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Magnetic field effects on the elastic behavior of polymeric piezoelectric cylinder reinforced with CNTs

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Abstract

Magnetic field effects on the elastic response of polymeric piezoelectric cylinder reinforced with carbon nanotubes (CNTs) is studied. The cylinder is subjected to internal pressure, a constant electric potential difference at the inner and outer surfaces, thermal and magnetic fields. The Mori-Tanaka model is used for obtaining the equivalent material properties of the cylinder. Based on the charge and equilibrium relations, the governing differential equation of the cylinder is derived and solved analytically. The main purpose of this paper is to investigate the effect of magnetic field on the stresses, the electric potential and radial displacement distributions of the polymeric piezoelectric cylinder. The presented results indicate that considering magnetic field can reduce the stresses of nano-composite cylinder.

Keywords: Magnetic field; CNT; Piezoelectric cylinder; Mori-Tanak model; Electric filed.

1. Introduction

In recent years there has been a resurgence of interest in piezoelectricity, motivated by advances in smart structures technology. The piezoelectric phenomenon has been exploited for decades. Classic piezoelectric devices include microphones and record players. More recent applications have focused on improving existing devices and transforming them into ‘‘smart structures.’’ For example, piezoelectric actuators can be used to modify the shape of an airfoil, thereby reducing transverse vortices, or to maintain proper tension with overhead electrical wires on a locomotive pantograph. In addition to being used as actuators, which respond to changes in an electric field by producing mechanical strain, they can also be used as sensors, which respond to a mechanical strain by producing an electrical signal. One notable civil engineering application of piezoelectric sensors is in structural health monitoring. A change in the level of strain will produce an electric charge and trigger sensors in the structure.

For homogeneous piezoelectric media, Ghorbanpour et al. [1] investigated the stress and electric potential fields in piezoelectric hollow spheres. Their results showed that an existing mechanical hoop stress distribution could be neutralized by a suitably applied electric field. Saadatfar and Razavi [2] analyzed the stress in piezoelectric hollow cylinder with thermal gradient. An analytic solution to the axisymmetric problem of an infinitely long, radially polarized, radially orthotropic piezoelectric hollow circular cylinder rotating about its axis at constant angular velocity was also developed by Galic and Horgan [3]. The analytical solution of a functionally graded piezothermoelastic hollow cylinder was presented by Chen and Shi [4]. They assumed that only the piezoelectric coefficient was varying quadratically in the radial direction while the other material parameters were constants. Babaei and chen [5] later presented the analytical solution for a radially FGPM rotating hollow shaft. The same as [4] and [5], the elastic and piezoelectric constants were assumed to vary as a power function of radius by Khoshgoftar et al. [6] who studied the behaviors of a thick walled cylinder made of FGPM subjected to the temperature gradient and inner and outer pressures. In [4] and [5] however, other in-homogeneity parameters such as thermal conduction coefficient and modulus of elasticity were neglected.

With respect to developmental works on stress analysis of the cylinders, it should be noted that none of the research mentioned above, have considered smart composites and their specific characteristics. Active control of laminated cylindrical shells using piezoelectric fiber reinforced composites was studied by Ray and Reddy [7] using Mori-Tanaka model. However, the reinforced materials used were CNTs which are not smart. Also, Mori–Tanaka models for the thermal conductivity of composites with interfacial resistance and particle size distributions were studied by Bohm and Nogales [8]. Micromechanical modeling which has the potential to take into account the electrical load was used by Tan and Tong [9] for studying an imperfect textile composite. However, neither the matrix nor the reinforced material used in the composite employed in this work was smart. Agglomeration effect on the electro-magneto-thermo elastic response of a hollow piezoelectric circular cylinder reinforced with carbon nanotubes (CNTs) was presented by Loghman and Cheraghbak [10].

In the present work, magnetic field effects on magneto-electro-thermo elastic behavior of piezoelectric nano-composite cylinder are considered. The cylinder is subjected to mechanical, magnetic and thermal loads, moreover an electric potential difference is also induced by electrodes attached to the inner and outer surfaces of the cylinder. Finally, using exact solution, the stresses, electric potential and displacement distributions of the piezoelectric cylinder are obtained.

2. Mori-Tanaka Model

In this section, the effective modulus of the composite shell reinforced by CNTs is developed. Different methods are available to obtain the average properties of a composite [11]. Due to its simplicity and accuracy even at high volume fractions of the inclusions, the Mori-Tanaka method [11] is employed in this section. The CNTs content are assumed to be aligned and straight with uniform dispersion in the polymer matrix. The matrix is assumed to be isotropic and elastic, with the Young's modulus E_m and the Poisson's ratio υ_m. The constitutive relations for a layer of the composite with the principal axes parallel to the r,θ and z directions are [10]:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} k+m & l & k-m & 0 & 0 & 0 \\ l & n & l & 0 & 0 & 0 \\ k-m & l & k+m & 0 & 0 & 0 \\ 0 & 0 & 0 & p & 0 & 0 \\ 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & p \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} \tag{1}$$

where $\sigma_{ij}, \varepsilon_{ij}, \gamma_{ij}, k, m, n, l, p$ are the stress components, the strain components and the stiffness coefficients respectively. According to the Mori-Tanaka method the stiffness coefficients are given by [10-12]:

$$\begin{aligned} k &= \frac{E_m\{E_m c_m + 2k_r(1+\nu_m)[1+c_r(1-2\nu_m)]\}}{2(1+\nu_m)[E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)]} \\ l &= \frac{E_m\{c_m\nu_m[E_m+2k_r(1+\nu_m)]+2c_r l_r(1-\nu_m^2)]\}}{(1+\nu_m)[E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)]} \\ n &= \frac{E_m^2 c_m(1+c_r-c_m\nu_m)+2c_m c_r(k_r n_r-l_r^2)(1+\nu_m)^2(1-2\nu_m)}{(1+\nu_m)[E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)]} \\ &\quad + \frac{E_m[2c_m^2 k_r(1-\nu_m)+c_r n_r(1+c_r-2\nu_m)-4c_m l_r\nu_m]}{E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)} \\ p &= \frac{E_m[E_m c_m+2p_r(1+\nu_m)(1+c_r)]}{2(1+\nu_m)[E_m(1+c_r)+2c_m p_r(1+\nu_m)]} \\ m &= \frac{E_m[E_m c_m+2m_r(1+\nu_m)(3+c_r-4\nu_m)]}{2(1+\nu_m)\{E_m[c_m+4c_r(1-\nu_m)]+2c_m m_r(3-\nu_m-4\nu_m^2)\}} \end{aligned} \tag{2}$$

where the subscripts m and r stand for matrix and reinforcement respectively. C_m and C_r are the volume fractions of the matrix and the CNTs respectively and k_r ، l_r ، n_r ، p_r , m_r are the Hills elastic modulus for the CNTs [10].

3. Governing equations

As presented in Fig. 1, the hollow circular piezoelectric cylinder considered here with inner and outer radius of a and b respectively, and is subjected to axisymmetric thermo-mechanical and electrical loadings. The governing equations for a homogeneous anisotropic piezoelectric cylinder can be written as ([13])

$$\begin{bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\\ \sigma_{zz}\\ \tau_{\theta z}\\ \tau_{rz}\\ \tau_{r\theta}\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\left(\begin{Bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\\ 2\varepsilon_{\theta z}\\ 2\varepsilon_{rz}\\ 2\varepsilon_{r\theta}\end{Bmatrix}-\begin{Bmatrix}\alpha_{rr}\\ \alpha_{\theta\theta}\\ \alpha_{zz}\\ 0\\ 0\\ 0\end{Bmatrix}T\right)-\begin{bmatrix}e_{11} & 0 & 0\\ e_{12} & 0 & 0\\ e_{13} & 0 & 0\\ 0 & e_{24} & 0\\ 0 & 0 & e_{35}\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}E_{rr}\\ E_{\theta\theta}\\ E_{zz}\end{Bmatrix}, \tag{3}$$

$$\begin{bmatrix}D_{rr}\\ D_{\theta\theta}\\ D_{zz}\end{bmatrix}=\begin{bmatrix}e_{11} & e_{12} & e_{13} & 0 & 0 & 0\\ 0 & 0 & 0 & e_{24} & 0 & 0\\ 0 & 0 & 0 & 0 & e_{35} & 0\end{bmatrix}\left(\begin{Bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\\ 2\varepsilon_{\theta z}\\ 2\varepsilon_{rz}\\ 2\varepsilon_{r\theta}\end{Bmatrix}-\begin{Bmatrix}\alpha_{rr}\\ \alpha_{\theta\theta}\\ \alpha_{zz}\\ 0\\ 0\\ 0\end{Bmatrix}T\right)+\begin{bmatrix}\in_{11} & 0 & 0\\ 0 & \in_{22} & 0\\ 0 & 0 & \in_{33}\end{bmatrix}\begin{Bmatrix}E_{rr}\\ E_{\theta\theta}\\ E_{zz}\end{Bmatrix}. \tag{4}$$

where ε_{ij} and σ_{ij} are mechanical strain and stress respectively, c_{ijkl} is the elastic compliance, D_m is the component of electric displacement (also referred to as charge density), E_m is the component of electric field, e_{mi} is the piezoelectric module, which relate the electrical and mechanical effects and $\in_{mk}$ is the dielectric permittivity constant at constant stress. It is also noted that the electric field E_m can be written in terms of electric potential ϕ as

$$E=-\nabla\phi. \tag{5}$$

In addition, the temperature distribution can be obtained as

$$T(r)=F_1\ln(\mathrm{r})+\mathrm{F}_2, \tag{6}$$

where F_1 and F_2 are obtained from the thermal boundary conditions at the inner and outer surfaces.
The components of displacement and electric potential are assumed as

$$\begin{aligned}u_r&=u(r),\\ u_z&=0,\\ u_\theta&=0,\\ \phi&=\phi(r).\end{aligned} \tag{7}$$

The equation of equilibrium and the Maxwell's equation for free electric charge density ([11] and [14]) are written as:

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}+f_z=0. \tag{8}$$

$$\frac{\partial D_{rr}}{\partial r}+\frac{D_{rr}}{r}=0. \tag{9}$$

where σ_{ii} $(i=r,\theta)$ is the stress tensor and D_{rr} is the radial electric displacement, respectively.
Furthermore, f_z is the Lorentz force which may be written as [15]

$$f_z=\mu H_z^{\,2}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right) \tag{10}$$

Radial and circumferential strains and the relation between electric field and electric potential are therefore reduced to

$$\varepsilon_{rr}=\frac{\partial u}{\partial r}, \tag{11}$$

$$\varepsilon_{\theta\theta}=\frac{u}{r}, \tag{12}$$

$$E_{rr}=-\frac{\partial\phi}{\partial r}. \tag{13}$$

The corresponding constitutive relations and components of radial electric displacement vector may be written as

$$\sigma_{rr}=C_{11}\left(\varepsilon_{rr}-\alpha_{rr}T(r)\right)+C_{12}\left(\varepsilon_{\theta\theta}-\alpha_{\theta\theta}T(r)\right)-e_{11}E_{rr}, \tag{14}$$

$$\sigma_{\theta\theta}=C_{12}\left(\varepsilon_{rr}-\alpha_{rr}T(r)\right)+C_{22}\left(\varepsilon_{\theta\theta}-\alpha_{\theta\theta}T(r)\right)-e_{12}E_{rr}, \tag{15}$$

$$\sigma_{zz}=C_{13}\left(\varepsilon_{rr}-\alpha_{rr}T(r)\right)+C_{23}\left(\varepsilon_{\theta\theta}-\alpha_{\theta\theta}T(r)\right)-e_{13}E_{rr}, \tag{16}$$

$$D_{rr}=e_{11}\left(\varepsilon_{rr}-\alpha_{rr}T(r)\right)+e_{12}\left(\varepsilon_{\theta\theta}-\alpha_{\theta\theta}T(r)\right)+\in_{11}E_{rr}, \tag{17}$$

To develop the analytical solution, the following dimensionless quantities are introduced

$$\sigma_r = \frac{\sigma_{rr}}{C_{22}}, \qquad \sigma_\theta = \frac{\sigma_{\theta\theta}}{C_{22}}, \qquad \sigma_z = \frac{\sigma_{zz}}{C_{22}}, \quad U = \frac{u}{a}, \qquad \chi = \frac{r}{a}, \quad \beta = \frac{b}{a}, \tag{18}$$
$$\Phi = \frac{\phi}{\phi_0}, \ \phi_0 = a\sqrt{\frac{C_{22}}{\in_{11}}}, \qquad \Psi_r = \frac{D_{rr}}{\sqrt{C_{22}\in_{11}}}, \ C_1 = \frac{C_{11}}{C_{22}}, C_3 = \frac{C_{13}}{C_{22}}, C_4 = \frac{C_{33}}{C_{22}},$$
$$C_2 = \frac{C_{12}}{C_{22}}, \qquad \Xi_1 = \frac{e_{11}}{\sqrt{C_{22}\in_{11}}}, \ \Xi_2 = \frac{e_{12}}{\sqrt{C_{22}\in_{11}}}, \Xi_3 = \frac{e_{13}}{\sqrt{C_{22}\in_{11}}}, \overline{H}_Z = \frac{\mu H_z^{\,2}}{C_{22}}$$

Using these dimensionless variables the constitutive equations may be rewritten in non-dimensional form as:

$$T(\chi) = F_1 \ln(\chi) + F_2, \tag{19}$$
$$\sigma_r = C_1\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + C_2\left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right) + \Xi_1\frac{d\Phi}{d\chi}, \tag{20}$$
$$\sigma_\theta = C_2\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + \left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right) + \Xi_2\frac{d\Phi}{d\chi}, \tag{21}$$
$$\sigma_z = C_3\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + C_4\left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right) + \Xi_3\frac{d\Phi}{d\chi}, \tag{22}$$
$$\Psi_r = \Xi_1\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + \Xi_2\left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right) - \frac{d\Phi}{d\chi}, \tag{23}$$
$$\frac{\partial\sigma_r}{\partial\chi} + \frac{\sigma_r - \sigma_\theta}{\chi} + \overline{H}_z\,\frac{\partial}{\partial\chi}\left(\frac{\partial U}{\partial\chi} + \frac{U}{\chi}\right) = 0. \tag{24}$$
$$\frac{\partial\Psi_r}{\partial\chi} + \frac{\Psi_r}{\chi} = 0. \tag{25}$$

The solution of Eq. (25) is $\Psi_r = F/\chi$ where F_3 is a constant. Substituting this equation into Eq. (23), we obtain

$$\frac{d\Phi}{d\chi} = -\frac{F_3}{\chi} + \Xi_1\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + \Xi_2\left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right). \tag{26}$$

Substituting Eq. (27) into Eqs. (20), (21) and (22) yields

$$\sigma_r = \left(C_1 + \Xi_1^2\right)\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + \left(C_2 + \Xi_2\Xi_1\right)\left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right) - F_3\Xi_1\chi^{-1}, \tag{27}$$
$$\sigma_\theta = \left(C_2 + \Xi_1\Xi_2\right)\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + \left(1 + \Xi_2^2\right)\left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right) - F_3\Xi_2\chi^{-1}, \tag{28}$$
$$\sigma_z = \left(C_3 + \Xi_1\Xi_2\right)\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + \left(C_4 + \Xi_2^2\right)\left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right) - F_3\Xi_4\chi^{-1}, \tag{29}$$

Finally, substituting Eqs. (27) and (28) into Eq. (24) yields the following non-homogeneous ordinary differential equation as:

$$\chi^2\frac{d^2U}{d\chi^2} + \chi L_1\frac{dU}{d\chi} + L_2 U = -L_3\chi - L_4\chi\ln(\chi) - L_5, \tag{30}$$

where

$$L_1 = 1, \tag{31}$$
$$L_2 = \frac{-\overline{H}_z - \Xi_2^2 - 1}{C_1 + \Xi_1^2 + \overline{H}_z}, \tag{32}$$
$$L_3 = \frac{-F_2\begin{pmatrix} C_2\alpha_\theta + C_2\alpha_r + C_1\alpha_r + \Xi_1^2\alpha_r - \\ \Xi_2^2\alpha_\theta - \alpha_\theta + \Xi_1\Xi_2\alpha_\theta - \Xi_1\Xi_2\alpha_r \end{pmatrix} - F_1\left(\Xi_1^2\alpha_r + C_2\alpha_\theta + \Xi_1\Xi_2\alpha_\theta + C_1\alpha_r\right)}{C_1 + \Xi_1^2 + \overline{H}_z}, \tag{33}$$
$$L_4 = \frac{F_1\left(\alpha_\theta + \Xi_2^2\alpha_\theta - C_2\alpha_\theta + \Xi_1\Xi_2\alpha_r - C_1\alpha_r - \Xi_1^2\alpha_r - \Xi_1\Xi_2\alpha_\theta\right)}{C_1 + \Xi_1^2 + \overline{H}_z}, \tag{34}$$
$$L_5 = \frac{F_3\Xi_2}{C_1 + \Xi_1^2 + \overline{H}_z}, \tag{35}$$

Equation (30), a non-homogeneous second-order ordinary differential equation, is the governing equation for displacement of the cylinder subjected to axisymmetric thermo-electro-magneto-mechanical loading. The corresponding general solution for the homogeneous differential equation can be written as

$$U_g = F_4 \underbrace{\chi^{\Gamma_1}}_{u_{g1}} + F_5 \underbrace{\chi^{\Gamma_2}}_{u_{g2}}, \tag{36}$$

in which F_4 and F_5 are integration constants to be determined by the boundary conditions. Γ_1 and Γ_2 are the roots of the corresponding characteristic equation of Eq. (36) and may be evaluated from

$$\Gamma_{1,2} = \frac{(1-L_1) \pm \sqrt{(L_1-1)^2 - 4L_2}}{2} \tag{37}$$

The particular solution for Eq. (30) is assumed to have the form

$$U_p = \chi^{\Gamma_1} U_{p1} + \chi^{\Gamma_2} U_{p2}, \tag{38}$$

Where

$$U_{p1} = -\int \frac{\chi^{\Gamma_2} R(\chi)}{W_p} d\chi, \tag{39}$$

$$U_{p2} = \int \frac{\chi^{\Gamma_1} R(\chi)}{W_p} d\chi, \tag{40}$$

In which

$$W_p = \begin{bmatrix} U_{g1} & U_{g2} \\ \left(U_{g1}\right)' & \left(U_{g2}\right)' \end{bmatrix}, \tag{41}$$

$$R(\chi) = -L_3\chi - L_4\chi\ln(\chi) - L_5, \tag{42}$$

Finally, the radial displacement is the sum of general and particular solution as

$$U = U_g + U_p, \tag{43}$$

Substituting U from Eq. (43) into Eq. (36) and performing the integration, Φ is obtained as

$$\Phi = \left[\int\left\{-\frac{F_3}{\chi} + \Xi_1\left(\frac{dU}{d\chi} - \alpha_{rr}\left(F_1\ln(\chi)+F_2\right)\right) + \Xi_2\left(\frac{U}{\chi} - \alpha_{\theta\theta}\left(F_1\ln(\chi)+F_2\right)\right)\right\}\mathrm{d}\chi\right] + \mathrm{F}_6. \tag{44}$$

Also, by substituting U from Eq. (43) into Eqs. (27), (28) and (29) expressions for radial, circumferential and axial stresses can be obtained. The boundary conditions at the inner and outer surfaces for each case can be written as follows:

$$\sigma_r(1) = -1,\ \ \sigma_r(\chi) = 0,\ \ \Phi(1) = 1,\ \ \Phi(\chi) = 1,\ \ T(1) = 1,\ \ T(\chi) = b, \tag{45}$$

4. Numerical results and discussion

The numerical results are drawn in Figs. 2-6 showing the effects of magnetic field effects on the variation of stress, electric potential and radial displacement across the thickness of the nano-composite cylinder. Presented results are for the boundary conditions described above in Eq. (68), with aspect ratios of $b/a = 2$. The plots in these figures correspond to $T(a) = 323$ K and $T(b) = 298$ K. The piezoelectric material PVDF has been selected with the following mechanical and electrical properties [16]

Table.1 Mechanical and electrical properties for PVDF

Properties	PVDF
c_{11}	238.24 (GPa)
c_{12}	3.98 (GPa)
c_{22}	23.6 (GPa)
e_{11}	-0.135 (C/m^2)
e_{12}	-0.145 (C/m^2)
e_{11}	1.1e-8 (C^2/Nm^2)
α_r	7.1e-5 (1/K)
α_θ	7.1e-5 (1/K)

The magnetic permittivity of CNTs is selected as $\mu = 4\pi \times 10^{-7} N/A^2$ and the magnetic field intensity is $H_z = 1\times10^8 A/m$.

The magnetic field effects on the graphs of radial stress, circumferential stress, axial stress, effective stress, electric potential and radial displacement along the radial direction are shown in Figs. 2-6 respectively for the proposed boundary condition. These figures indicate clearly that the magnetic field has a major effect on the electro-thermoelastic stresses, radial displacement and electric potential. Fig. 2 depicts the distribution of radial stress along the radius. As can be seen, the radial stresses at the internal and external surfaces of the cylinder satisfy the given boundary conditions. Moreover, considering the magnetic field decreases the radial stress.

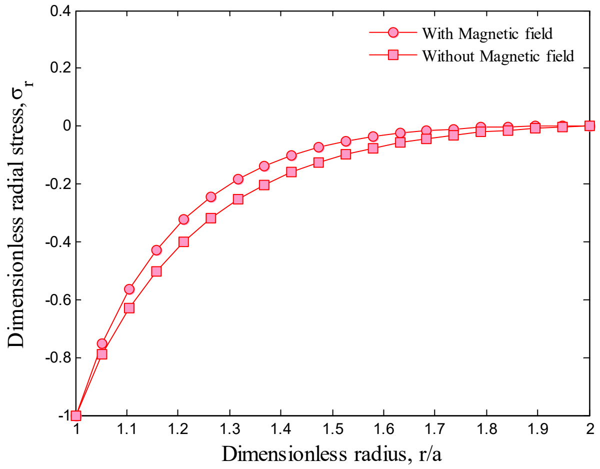

Fig. 2 Magnetic field effects on the redial stress

The distributions of the hoop and effective stresses are displayed in Figs. 3 and 4, where the hoop and effective stresses are monotonically changed from the inner to the outer radius. Furthermore, the hoop and effective stresses decrease with considering Magnetic field. It is due to the fact that with considering Magnetic field, the stiffness of cylinder increases.

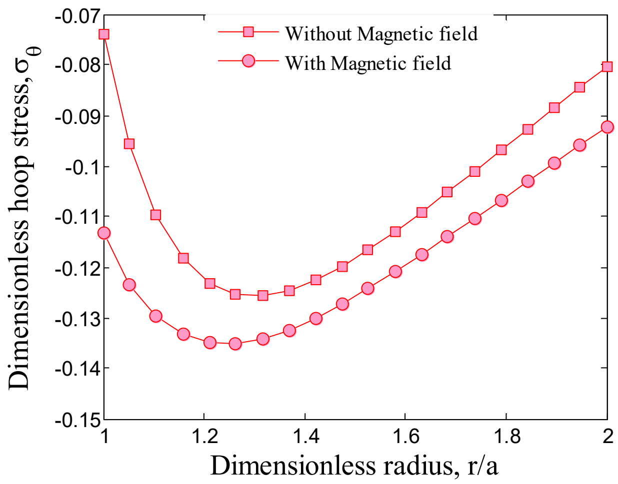

Fig. 3 Magnetic field effects on the hoop stress

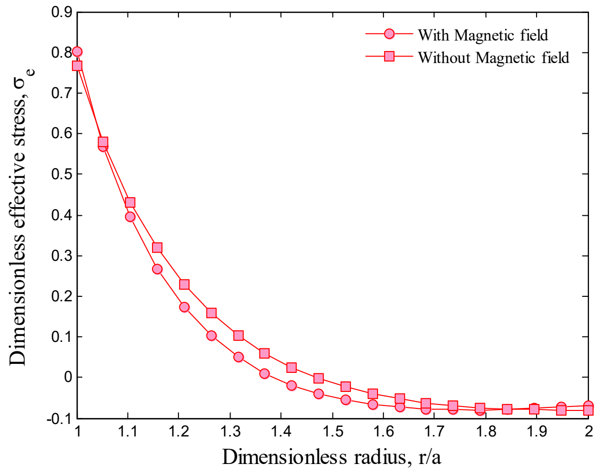

Fig. 4 Magnetic field effects on the effective stress

Fig. 5 shows the electric potential through the thickness of the nano-composite cylinder. It is seen from this figure that the electric potential satisfies the boundary conditions and increases with considering Magnetic field.

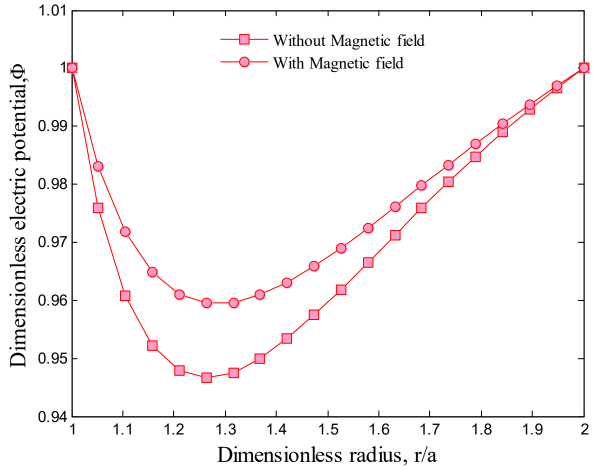

Fig. 5 Magnetic field effects on the electric potential

The variations of the radial displacement along the radius are demonstrated in Fig. 6 which indicates that the radial displacement decreases considering magnetic field and its maximum value occurs at the outer radius. It is because that with considering magnetic field, the stability of structure increases and hence the radial displacement decreases.

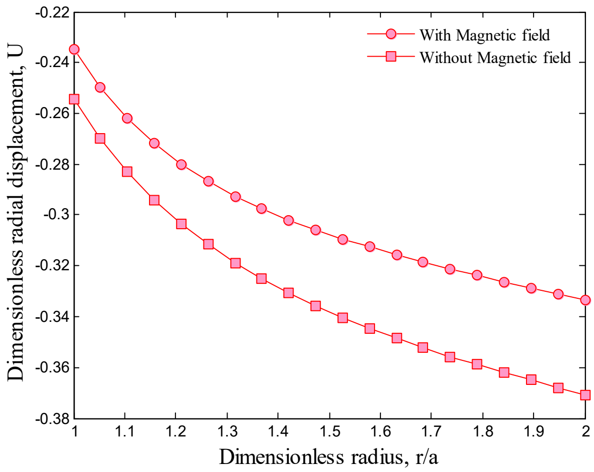

Fig. 6 Magnetic field effects on the radial displacement

5. Conclusions

Magnetic field effects on the electro-magneto-thermo-mechanical analysis of piezoelectric cylinders reinforced with CNTs were the main contribution of this work. The equivalent material properties of system are obtained using Mori-Tanaka model. The coupled governing equation is derived based on Maxwell and equilibrium equations. In order to obtain the stresses, electric potential and radial displacement distributions, an analytical method is applied. Results indicate that with considering magnetic field, the stresses reduce. Furthermore, the tensile hoop and effective stresses were monotonically changed from the inner to the outer radius.

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