Flow and Heat Transfer Analysis of the Sodium Alginate Conveying Copper Nanoparticles between Two Parallel Plates

Akin T. Akinshilo¹, Joseph O. Olofinkua², Osamudiamen Olaye³

¹ Mechanical Engineering Department; University of Lagos, Akoka-Yaba, 100001, Nigeria. ta.akinshilo@gmail.com
² Mechanical Engineering Department; University of Lagos, Akoka-Yaba, 100001, Nigeria. josepholofinkua@yahoo.com
³ Mechanical Engineering Department; University of Benin, Benin City, 300271, Nigeria. osamudiamen.olaye@uniben.edu

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Corresponding author: Akin T. Akinshilo, ta.akinshilo@gmail.com
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Abstract. In this study, the steady incompressible flow of a non-Newtonian sodium alginate (SA) fluid conveying copper nanoparticles (Cu) which flow within two vertical parallel plates is investigated by using the homotopy perturbation analytical scheme to solve the coupled nonlinear ordinary equations arising from the mechanics of the fluid. The developed analytical solutions are used to investigate the effect of the fluid flow and heat transfer parameters such as the nanoparticle concentration, the non-Newtonian parameter and the viscosity variation parameter. The obtained analytical results as compared to existing works in literature are in satisfactory agreements. Moreover, the results obtained from the present study can be used for further analysis of the behavior of the sodium alginate in applications such as food processing and chemical and pharmaceutical industries.

Keywords: Sodium alginate; Copper; Parallel plates; Perturbation; Nano fluid; Non-Newtonian.

1. Introduction

The sodium alginate is a sodium salt of the alginic acid which exists in nature as a white or yellowish brown filament, granule or powder form which a viscous solution is formed when dissolved in water. The study of the flow of the sodium alginate, a non-Newtonian fluid, has been the area of interest over recent years due to its extensive applications in manufacturing and biomedicine. This has led to a renewed effort to enhance the flow and heat transfer of the fluid and to investigate physical phenomena which are described by nonlinear differential equations and commonly are found in engineering applications such as textile, paper production, pharmaceuticals and tissue formation engineering.

Regarding previous efforts to analyze such flow processes, Hatami and Ganji [1] analyzed the flow of the sodium alginate conveying the titanium dioxide, a nanoparticle within porous media between two coaxial cylinders, which was an innovative way of improving the heat transfer and increasing the overall energy and the thermal capacity of the fluid, shortly after which Hatami and Ganji [2] extended their study to investigate the effect of the natural convection on the flow of the sodium alginate between two vertical parallel plates. Ganji and Kachapi [3] analyzed nonlinear equations in fluids by using numerical and analytical schemes for solving coupled nonlinear differential equations. The differential transformation method was applied by Hatami and Jing [4] to compare the flow analysis of the Newtonian nano-fluid between two horizontal plates and the non-Newtonian nano-fluid between two vertical plates; the obtained solution were compared to the numerical solutions which were derived through fourth order Runge-Kutta method. Ziabakhsh and Domairry [5] investigated the effects of natural convections...
on the flow of a non-Newtonian fluid between two vertical parallel plates by using the homotopy analytical method. A comprehensive analysis of the fluid flow and the heat transfer of the nano-fluid flowing unsteadily through a flat plate was presented by Ahmadi et al. [6]. The study of the nano-fluid flow and the heat transfer between two parallel plates by considering Brownian motion was presented by Sheikholeslami and Ganji [7] using the differential transformation method. Later, Sheikholeslami et al. [8] extended their research on the nano-fluid flow by considering thermo-phoresis and Brownian motion effects. Pourmehran et al. [9] investigated the effect of squeezing on the unsteady nano-fluid flow between two parallel plates using numerical schemes. The mixed convection boundary layer flow was presented by Mandy [10] where he considered the nano-fluid to flow unsteadily due to the sheet stretching. Hamad et al. [11] studied the effects of the free convection flow of a nano-fluid through a vertical semi-infinite plate by considering the effect of the magnetic field. Mustafa et al. [12] studied the heat and mass transfer effect of the unsteady squeezing flow between two parallel plates. The unsteady squeezing flow of the viscous MHD fluid was investigated by Siddiqui et al. [13] where parallel plates were considered as the fluid flow medium. Afify and Abdel-Azizi [14] applied the Lie group analysis to study the steady two-dimensional flows and the heat transfer of a non-Newtonian power law fluid over a stretching sheet under the convective boundary conditions. The two-phase simulation of the flow and heat transfer of the nano-fluid flow through an annulus were presented by Sheikholeslami and Abelman [16] while Madaki et al. [17] investigated the effect of squeezing on the unsteady nano-fluid flow under the thermal radiation by applying the homotopy perturbation and fourth order runge kutta schemes. The thermodynamic analysis of the third grade fluid was studied by Adesanya and Falade [18] where they presented the rate of the entropy generation of the fluid flow between horizontal parallel plates saturated with porous materials. The unsteady incompressible flow between parallel plates was examined by Hoshyar et al. [19] using the homotopy analysis method. The effect of variable viscosity on the fluid flow between parallel plates with the shear heating was presented by Myers et al. [20]. Besides, Kargar and Akbarzade [21] investigated the non-Newtonian fluid flow between two infinitely long vertical parallel plates by using both analytical and numerical approaches.

Getting motivated by the above-mentioned research, this study aims to investigate the flow and heat transfer of the sodium alginate (SA) which conveys copper (Cu) nanoparticles by using the homotopy perturbation analytical scheme. For this purpose, the effects of various fluid parameters such as the viscosity variation parameter, the non-Newtonian parameter, and the nanoparticle concentration term are discussed in detail.

2. Problem Description and Mathematical Formulation

The proposed nano-fluid is a non-Newtonian sodium alginate based fluid with copper nanoparticles which flows steadily under the natural convection between two plates which are placed vertically against each other as shown in the physical model of the problem (Fig. 1).

![Fig. 1. Physical model of the problem](image)

The velocity at both walls of the plates are equal to zero which connotes that the nano-fluid and the wall of the plates are at equal velocity; the nano-fluid that is close to the wall surface sticks which satisfies the no slip condition. The left wall is held at 0.5 and the right wall is held at -0.5 which connotes a constant temperature but a different magnitude which causes a rise in the nano-fluid close to the left wall and a fall in the fluid level close to the right wall. The formulation of the development model of the two mixed components is done based on the assumptions that the fluid is incompressible, the base fluid and nanoparticles are in the thermal equilibrium to each other and the wall temperature is constant. Therefore, according to the above-mentioned assumptions, the momentum and energy equations can be reduced to ordinary pairs of differential equations. The details of the governing equations and non-dimensional parameters have been described in [4]. Following the notation of the [4] study on incorporating the variational viscosity parameter, which measures the effect of varying viscosity on the flow and heat transfer of the nano-fluid, the governing equations and the corresponding boundary conditions are introduced as follows:

\[
\frac{d^2V}{dX^2} + 6\delta(1-\phi)^{2.5}(\gamma + \frac{d^2V}{dX^2})^2 + \theta
\]
\[
\frac{d^2 \theta}{dX^2} + Ec \cdot Pr \left( \frac{1}{A_1} \right) \left[ \frac{d^2 \psi}{dX^2} \right]^2 + 2\delta Ec \cdot Pr \left( \frac{1}{A_1} \right) \left[ \frac{d\psi}{dX} \right]^4 = 0
\] (2)

The boundary conditions are as follows:

\[ v = 0, \theta = 0.5 \quad \text{at} \quad x = -1 \] (3)

\[ v = 0, \theta = -0.5 \quad \text{at} \quad x = 1 \] (4)

where the effective density, \( \rho_{nf} \), the effective dynamic viscosity, \( \mu_{nf} \), the heat capacitance, \( (\rho C_p)_{nf} \), and the thermal conductivity of the nano-fluid, \( k_{nf} \), are defined as follows [4]:

\[ \rho_{nf} = (1-\phi)\rho_f + \phi \rho_s \] (5)

\[ (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi (\rho C_p)_s \] (6)

\[ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \] (7)

\[ A_1 = \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f-k_s)}{k_s + 2k_f - \phi(k_f-k_s)} \] (8)

### Table 1. Thermo-physical properties of the sodium alginate and the copper nanoparticle [2].

<table>
<thead>
<tr>
<th></th>
<th>Density ( (\text{Kg/m}^3) )</th>
<th>Specific heat capacity ( (\text{J/kgK}) )</th>
<th>Thermal conductivity ( (\text{W/mk}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sodium Alginate (SA)</td>
<td>989</td>
<td>4175</td>
<td>0.637</td>
</tr>
<tr>
<td>Copper</td>
<td>8933</td>
<td>385</td>
<td>401</td>
</tr>
</tbody>
</table>

According to the homotopy perturbation scheme, the homotopy of Eq. (1) and (2) can be expressed as:

\[ v(p, \eta) = (1-p) \left[ \frac{d^2 \psi}{dX^2} + \theta \right] + p \left[ \frac{d^2 \psi}{dX^2} + 6\delta (1-\phi)^{2.5} \left( \gamma + \frac{d^2 \psi}{dX^2} \right)^2 \right] + \theta \] (9)

\[ \theta(p, \eta) = (1-p) \left[ \frac{d^2 \theta}{dX^2} \right] + p \left[ \frac{d^2 \theta}{dX^2} + Ec \cdot Pr \left( \frac{1}{A_1} \right) \left[ \frac{d\psi}{dX} \right]^4 \right] + \theta \] (10)

Considering series solutions for the velocity and the temperature, the following equations are obtained:

\[ v = v_0 + pv_1 + O(p^2) \] (11)

\[ \theta = \theta_0 + p\theta_1 + O(p^2) \] (12)

Substituting Eqs. (11) and (12) into Eq. (9) yields:

\[ O(p^0): \frac{d^2 \psi_0}{dX^2} + \theta_0 \] (13)

\[ O(p^1): \frac{d^2 \psi}{dX^2} + 6\delta (1-\phi)^{2.5} \left( \gamma + \frac{d^2 \psi}{dX^2} \right)^2 + \theta_1 \] (14)

Substituting Eqs. (11) and (12) into Eq. (10) yields:

\[ O(p^0): \frac{d^2 \theta_0}{dX^2} \] (15)

\[ O(p^1): \frac{d^2 \theta}{dX^2} \left[ Ec \cdot Pr \left( \frac{d\psi_0}{dX} \right)^2 \right] / A_1 (1-\phi)^{2.5} + 2Ec \cdot Pr \delta \left( \frac{d\psi_0}{dX} \right)^4 / A_1 \] (16)
The leading order boundary condition is:

\[ v_0(0) = 0, \theta_0(0) = 0, v_0(1) = 0, \theta_0(1) = 0.5 \]  
(17)

Simplifying Eq. (13) and applying the leading order boundary condition (Eq. (17)) yields:

\[ v_0 = \frac{x}{12} (x^2 - 1) \]  
(18)

Simplifying Eq. (15) and applying the leading order boundary condition (Eq. (17)) yields:

\[ \theta_0 = -\frac{x}{2} \]  
(19)

The first order boundary condition is:

\[ v_1(0) = 0, \theta_1(0) = 0, v_1(1) = 0, \theta_1(1) = 0.5 \]  
(20)

Simplifying Eq. (14) and applying the first order boundary condition (Eq. (20)) yields:

\[ v_1 = x^6 \frac{90 \gamma \Delta \phi^6 - 18 \gamma \Delta \phi^5 - 180 \gamma \Delta \phi^4 + 180 \gamma \Delta \phi^3 - 90 \gamma \Delta \phi + 18 \gamma \phi^2}{288(1 - \phi)^{2.5}} - \frac{Ec \cdot Pr}{144 \Delta (1 - \phi)^{2.5}} \]

\[ x^2 \left[ \frac{Ec \cdot Pr}{240 \Delta (1 - \phi)^{2.5}} \cdot \frac{Ec \cdot Pr}{16 \Delta (1 - \phi)^{2.5}} + \frac{Ec \cdot Pr}{48 \Delta} \right] + x^3 \left[ \frac{Ec \cdot Pr}{12 \Delta (1 - \phi)^{2.5}} + \frac{Ec \cdot Pr}{40 \Delta} \right] + \left[ \frac{5 \gamma \Delta \phi^4 - 10 \gamma \Delta \phi^3 + 10 \gamma \Delta \phi^2 - 5 \gamma \Delta \phi + \phi}{1024(1 - \phi)^{2.5}} \right] - \frac{x^5}{288(1 - \phi)^{2.5}} \left[ 90 \gamma \Delta \phi^4 - 18 \gamma \Delta \phi^3 - 180 \gamma \Delta \phi^2 + 180 \gamma \Delta \phi - 90 \gamma \Delta \phi + 18 \gamma \phi^2 \right]

\[ + x^6 \left[ \frac{30 \gamma \Delta \phi^6 - 6 \gamma \Delta \phi^5 - 60 \gamma \Delta \phi^4 + 60 \gamma \Delta \phi^3 - 30 \gamma \Delta \phi + 6 \phi}{1441(1 - \phi)^{2.5}} \right] - \frac{x^7}{288(1 - \phi)^{2.5}} \left[ 45 \gamma \Delta \phi^4 - 9 \gamma \Delta \phi^3 + 90 \gamma \Delta \phi^2 - 45 \gamma \Delta \phi + 9 \phi \right]

\[ + x^8 \left[ \frac{60 \gamma \Delta \phi^6 - 12 \gamma \Delta \phi^5 - 120 \gamma \Delta \phi^4 + 120 \gamma \Delta \phi^3 - 60 \gamma \Delta \phi + 12 \gamma \phi}{144(1 - \phi)^{2.5}} \right] + \frac{Ec \cdot Pr \cdot \phi^7}{112 \Delta} \]

\[ x^2 \left[ \frac{5 \gamma \Delta \phi^4 - 10 \gamma \Delta \phi^3 + 10 \gamma \Delta \phi^2 - 5 \gamma \Delta \phi + \phi}{96(1 - \phi)^{2.5}} \right] - \frac{x^5}{240(1 - \phi)^{2.5}} \left[ 30 \gamma \Delta \phi^6 - 6 \gamma \Delta \phi^5 - 60 \gamma \Delta \phi^4 + 60 \gamma \Delta \phi^3 - 30 \gamma \Delta \phi + 6 \phi \right]

\[ + x^7 \left[ \frac{45 \gamma \Delta \phi^4 - 9 \gamma \Delta \phi^3 + 90 \gamma \Delta \phi^2 - 45 \gamma \Delta \phi + 9 \phi}{366(1 - \phi)^{2.5}} \right] + x^8 \left[ \frac{10 \gamma \Delta \phi^6 - 24 \gamma \Delta \phi^5 - 20 \gamma \Delta \phi^4 + 20 \gamma \Delta \phi^3 - 10 \gamma \Delta \phi + 2 \gamma \phi}{96(1 - \phi)^{2.5}} \right]

\[ - \frac{7 \gamma \Delta \phi^4}{192(1 - \phi)^{2.5}} + \frac{9 \gamma \Delta \phi^4}{112 \Delta} \]

\[ + \frac{10 \gamma \Delta \phi^4 - 2 \gamma \Delta \phi^3 - 20 \gamma \Delta \phi^2 - 10 \gamma \Delta \phi + 2 \gamma \phi}{96(1 - \phi)^{2.5}} - \frac{60 \gamma \Delta \phi^6 - 12 \gamma \Delta \phi^5 - 180 \gamma \Delta \phi^4 + 180 \gamma \Delta \phi^3 - 90 \gamma \Delta \phi + 18 \gamma \phi}{1440(1 - \phi)^{2.5}}

\[ = \frac{Ec \cdot Pr \cdot \phi^8}{384 \Delta} \]

Simplifying Eq. (16) and applying the first order boundary condition (Eq. (20)) yields:

\[ \theta_1 = \frac{Ec \cdot Pr}{48 \Delta (1 - \phi)^{2.5}} - \frac{Ec \cdot Pr \cdot \phi^4}{12 \Delta (1 - \phi)^{2.5}} + \frac{Ec \cdot Pr \cdot \phi \cdot \Delta}{240 \Delta} + \frac{Ec \cdot Pr \cdot \phi \cdot \Delta^5}{16 \Delta (1 - \phi)^{2.5}} + \frac{Ec \cdot Pr \cdot \phi^6}{48 \Delta} \]

(22)

Substituting Eqs. (18) and (21) into the series solution (Eq. (11)) leads to the following equations:
Prandtl’s number (Pr) on the velocity profile. It can be seen that the velocity distribution increases slightly as Pr increases. This

\[ \frac{\theta}{\theta_0} = \frac{\phi_0}{\phi} + \frac{\phi_0}{\phi} N \]

where \( \theta \) is the temperature difference, \( \phi \) is the dimensionless temperature, \( \phi_0 \) is the initial temperature, and \( N \) is the characteristic dimensionless number. The solution to this equation is given by

\[ \theta = \frac{\phi_0}{\phi} \left[ 1 + N \right] \]

Substituting Eqs. (19) and (22) into the series solution (Eq. (12)) leads to the following equations:

\[ \theta = \frac{\phi_0}{\phi} \left[ 1 + N \right] \]

This section discusses the results obtained from the analytical solutions which are reported graphically. Here, the effect of fluid parameters at various values on the velocity and the temperature profiles is presented. Fig. 2 shows the effect of the Prandtl’s number (Pr) on the velocity profile. It can be seen that the velocity distribution increases slightly as Pr increases. This
can be physically explained as a result of an increase in the momentum boundary layer thickness.

Fig. 2. Effect of the Prandtl’s number (Pr) on the velocity profile when Ec=γ=Q=1 and ϕ=0.01.

Fig. 3. Effect of the non-Newtonian parameter (δ) on the velocity profile when Ec=γ=Q=1 and ϕ=0.01.

Fig. 4. Effect of the viscosity variational parameter (γ) on the velocity profile when Ec=δ=Q=1 and ϕ=0.01.

Fig. 5. Effect of the nanoparticle concentration (ϕ) on the velocity profile when Ec=δ=γ=Q=1.
The effect of the non-Newtonian parameter ($\delta$) on the velocity distribution is demonstrated in Fig. 3. As can be observed, by increasing values of $\delta$, the velocity profile decreases because the rate of shear increases and causes a decrease in the boundary layer thickness. The viscosity variation parameter ($\gamma$) effect on the velocity profile is presented in the Fig. 4. It can be observed that the velocity distribution increases significantly by increasing values of $\gamma$ which can be physically explained as a result of increasing the fluid activation energy which causes an increase in the molecule collision and makes the fluid behave less viscous that leads to the rapid fluid motion. The effect of the nanoparticle concentration ($\phi$) is presented in the Fig. 5. As it is shown, the velocity distribution increases slightly by increasing values of $\phi$ due to the improvement in the energy exchange rate and increased the heat dissipation.

![Fig. 6](image1.png)  
**Fig. 6.** Effect of the non-Newtonian parameter ($\delta$) on the temperature distribution when $Ec=\gamma=Q=0.5$ and $\phi=0.05$.

![Fig. 7](image2.png)  
**Fig. 7.** Effect of the Prandtl’s number (Pr) on the temperature profile when $\delta=\gamma=Q=0.5$ and $\phi=0.05$.

![Fig. 8](image3.png)  
**Fig. 8.** Effect of the viscosity variational parameter ($\gamma$) on the temperature distribution when $Ec=\delta=Q=0.5$ and $\phi=0.05$. 
Fig. 9. Effect of the nanoparticle concentration ($\phi$) on the temperature distribution when $Ec=0.5, \gamma=Q=0.05$.

As the non-Newtonian parameter ($\delta$) increases (Fig. 6), the temperature distribution increases due to the decrease in the thermal boundary layer thickness. The influence of the Prandtl’s number (Pr) on the temperature profile is shown in Fig. 7 which indicates that the temperature increases significantly by increasing values of Pr which is caused by an increase in the thermal boundary layer. While by increasing values of the viscosity variational parameter ($\gamma$), the flow of the nano-fluid is enhanced and improves the heat transfer rate, therefore, a corresponding increase in the temperature distribution is observed in Fig. 8. Besides, the nanoparticle concentration ($\phi$) effect is demonstrated in Fig. 9 where it is shown that the temperature distribution decreases by increasing values of $\phi$ which can be physically explained by the increase in viscosity due to a corresponding increase in the rate of shear at the walls.

4. Conclusion

In this study, the nonlinear equations arising from the flow and heat transfer of the sodium alginate (SA) which conveys copper (Cu) nanoparticles was analyzed using the homotopy perturbation analytical technique. On the other hand, the approximate analytical solutions were used to demonstrate the effect of various flow and heat transfer parameters which were provided at various values. Moreover, the effect of parameters such as the Prandtl’s number, the nanoparticle concentration and the viscosity variational parameter was examined. The obtained results can be used for further studies of the sodium alginate in processes such as manufacturing and biomedical applications.

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$A_1$</td>
<td>Dimensionless thermal conductivity</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Eckert number</td>
</tr>
<tr>
<td>$G$</td>
<td>Material constant</td>
</tr>
<tr>
<td>$k_f$</td>
<td>Base fluid thermal conductivity</td>
</tr>
<tr>
<td>$k_{nf}$</td>
<td>Effective thermal conductivity</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Nanoparticle thermal conductivity</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity in x direction</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity in y Direction</td>
</tr>
<tr>
<td>$V$</td>
<td>Dimensionless velocity in x direction Greek symbols</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Viscosity variational parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Activation energy parameter</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Dimensionless non-Newtonian parameter</td>
</tr>
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</tr>
<tr>
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<td>$\varphi$</td>
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References


