$$J = (W - Y)^T (W - Y) + \alpha U^T U \tag{22}$$

The minimum of cost function, assuming there are no constraints, can found by making the gradient of J equal to zero, which leads to

$$\frac{\partial J}{\partial U} = -\Omega^T (W - \Phi Y_{-} - \Pi U_{-} - \Omega U) + \alpha U = 0 \ \rightarrow \ U = -(\Omega^T \Omega + \alpha I)^{-1} \Omega^T (W - \Phi Y_{-} - \Pi U_{-}) \tag{23}$$

Notice that the control signal that is actually sent to the plant is the first element of vector U, according to receding horizon strategy.

To implement the above relations, we first need to linearize the state space equations, and then convert linear state space model to the transfer function. By linearizing non-linear state space around the equilibrium point $(0,0)$, the linear will be as follows.

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX + DU \end{cases} \tag{24}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0.1667 & -0.18 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \ D = 0 \tag{25}$$

Notice that the system output is defined equal the second state variable here, and since $\dot{x} = y$, by output tracking we can control both state variables. Also pay attention that to stablish the receding horizon strategy it is necessary that $D = 0$.

After obtaining the linear state space model, by using the relationship between state space and transfer function as

$$Y(s) = (C(sI - A)^{-1} B + D) U(s) \tag{26}$$

we can obtain the transfer function model of the system as follows.

$$\frac{Y(s)}{U(s)} = \frac{s}{s^2 + 0.18s - 0.1667} \tag{27}$$

Considering sampling time as 0.01 seconds, the discrete transfer function model will be as follows.

$$\frac{Y(z^{-1})}{U(z^{-1})} = \frac{0.0099z^{-1} - 0.0099z^{-2}}{1 - 1.998z^{-1} + 0.9982z^{-2}} \tag{28}$$

4. Sliding Mode Controller (SMC)

In this section, stabilizing the state variables of the gear transmission system by using sliding mode controller based on Lyapanov stability theorem will be investigated. Consider the perturbed system defined by equation (11). We can show these equations as follows.

$$\begin{cases} \dot{x} = y \\ \dot{y} = f_{\text{nominal}} + f_{\text{uncertainty}} + u \end{cases} \tag{29}$$

where

$$f_{nom} = -0.18y + 0.1667x - 0.1667x^3 \tag{30}$$

and

$$f_{un} = \varepsilon (f_m + f_e \Omega_e^2 \cos(\Omega_e \tau + \varphi_e)) \tag{31}$$

is perturbation term that can be considered as model uncertainty. By defining sliding surface as sum of weighted errors as

$$s = y + \lambda x \tag{32}$$

the input signal for stabilizing the state variables can be considered as follows.

$$u = u_{\text{equal}} + u_{\text{reach}} \tag{33}$$

Where u_{eq} and u_r are to removing certain term f_{nom} and putting down the uncertainties f_{un} respectively that are define as follows.

$$u_{eq} = -f_{nom} = 0.18y - 0.1667x + 0.1667x^3 \tag{34}$$

$$u_r \leq -k\frac{s}{|s|},\ k = \zeta + \mu \tag{35}$$

Theorem: Consider the system given in (29). By defining the sliding surface as (32), the suggested controller structure given in (33) makes the tracking error converge to zero and all signals in the closed loop system bounded as well.
Proof: To prove the stability of the closed loop system, the time derivative of the slide variable is needed.

$$\dot{s} = \dot{y} + \lambda\dot{x} = f_{nom} + f_{un} + u \tag{36}$$

The control input u is appeared in first derivative of sliding surface, so the candidate Lyapanov function is.

$$v = \frac{1}{2}s^2 \tag{37}$$

By derivative of Lyapanov function we have.

$$\dot{v} = s\dot{s} \tag{38}$$

For finite time stability, the following condition should be satisfied.

$$\dot{v} = s\dot{s} \leq -\zeta|s(t)|,\ \zeta > 0 \tag{39}$$

Above equation can rewritten as follows.

$$s\dot{s} \leq -\zeta|s| \rightarrow \frac{s}{|s|}\dot{s} \leq -\zeta \rightarrow \int_{s(0)}^{0}\frac{s}{|s|}ds \leq \int_{0}^{t_r} -\zeta dt \tag{40}$$

Therefore, the controller is finite time and it is proofed according to (40) that this time must satisfies the following inequality.

$$t_r \leq \frac{|s(0)|}{\zeta} \tag{41}$$

By considering (29) and replacing (34) and (35) into (38) we have

$$\begin{aligned}&\dot{v} = s\dot{s} = s(\dot{y} + \lambda\dot{x}) = s(f_{nom} + f_{un} + u) \leq -\zeta|s(t)| \\ &\rightarrow f_{un}\frac{s}{|s|} + u_r\frac{s}{|s|} + \zeta \leq 0\end{aligned} \tag{42}$$

If μ is upper band of uncertainty profile such that $|f_{un}| < \mu$, then upper band of u_r is

$$u_r \leq -(\zeta + \mu)\frac{s}{|s|} \tag{43}$$

and the control input for putting down the uncertainties can be considered as follows.

$$u_r = -(\zeta + \mu)\frac{s}{|s|} = -k\,\text{sgn}(s) \tag{44}$$

Now by imposing the controller (33) to the system it is guaranteed that sliding surface converges to zero in finite time. Here the proof is completed.
Remark: boundary layer method can used to eliminate the chattering phenomenon in sliding mode controllers. So u_r can defined as follow

$$u_r = -(\zeta + \mu)\text{sat}(\frac{s(t)}{\varphi}),\ \ \text{k} = \zeta + \mu \tag{45}$$

Where φ determines the width of the boundary layer and $\text{sat}(.)$ is saturation function. In addition, we can use the continuous function $\tanh(s/\varphi)$ for better elimination of chattering. Notice that parameter k must be obtained for worst condition that depends on the uncertainties.

5. Results

In this section, numerical simulations presented to demonstrate the accuracy of the theoretical predictions, and to investigate the performance of the proposed controllers for chaos control in non-linear model of gear transmission system.

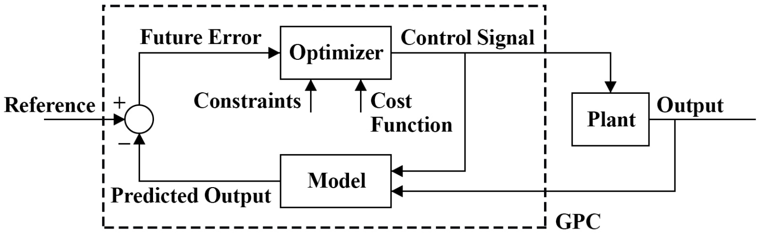

Fig. 6. GPC Controller Structure

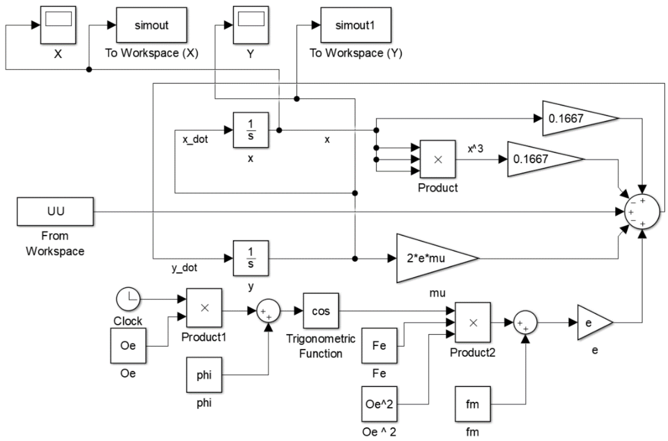

Fig. 7. Gear transmission system in MATLAB Simulink

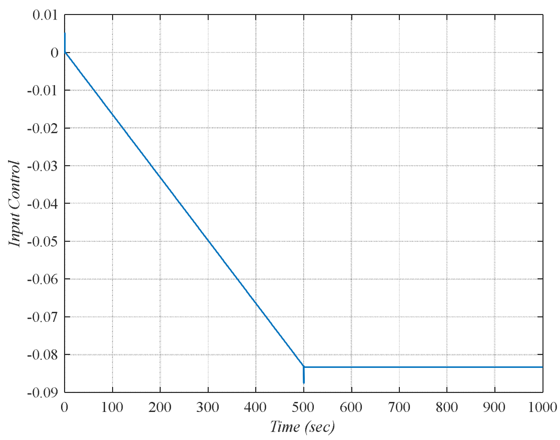

Fig. 8. GPC Algorithm control input

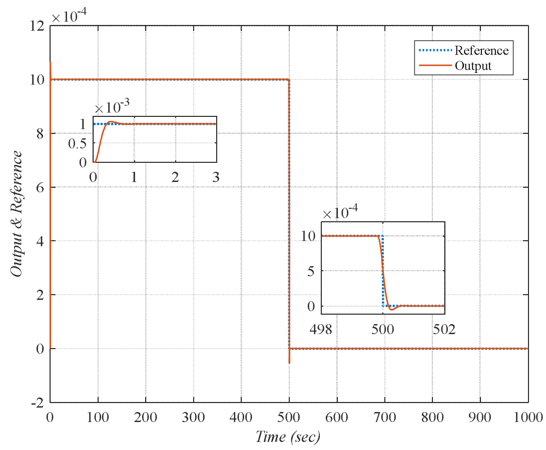

Fig. 9. Reference tracking after applying control input

5.1 GPC

Consider the GPC controller designed in section 3, that its structure is shown in following block diagram (Fig. 6). It is clear that control signal is generated by optimizing the error between reference and predicted output considering the cost function and possible constraints. Then by applying this control input to the plant and its model, the output prediction is updated and the optimization process is repeated again.

In order to implement the GPC controller designed in section 3, prediction and control horizon are chosen as 20 and 5 respectively. The reference signal amplitude initially considered 0.001 and then it changed to zero. Controller designed for a period of 1000 seconds, which applied to the non-linear model simulated in the MATLAB Simulink as shown in Fig. 7, after 500 seconds. Therefore, the simulation time considered 1500 seconds.

GPC control input and reference tracking are shown in Fig. 8 and Fig. 9 respectively.

State space variables time history and phase plane are shown Fig. 10 and Fig. 11, respectively. It is clear that chaotic behavior eliminated and phase plane remains on the closed orbit around the fixed point.

3D phase plane can be help us for better review of system performance that is shown in Fig. 12. It is clear that after 500 seconds, control signal calculated by the GPC algorithm applied to the system and chaotic behavior eliminates over time. About 500 seconds after that, the system placed on the closed orbit around the fixed point.

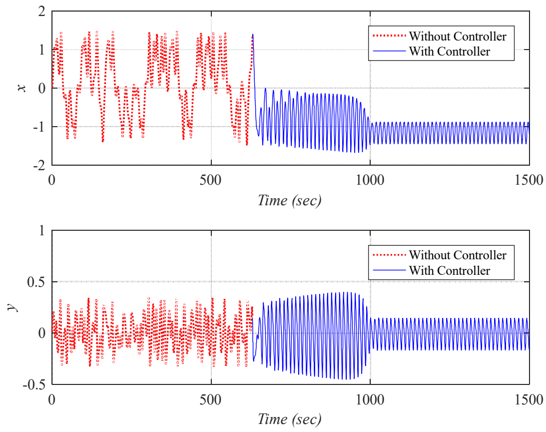

Fig. 10. Time history of state space variables with $f_e = 30$, after applying GPC control input

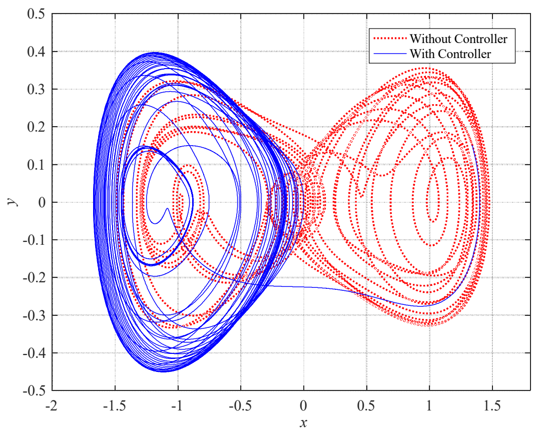

Fig. 11. Phase plane with $f_e = 30$, after applying GPC control input

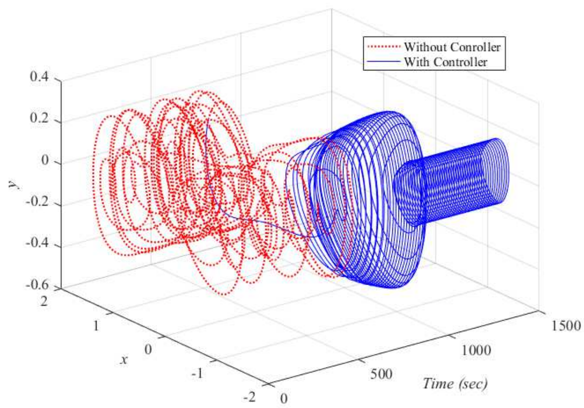

Fig. 12. 3D Phase plane with $f_e = 30$, after applying GPC control input

5.2 SMC

Consider the sliding mode controller designed in section 4, that its structure is shown in following block diagram.

By considering, $\lambda = 0.1$ Controller designed for a period of 1000 seconds, which applied to the non-linear model simulated in the MATLAB Simulink as shown in Fig. 14, after 500 seconds. Therefore, the simulation time considered 1500 seconds. Sliding mode control input and state space variables time history are shown in Fig. 15 and Fig. 16 respectively. It is clear that after 500 seconds, SMC control input applied to the system, and chaotic behavior eliminates over time and the system placed on the closed orbit around the fixed point.

We can also use 3D phase plane for better review of system performance that shown in Fig. 17.

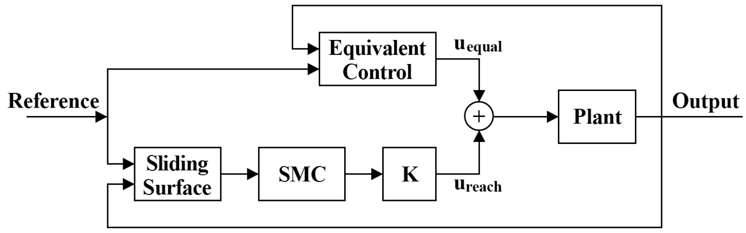

Fig. 13. SMC Controller Structure

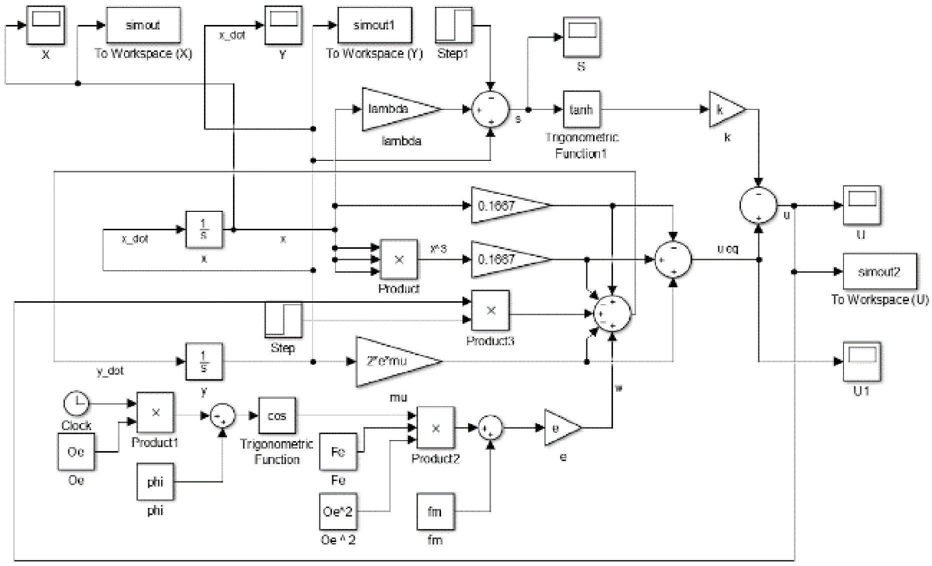

Fig. 14. Gear transmission system and sliding mode controller in MATLAB Simulink

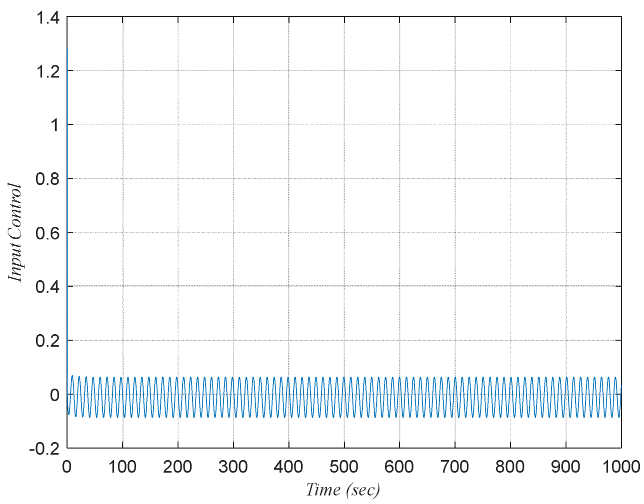

Fig. 15. SMC control input

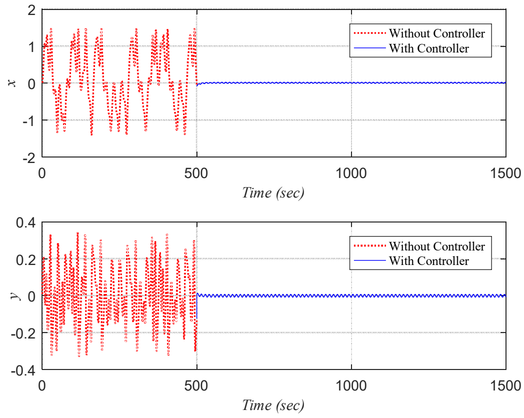

Fig. 16. Time history of state space variables with $f_e = 30$, after applying SMC control input

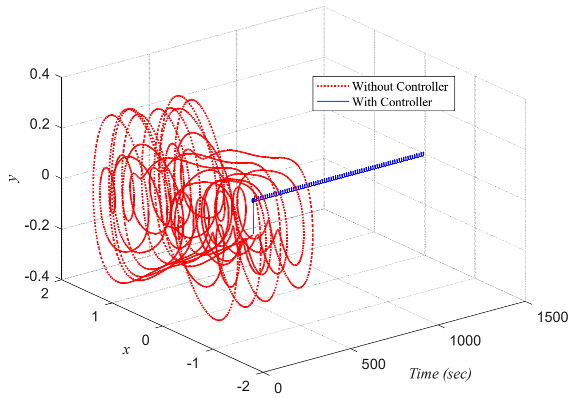

Fig. 17. 3D Phase plane with $f_e = 30$, after applying SMC control input

6. Conclusion

In this paper, the chaos control in the gear transmission system was discussed using GPC and SMC controllers that control structure can be done by applying an additional excitation torque to the driver gear. Simulation results show the effectiveness of both proposed control systems. One of the primary purposes of this study was to compare two control strategies for eliminating chaotic behavior. By comparing Fig. 10, and Fig. 16 it is clear that the oscillating amplitude of state variables in the SMC controller is less than in GPC, but the control input of SMC is also oscillating that can be harmful to the system. In GPC, any sample of control signal was updated by minimizing the error between output and prediction, so it is not an oscillating signal (Fig. 8 and Fig. 15). Notice that the implementation of GPC is much simpler than SMC, and there is no need for parameter tuning after designing the controller. By applying GPC control input, the chaotic behavior of the system was converted to periodic behavior at any moment. Therefore, both controllers can be used as an effective way for chaos control in the gear transmission system.

Author Contributions

M. Gharagozloo planned the scheme, initiated the project and suggested to design MPC for gear system and compare this controller with other control methods. Controller design and simulations were done by M. Gharagozloo. Mathematical modeling, model approximation, theory validation and reviewing the results were done by A. Shahmansoorian. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

Acknowledgments

Not applicable.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

Funding

The authors received no financial support for the research, authorship, and publication of this article.

Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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How to cite this article: Gharagozloo M., Shahmansoorian A. Chaos Control in Gear Transmission System using GPC and SMC Controllers, *J. Appl. Comput. Mech.*, 8(2), 2022, 545–556. https://doi.org/10.22055/JACM.2020.32499.2028