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Research Paper

Entropy Analysis of a Radiating Magnetic Liquid Film along a Slippery Inclined Heated Surface with Convective Cooling

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Abstract. The hydromagnetic flow of a Newtonian incompressible and electrically conducting variable-viscosity liquid film along an inclined isothermal or isoflux hydrophobic surface is investigated numerically. It is assumed that the fluid is subjected to a convective cooling at the free surface in the presence of a transversely imposed magnetic field. We incorporated in the energy equation, the viscous dissipation, Joule heating due to the magnetic field and the local radiative heat flux term for optically thick gray fluid reported by Roseland approximation. The governing non-linear differential equations are obtained and solved numerically using the shooting method coupled with a fourth-order Runge-Kutta method. The effects of some parameters on velocity and temperature profiles, skin friction, Nusselt number entropy generation rate, and the Bejan number profiles are analyzed graphically and discussed.

Keywords: Inclined slippery surface; Magnetic liquid film; Variable viscosity; Thermal radiation; Convective cooling; Entropy analysis

1. Introduction

The liquid film is a tinny layer of liquid flowing over a wall. Liquid films have many applications, which include flow behavior of paints, coating flows, chemical, and nuclear reactor design, agrochemical applications, biofluid applications, microfluidic applications, medical applications, food, and chemical industries. The liquid film is also important in designing some of the heat transfer equipment. By virtue of these applications, many researchers have carried out various investigations on liquid film. Sadiq and Usha [1] investigated linear instabilities analysis based on a constant temperature gradient of a thin layer of viscoelastic fluid flowing down a non-uniformly heated inclined plane. Their result revealed that the fluid turns out to be warmer downward and the thermocapillary forces seen at the interface acted in the opposite direction to the gravitational acceleration. Thiele and Knobloch [2] considered a thin liquid film on a uniformly heated substrate with the hope of multiplicity of solutions to the nonlinear evolution equation and their stability properties and also understanding small inclination effects of the substrate. Makinde [3] studied the intrinsic irreversibility and thermal stability in a gravity-driven temperature-dependent variable viscosity thin liquid film along with an inclined heated plate alongside convective cooling. The result divulged that a minute entropy generation system could be designed with a suitable choice and proper combination of the various thermophysical parameters. Makinde [4] researched the thermal criticality for a reactive gravity-driven thin film flow of a third-grade fluid with adiabatic free surface down an inclined isothermal plane. The analysis uncovered that the non-Newtonian third-grade reactive fluid is thermally more stable than the Newtonian reactive fluid in industrial practices. Makinde [5] considered the thermal stability of boundary layer flows of a temperature-dependent viscosity liquid film with the adiabatic free surface along an inclined heated plate using a special type of Hermite–Pade´ approximation method. His results showed the analytical structure of the solution function and the imperative properties of the overall flow structure. Kabova et al. [6] performed theoretical and numerical investigations of the heat transfer and hydrodynamics in a liquid film flowing along an inclined substrate under the action of gravity with a local heat source. Their analysis exposed that the viscosity effect leads to the decrease of the film thickness. Adesanya et al. [7] presented a gravity-aided thin couple stress liquid film in an inclined heated substrate and demonstrated the effects of various parameter values. They concluded that both slip and porous permeability parameters had significant effects on the entropy generation rate. Zahir Shah et al. [8] considered the impacts of non-linear radiation and a uniform magnetic field on the thin fluid flow of viscous nanofluid. They concluded that increasing the magnetic field decrease the velocity profile and increase the temperature profile. Samanta et al. [9] presented a comprehensive study on the effects of surface slippage on the flow dynamics of a thin film over an inclined plane. The theoretical investigation of a thin liquid film flow between a hydrophobic surface was performed by Vinogradova [10].

Entropy analysis of the system is used to evaluate and optimize different industrial projects and industrial equipment with the objective of reducing the entropy generation rate. Bejan [11] was first to gather the entropy generation process and analyzed local entropy generation owed to heat transfer and viscous effects for convective heat transfer in a channel. He showed how the geometrical parameters of the flow might be selected in order to minimize the irreversibility associated with a specific convective heat transfer process. After Bejan, several other researchers have carried out works on different kinds of entropy generation for various geometries. Eegunjobi and Makinde [12] explored the entropy generation of a variable viscosity Hartmann flow numerically through a rotating channel with Hall effects. Mkwizu and Makinde [13] investigated the joint effects of thermophoresis, Brownian motion, and variable viscosity on entropy generation in an unsteady flow of water-based nanofluids restricted between two parallel plates with convective heat transfer exchange and ambient surrounding at the walls. Eegunjobi and Makinde [14] examined the inherent irreversibility in steady hydromagnetic permeable channel flow of a conducting fluid with variable electrical conductivity and asymmetric Navier slip at the channel walls in the presence of an induced electric field.

In this present study, the analysis of the entropy generation rate in a radiating magnetic liquid film along a slippery inclined heated surface with convective cooling is carried out. Understanding the effects of thermal radiation [15] and surface slip [9] on entropy production in liquid film flow are very relevant and momentous in numerous natural and industrial settings such as lubrication, microfluidics polymer melt, etc., in order to achieve efficient operation. The hydrophobic surface is commonly denoted by a slip boundary condition on the surface, while the absorption or emission of electromagnetic heat waves at the surface is epitomized by radiative heat flux. In the subsequent sections, the problem is formulated, numerically analyzed, and solved. Some of the significant results are presented graphically and discussed.

2. Mathematical Formulation

The hydromagnetic flow of a Newtonian incompressible and electrically conducting variable-viscosity liquid film along an inclined isothermal or isoflux hydrophobic slippery surface under the action of a transversely imposed uniform magnetic field of strength B_0 as shown in Fig. 1 is considered. It is assumed that the fluid viscosity is exponentially dependent on temperature, and the slight variation in the fluid density due to temperature difference is assumed to follow Boussinesq approximation. In contrast, the rest of the fluid properties remain constant.

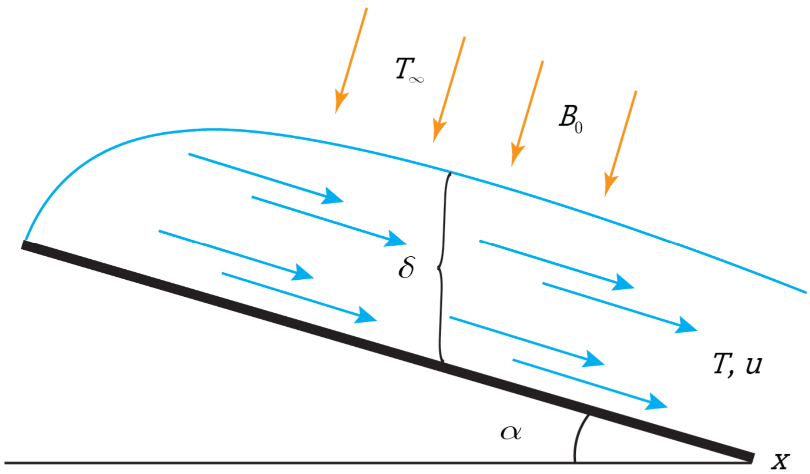

Fig. 1. Problem geometry

Following [3], the temperature-dependent liquid film viscosity (μ) can be expressed as:

$$\mu = \mu_0 e^{-\gamma(T-T_\infty)}, \tag{1}$$

where μ_0 is the dynamic viscosity at the ambient temperature T_∞, γ is the viscosity variation parameter, and T is the liquid film temperature. Following the Roseland approximation [15-17], the local radiative heat flux term for optically thick gray fluid is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \approx -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y}, \tag{2}$$

where $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$ (by Taylor series approximation), k^* is mean absorption coefficient and σ^* is Stefan-Boltzmann constant. Consequently:

$$\frac{\partial q_r}{\partial y} \approx -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

Under these assumptions, the continuity, momentum, energy, and volumetric entropy generation equations take the form [3-5, 18-20]:

$$\frac{\partial u}{\partial x} = 0, \tag{4}$$

$$\frac{d}{dy}\left(\mu\frac{du}{dy}\right) - \sigma B_0^2 u + (\rho g \beta)(T - T_\infty)\sin(\alpha) = 0, \tag{5}$$

$$\left(1+\frac{16\sigma^{*}T_{\infty}^{3}}{3kk^{*}}\right)\frac{\partial^{2}T}{\partial y^{2}}+\frac{\mu}{k}\left(\frac{du}{dy}\right)^{2}+\frac{\sigma B_{0}^{2}}{k}u^{2}=0, \tag{6}$$

$$E_{g}=\frac{k}{(T_{w}-T_{\infty})^{2}}\left(1+\frac{16\sigma^{*}T_{\infty}^{3}}{3kk^{*}}\right)\left(\frac{dT}{dy}\right)^{2}+\frac{\mu}{(T_{w}-T_{\infty})}\left(\frac{du}{dy}\right)^{2}+\frac{\sigma B_{0}^{2}u^{2}}{(T_{w}-T_{\infty})}, \tag{7}$$

The boundary conditions for us to explore both isothermal and isoflux heated in an inclined wall surface are written as:

$$u=\frac{\mu}{L}\frac{du}{dy},\quad ak\frac{\partial T}{\partial y}=\frac{kb}{\delta}(T-T_{\infty})-\frac{k}{\delta}(T_{w}-T_{\infty})\quad at\quad y=0, \tag{8}$$

$$\frac{du}{dy}=0,\quad -k\frac{\partial T}{\partial y}=h_{f}(T-T_{\infty})\quad at\quad y=\delta \tag{9}$$

where u represents axial velocity, δ is the thickness of the liquid film, L is the slip length parameter, α is the angle of inclination, B_0 is the uniform magnetic field acting perpendicular to the liquid film flow, T_w is the wall temperature. Furthermore, h_f represents heat transfer coefficient, k is the thermal conductivity, (x,y) are the axial and normal coordinates to the incline plane surface, c_p is the specific heat at constant pressure, ρ is the liquid film density, g is the gravitational acceleration, β is the volumetric thermal expansion coefficient. Eg, is the entropy generation rate, while a and b are inclined wall heating parameters. Isoflux heating occurs where a = 1, b = 0 and isothermal heating take place where a = 0, b = 1. We introduce the following variables and parameters:

$$\left.\begin{array}{l}\eta=\dfrac{y}{\delta},\quad \theta=\dfrac{T-T_{\infty}}{T_{w}-T_{\infty}},\quad W=\dfrac{u\delta}{\upsilon_{0}},\quad \upsilon_{0}=\dfrac{\mu_{0}}{\rho},\quad \lambda=\gamma(T_{w}-T_{\infty}),\\ M=\dfrac{\sigma B_{0}^{2}\delta^{2}}{\mu_{0}},\quad Ra=\dfrac{g\beta(T_{w}-T_{\infty})\delta^{3}}{\upsilon_{0}^{2}},\quad Bi=\dfrac{h_{f}\delta}{k},\quad Nr=\dfrac{16\sigma^{*}T_{\infty}^{3}}{3kk^{*}},\\ Pr=\dfrac{\mu_{0}c_{p}}{k},\quad Ec=\dfrac{\upsilon_{0}^{2}}{c_{p}\delta^{2}(T_{w}-T_{\infty})},\quad Ns=\dfrac{\delta^{2}E_{g}}{k},\quad S=\dfrac{\mu_{0}}{\delta L}.\end{array}\right\} \tag{10}$$

The dimensionless governing equations are obtained as:

$$\frac{d^{2}W(\eta)}{d\eta^{2}}-\lambda\frac{d\theta(\eta)}{d\eta}\frac{dW(\eta)}{d\eta}-MW(\eta)e^{\lambda\theta(\eta)}+Ra\theta(\eta)e^{\lambda\theta(\eta)}\sin(\alpha)=0, \tag{11}$$

$$(1+Nr)\frac{d^{2}\theta(\eta)}{d\eta^{2}}+Ec\,Pr\left(\frac{dW(\eta)}{d\eta}\right)^{2}e^{-\lambda\theta(\eta)}+Ec\,Pr\,MW(\eta)^{2}=0, \tag{12}$$

$$Ns=(1+Nr)\left(\frac{d\theta(\eta)}{d\eta}\right)^{2}+Ec\,Pr\,e^{-\lambda\theta(\eta)}\left(\frac{dW(\eta)}{d\eta}\right)^{2}+Ec\,Pr\,MW(\eta)^{2}, \tag{13}$$

with

$$\left.\begin{array}{l}W=Se^{-\lambda\theta}\dfrac{dW}{d\eta},\quad a\dfrac{d\theta}{d\eta}=b\theta-1,\quad at\quad \eta=0\\ \dfrac{dW}{d\eta}=0,\quad \dfrac{d\theta}{d\eta}=-Bi\theta,\quad at\quad \eta=1\end{array}\right\} \tag{14}$$

where Bi denotes Biot number, S is the slip parameter, Pr is the Prandtl number, Ec is the Eckert number, M is the magnetic field parameter, Ra is the Rayleigh number, Nr is the radiation parameter and λ is the variable viscosity parameter. Other parameters of interests are skin friction coefficients (C_f), Nusselt number (Nu), and Bejan number defined as:

$$C_{f}=\frac{\delta^{2}\tau_{w}}{\rho\upsilon_{0}^{2}}=e^{-\lambda\theta}\left.\frac{dW}{d\eta}\right|_{\eta=0},\quad Nu=\frac{\delta q_{w}}{k(T_{w}-T_{\infty})}=-(1+Nr)\left.\frac{d\theta}{d\eta}\right|_{\eta=0},Be=\frac{N_{1}}{Ns}=\frac{1}{1+F}, \tag{15}$$

where the wall shear stress τ_w, the wall heat flux q_w and the irreversibility ratio F are given as:

$$\left.\begin{array}{l}\tau_{w}=\mu\left.\dfrac{du}{dy}\right|_{y=0},\quad q_{w}=-k\left(1+\dfrac{16\sigma^{*}T_{\infty}^{3}}{3kk^{*}}\right)\left.\dfrac{dT}{dy}\right|_{y=0},\quad N_{1}=(1+Nr)\left(\dfrac{d\theta(\eta)}{d\eta}\right)^{2},\quad F=\dfrac{N_{2}}{N_{1}},\\ N_{2}=Ec\,Pr\,e^{-\lambda\theta(\eta)}\left(\dfrac{dW(\eta)}{d\eta}\right)^{2}+Ec\,Pr\,MW(\eta)^{2},\end{array}\right\} \tag{16}$$

and N_1 represents irreversibility due to heat transfer while N_2 denotes entropy due to viscous dissipation.

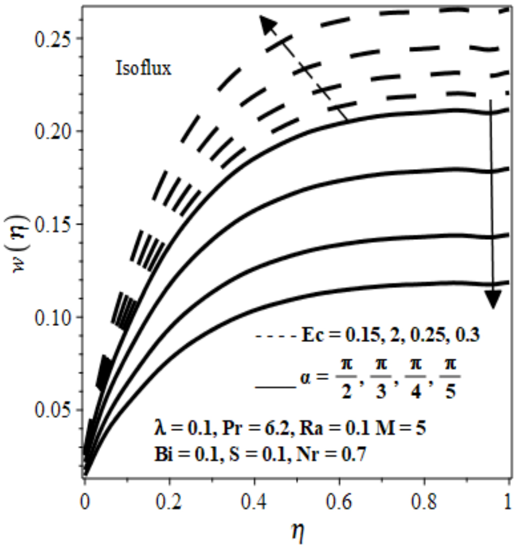

Fig. 2. Velocity profile with increasing α and Ec

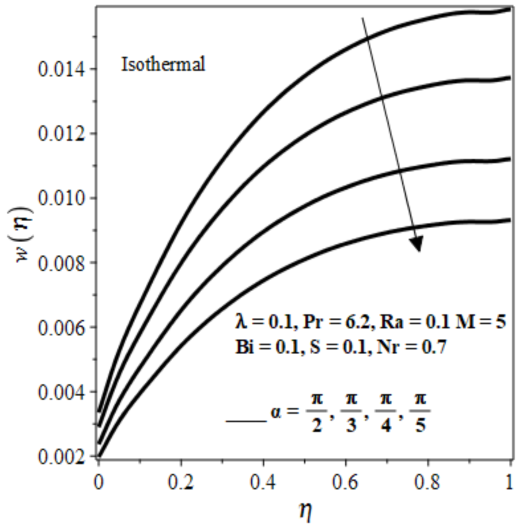

Fig. 3. Velocity profile with increasing α

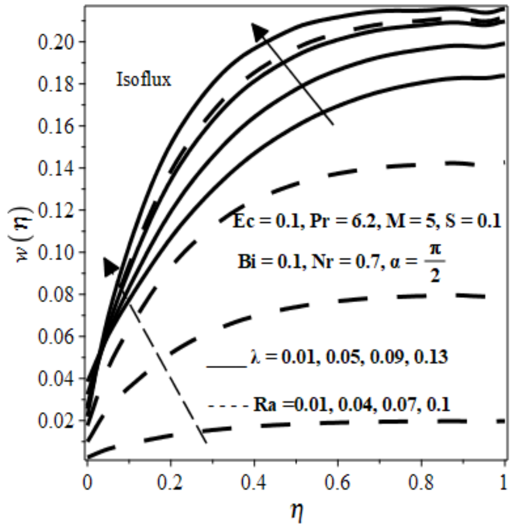

Fig. 4. Velocity profile with increasing λ and Ra

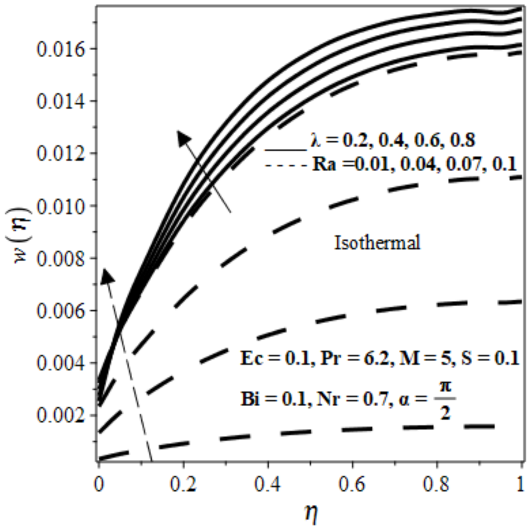

Fig. 5. Velocity profile with increasing λ and Ra

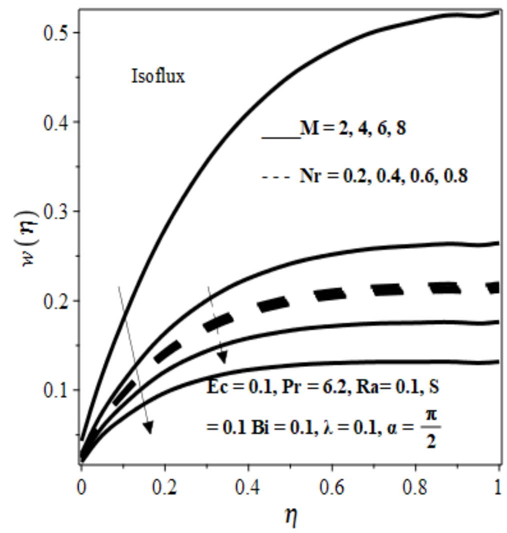

Fig. 6. Velocity profile with increasing M and Nr

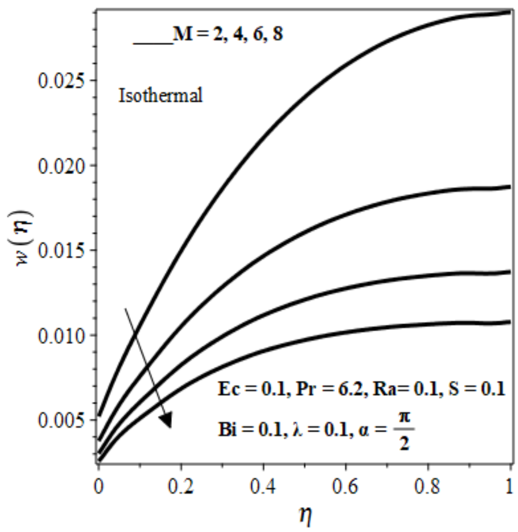

Fig. 7. Velocity profile with increasing M

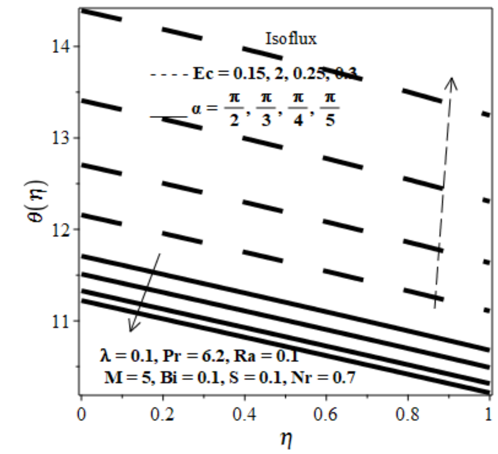

Fig. 8. Temperature profile with increasing α and Ec

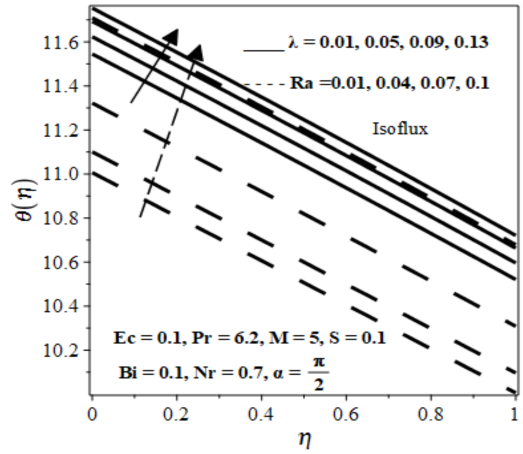

Fig. 9. Temperature profile with increasing λ and Ra

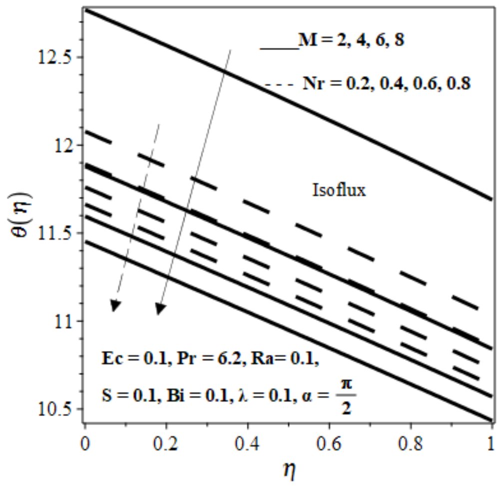

Fig. 10. Temperature profile with increasing M and Nr

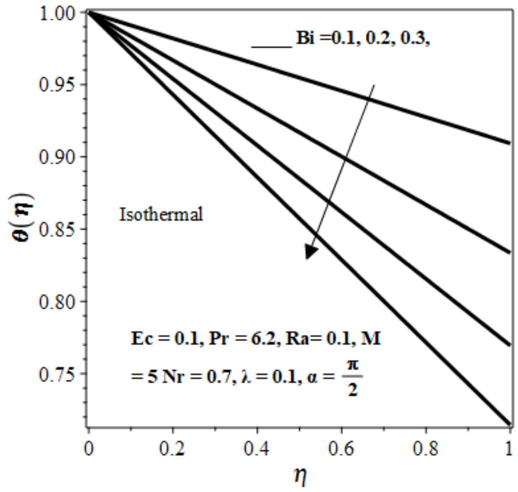

Fig. 11. Temperature profile with increasing Bi

3. Numerical Procedure

The dimensionless model equations (11-12) alongside boundary conditions (14) expressing the boundary value problem (BVP). We convert these equations into sets of nonlinear first-order ordinary differential equations as well as unspecified initial conditions to be found by shooting techniques. We assume

$$w(\eta) = k_1, \qquad w'(\eta) = k_2, \qquad \theta(\eta) = k_3, \quad \theta'(\eta) = k_4 \tag{17}$$

Consequently, $k_1' = k_2, k_3' = k_4$. The dimensionless governing equations (11-12) now become:

$$\left.\begin{aligned} & k_2' = \lambda k_4 k_2 + M k_1 e^{\lambda k_3} - Ra k_3 e^{\lambda k_3} \sin(\alpha) \\ & (1+Nr)k_4' + Ec \Pr k_2^2 e^{-\lambda k_3} + Ec \Pr M k_1^2 = 0 \end{aligned}\right\} \tag{18}$$

with initial conditions

$$k_1(0) = S k_2(0) e^{-\lambda k_3(0)}, \; k_2(0) = p_1, \; k_3(0) = 0, \; k_4(0) = p_2 \tag{19}$$

where p_1 and p_2 are assumed unspecified initial conditions and thereafter determined by using the Newton Raphson method based on the prescribed boundary conditions for each set of parameter values. The outcome of the initial value problems is solved numerically by utilizing the Runge-Kutta-Fehlberg integration outline [21].

4. Results and Discussion

We carried out the numerical solution in lieu of velocity profile, temperature profile, skin friction profile, Nusselt number, entropy generation rate profile, and the Bejan number by ascribing some randomly selected specific values to different thermophysical parameters regulating the flow structure. The numerical results obtained for a special case of this present study (Nr=0, S=0) is compared with that of [3], as showing in Figs. 26-27, and an excellent agreement is achieved. This validates the accuracy of our numeral procedure and benchmark our present numerical results with the one already in the literature. Figs. 2-7 illustrate the effects of various thermophysical parameters on the velocity profile for both isothermal and isoflux walls. It can be noted in Figs. 2-3 that as the angle of inclination is decreasing for both isoflux and isothermal walls, the fluid velocity profiles decrease and attained maximum fluid velocity at the free surface with a higher angle of inclination. The decrease may be attributed to the presence of viscous forces in the fluid velocity equation. It is also discovered in Fig. 2 that the fluid velocity profile increases with increasing Eckert number at the free surface. In Figs. 4-5, we observed that the fluid velocity profiles for both isoflux and isothermal increases with the increase in the Rayleigh number and variable viscosity parameter. This may be attributed to the fact that an increase in Rayleigh number creates an avenue to overcome the viscous forces and therefore improve the fluid velocity profiles. The effects of the magnetic field parameter and radiation parameter are shown in Figs. 6-7. It is shown in Fig. 7 that the radiation parameter has no effects on fluid velocity in the isothermal wall, but increasing these parameters tend to decrease the fluid velocity profiles at the free surfaces.

Figs. 8-11 depict the effects of some of these thermophysical parameters on the temperature profiles. As can be seen in Fig. 8, the temperature profile improved with increasing the Eckert number and slow down the temperature profile with decreasing the angle of inclination for the isoflux wall. Fig. 9 presents the effects of the viscosity parameter and Rayleigh number in the temperature profile. It is observed that the temperature profile rises with increasing these parameters. The impacts of the magnetic field and radiation parameter on the temperature profile id presented in Fig. 10. It can be seen from this figure that temperature profile decrease as the magnetic field parameter and radiation parameter is increasing for the isoflux wall. As these parameters are increasing, it slows down the particle in a less sporadic rate and hence reduce the temperature profile. Fig. 11 shows the influence of the Biot number on the temperature profile. As the Biot number increases, the fluid temperature unites at the wall surface and gradually decreasing towards the free surface. This decrease in temperature profile is noticed due to a slippery convective cooling in an inclined liquid film.

The impacts of some of these thermophysical parameters on the entropy generation rate profile are displayed in Figs. 12-17. The effects of the angle of inclination and the Eckert number are presented in Figs. 12-13 for both isothermal and isoflux walls. It is noticed at these figures that increase in Eckert number caused the entropy generation rate to be increased. This may invariably inhibit the system efficient operation since the Eckert number enhances the heat transfer irreversibility. In the same figures, as the angle of inclination decreases, we noticed retardation in the entropy generation rate. This is expected since the fluid friction irreversibility lessens with a reduction in inclination angle; consequently, the entropy production rate drops.

The impact of variable viscosity parameter and Rayleigh number for both isoflux and isothermal on entropy generation rate are shown in Figs. 14- 15. It is clearly seen in these figures that an increase in each of these parameters improved the entropy generation rate profiles but more pronounced at the free surface. This may be attributed to an escalation in the heat transfer irreversibility at the free surface due to convective cooling, hence enhances the entropy generation rate. Figs. 16-17 depict the influences of the magnetic field parameter and radiation parameter for both isoflux and isothermal wall on entropy generation rate. It is noticed that the increase in radiation parameter creates more restrictive forces that boost the entropy generation rate while an increase in the magnetic field parameter prevents restrictive forces and thereby reduces the entropy generation rate. Moreover, It is noteworthy that the presence of thermal radiation boosts the heat transfer irreversibility effect while the imposed magnetic field serves as a flow control that lessens the fluid friction irreversibility.

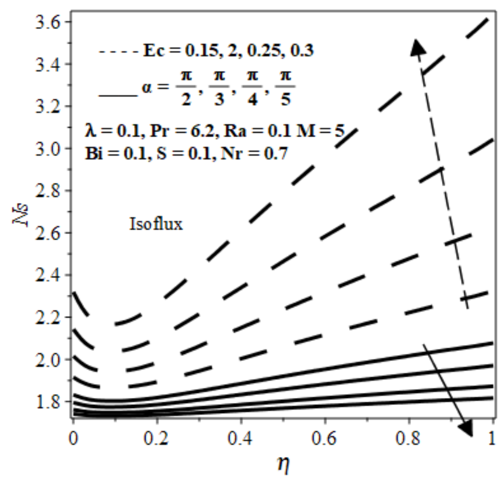

Fig. 12. Entropy profile with increasing α and Ec

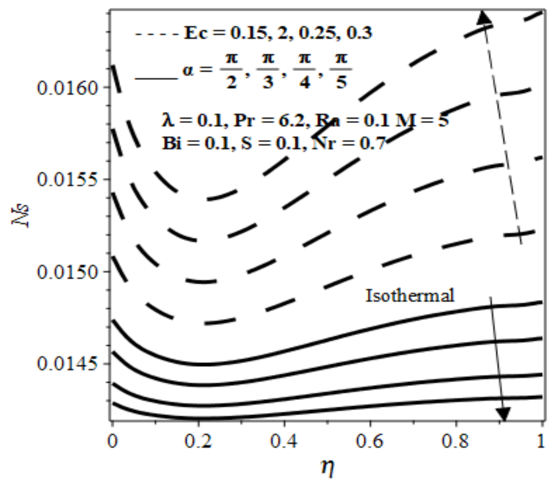

Fig. 13. Entropy profile with increasing α and Ec

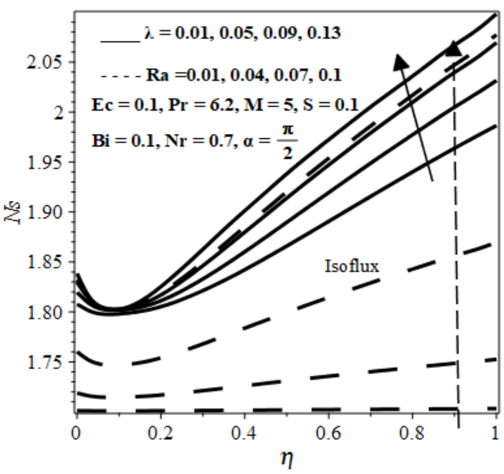

Fig. 14. Entropy profile with increasing λ and Ra

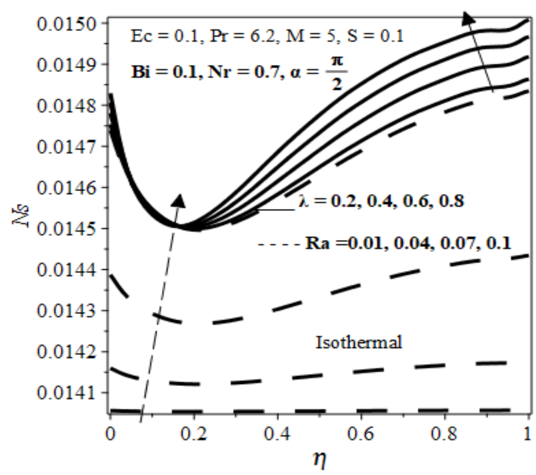

Fig. 15. Entropy profile with increasing λ and Ra

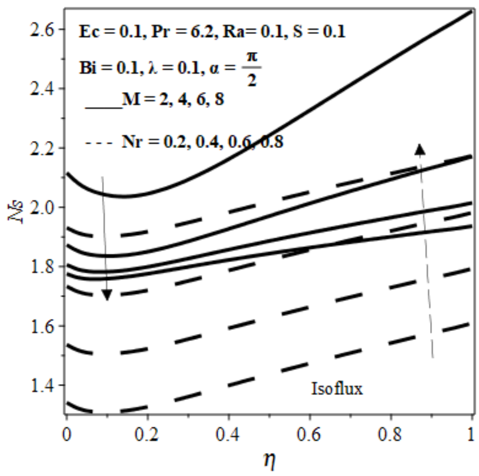

Fig. 16. Entropy profile with increasing M and Nr

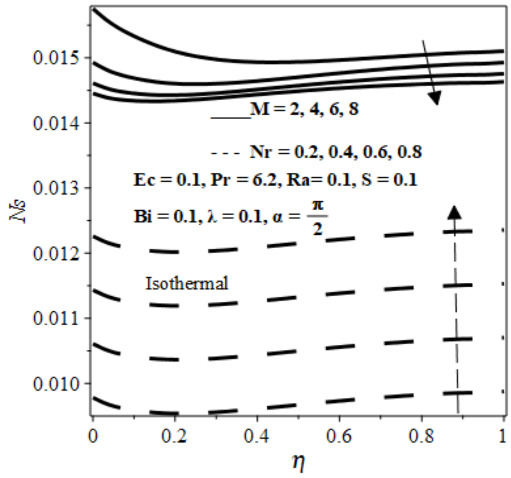

Fig. 17. Entropy profile with increasing M and Nr

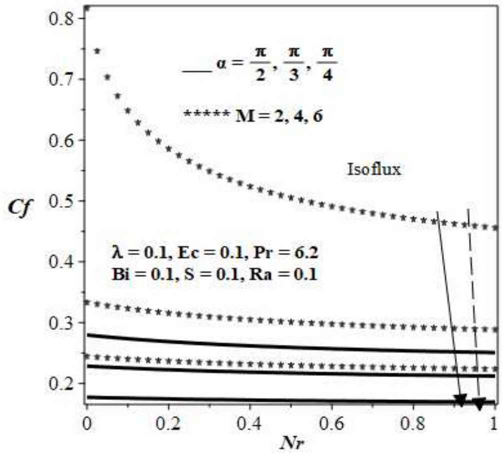

Fig. 18. Skin friction with increasing Nr versus α and M

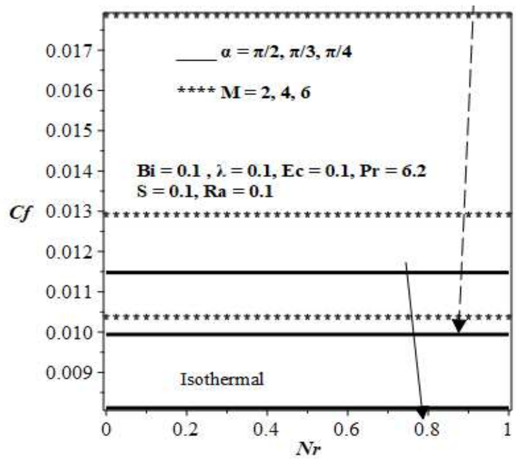

Fig. 19. Skin friction with increasing Nr versus α and M

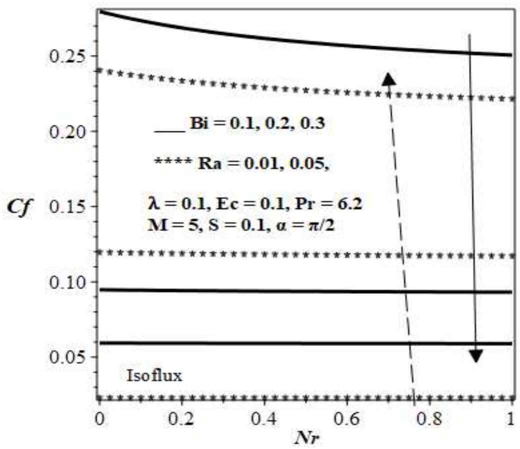

Fig. 20. Skin friction with increasing Nr versus Bi and Ra

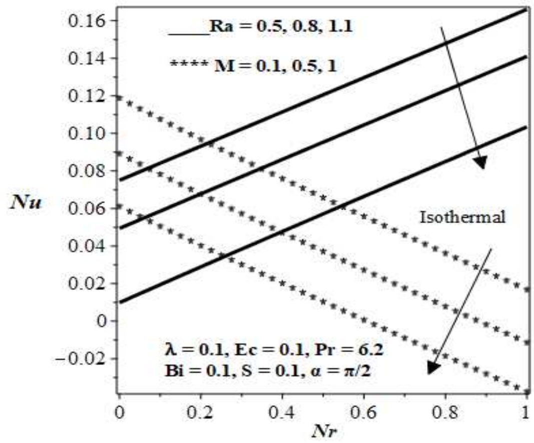

Fig. 21. Nusselt with increasing Nr versus Ra and M

The variation of skin friction profiles and Nusselt number profile on different values of thermophysical parameters are displayed in Figs. 18-21. The impacts of the angle of inclination and magnetic field parameters are revealed in Figs. 18-19. It is noted in these figures that skin friction decreases with increasing angle of inclination and magnetic field parameter at both wall surfaces and free surfaces walls while in Fig. 20, an increase in Rayleigh number enhanced the skin friction profile, but an increase in Biot number slow down the skin friction. In Fig. 21, the influences of the Rayleigh number and magnetic field parameter on Nusselt number are disclosed. It is seen in this figure that increases in these parameters slow down the Nuselt number.
The behaviors of various involved parameters on the Bejan number are shown in Figs. 22-25. Figs. 22-23 concealed effects of Eckert number and angle of inclination on the Bejan number. It is observed that the Bejan number decreases as the Eckert number is increasing while decreasing angle of inclination leads to an increase in the Bejan number profiles. This implies that irreversibility due to heat transfer dominates as the angle of inclination decreases and entropy due to viscous dissipation dominates with an increase in Eckert number. In Fig. 24, the Bejan number decreases with increasing variable viscosity parameter and Rayleigh number. Hence, entropy due to viscous dissipation dictates the flow structure. Meanwhile, Fig.25 reflected the increase in the magnetic parameter and radiation parameter on the Bejan number. It is observed that the Bejan number increase as these parameters is increasing. This led to the dominant effect of irreversibility heat transfer on the flow.

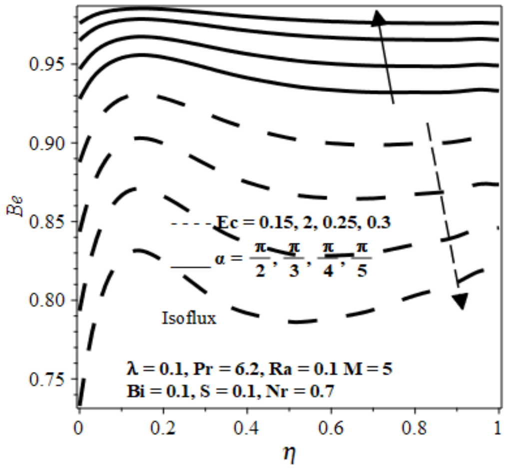

Fig. 22. The Bejan profile with increasing α and Ec

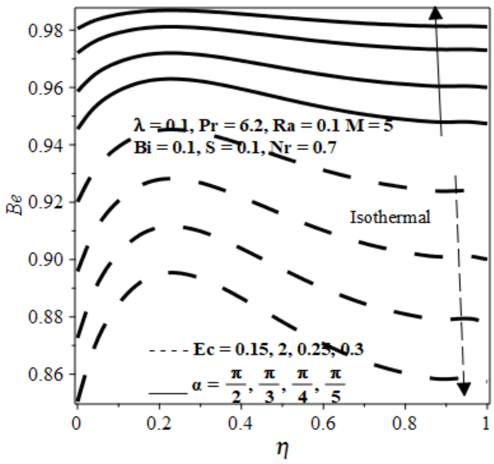

Fig. 23. The Bejan profile with increasing α and Ec

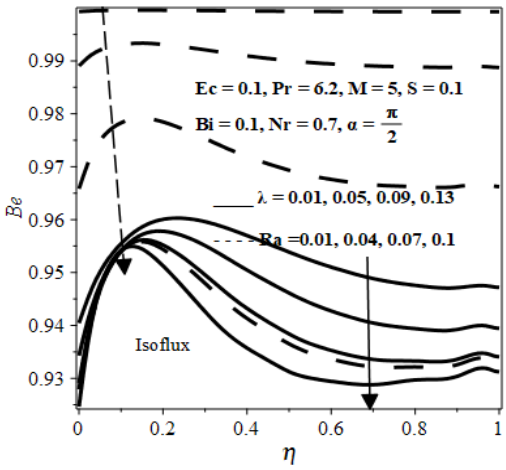

Fig. 24. The Bejan profile with increasing λ and Ra

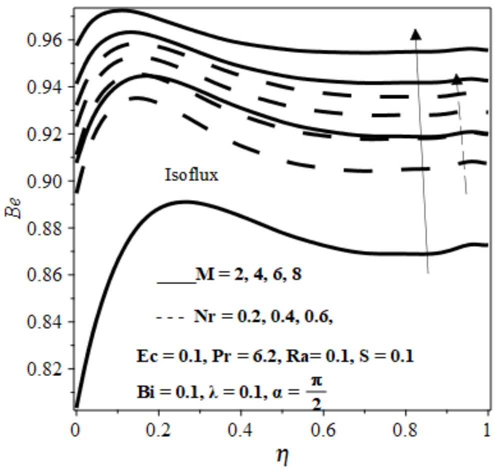

Fig. 25. The Bejan profile with increasing M and Nr

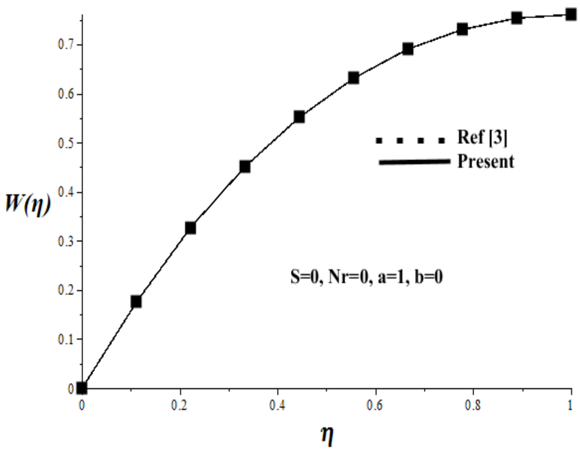

Fig. 26. Velocity profile (Ref. [3] and present analysis)

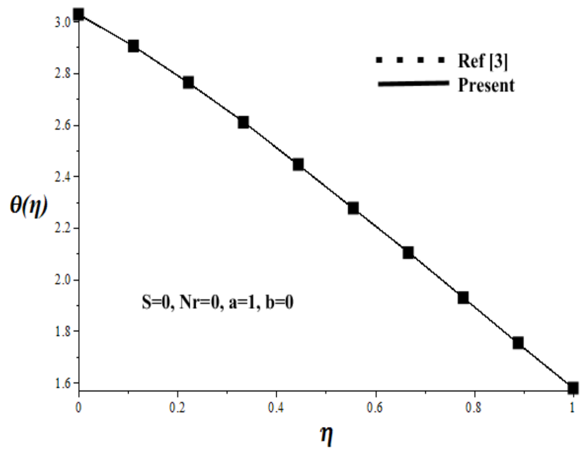

Fig. 27. Temperature profile (Ref. [3] and present analysis)

5. Conclusion

The hydromagnetic flow of a Newtonian incompressible and electrically conducting liquid film along a slippery inclined heated convective cooling was numerically investigated. Using the Newton Raphson method, together with the Runge-Kutta-Fehlberg integration scheme, the nonlinear model initial boundary value problem was solved. A few results obtained can be summarised as follow:

- Velocity profiles increase with rising in Eckert number, variable viscosity parameter, and Rayleigh number while increasing magnetic field parameter, radiation parameter, and decrease in angle of inclination reduce the velocity profiles.
- Temperature profiles increase by increasing the Eckert number, variable viscosity parameter, and Rayleigh number while increasing the magnetic field parameter, radiation parameter, Biot number. Moreover, the decrease in angle of inclination

reduces the temperature profiles.

- An increase in the Eckert number, variable viscosity parameter, radiation parameter, and Rayleigh number enhanced the entropy profile while increasing the magnetic field parameter and decreasing the angle of inclination decrease the entropy profiles.
- The skin friction increases with increasing Rayleigh number and decreases by increasing magnetic field parameter, Boit number, and decreasing angle of inclination.
- The Nusselt number increases with increasing the Rayleigh number and decreases with increasing the magnetic field parameter.
- The Bejan number profiles decrease with increasing Eckert number, variable viscosity parameter, and Rayleigh number while it upsurges with the increasing magnetic field parameter, radiation parameter, and decreasing angle of inclination.

Finally, adequate knowledge of combined effects of surface slip and thermal radiation on entropy generation rate is very relevant and significant, in several natural and industrial settings for efficient operation, the hydrophobic surface is usually represented by a slip boundary condition on the surface, and there are several applications such as lubrication, microfluidics polymer melt, etc., where the velocity of a viscous fluid exhibits a tangential slip on the wall.

Author Contributions

Both authors planned the work, formulated the models and carry out the analysis. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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