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Reliability Analysis of Nanocomposite Beams Reinforced with CNTs under Buckling Forces Using the Conjugate HL-RF

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Abstract

In this paper, the nonlinear conjugate map is applied based on the conjugate Hasofer-Lind and Rackwitz-Fiessler (CHL-RF) method to evaluate the reliability index using the first order reliability method of the embedded nanocomposite beam, which is made of a polymer reinforced with carbon nanotubes (CNTs). The structure is simulated with the Timoshenko beam model. The Mori-Tanaka model is applied for calculating the effective material properties of the nanocomposite beam and the surrounding elastic medium is considered as spring and shear constants. The governing equations are derived based on the energy method and the Hamilton's principle. Moreover, using an analytical method, the buckling performance function of the structure is obtained. The effects of the basic random variables including the length-to-thickness ratio of the beam (L/h), the spring constant, and the shear constant of the foundation with respect to the volume fraction of CNTs are investigated based on the reliability index of the nanocomposite beam which is subjected to an axial force of 20 GPa. The results indicate that the failure probabilities of the studied nanocomposite beams are sensitive to the length-to-thickness ratio of the beam (L/h) and the spring constant of the foundation variables.

Keywords: Nanocomposite beam; Conjugate HL-RF; The first order reliability method; Timoshenko beam model.

1. Introduction

Research and development in composites have advanced at an unimaginable rate in the past decade. Indeed, the nanoscale can lead to new phenomena, providing opportunities for novel multifunctional materials applications. Nanocomposites have become a new class of materials that circumvent classic composite material performance by accessing new properties and exploiting unique synergism between materials.

The buckling of the beams has been investigated by many researchers. The first scholar who theoretically studied the buckling stability of the elasto-plastic beams appeared to be Engesser [1]. He raised the important question of how the beam unloads and suggested that the buckling load of an inelastic beam must be obtained from Euler's formulae. This question was only correctly resolved later on by Shanley [2] who, with the help of the simple theoretical model and various experiments, showed that the buckling of an elastic-plastic beam occurs at the so-called tangent critical load. Mau [3] and Mau and El-Mabsout [4] developed a beam-beam element for inelastic buckling analysis of the finite element to determine load-carrying capacity of the beam. Pantazopoulou [5] compiled data from the literature of over 300 beam tests and developed requirements for reinforcement stability that recognize the interaction between displacement ductility demand in critical section, tie effectiveness, limiting strain, bar size, and tie spacing. Dhakal and Maekawa [6] used finite element analyses of a fiber to present an average compressive stress-strain relation for reinforcing bars as a function of the slenderness ratio and the yield strength. Later, Bae et al. [7] conducted an experimental program study on the bar buckling and examined the effects of three important bar parameters: the L/D ratio (length over bar diameter), e/D (initial imperfection over bar diameter) ratio, and the ratio of the ultimate strength to the yield strength. Dhakal and Maekawa [8] derived a method to predict the buckling length of the longitudinal reinforcing bars using an energy method. The exact analytical solutions for the buckling loads of a reinforced Euler-type beam are presented in e.g. Krauberger et al. [9], where the effect of the material non-linearity on the buckling load is fully assessed. The development of a finite element model for the geometric and material nonlinear analysis of the bonded prestressed continuous beams was presented by Lou et al. [10]. Bajc et al. [11] derived a new semi-analytical procedure for the determination of the buckling of the reinforced beam exposed to fire. An experimental investigation on the behaviour of geopolymer composite beams which were reinforced with conventional steel bars and various types of fibres namely steel, polypropylene, and glass in different volume fractions under flexural loading was presented by Vijai et al. [12].

The reliability analysis can be monitored by the safety levels of the complex real engineering problems. The first order reliability method (FORM) is commonly applied to estimate the failure probability based on the reliability index using the most probability failure point (MPP). FORM is a basic reliability approach for reliability-based design code [13] and reliability–based design optimization [14, 15]. There were developed robust and efficient algorithms to search MPP based on the conjugate search direction [13, 16-19]. Keshtegar et al. [13, 16-18] introduced the conjugate algorithms-based Fletcher–Reeves method [20]. The stability transformation method and the chaos control approach are improved using the chaotic conjugate search direction [16, 17]. The conjugate search direction is improved based on the limited scalar conjugate factor in the limited conjugate first order reliability method [18]. The sufficient descent condition in the proposed approach in the Refs [16, 18] and the Armijo rule in the Ref [13, 17] are applied to achieve the stabilization of the iterative FORM formula. Keshtegar [16-18] showed that the conjugate search direction can be improved by the robustness and efficiency of FORM formula compared with the reliability algorithm–based steepest descent search direction [21, 22] for highly nonlinear engineering problems. The conjugate Hasofer-Lind and Rackwitz-Fiessler (CHL-RF) method [13] is a simple and robust FORM formula which is developed based on the conjugate discrete map to improve the instability of the HL-RF method. The CHL-RF method is applied to successfully approximate the failure probabilities of corroded pipe in the Ref [13].

In this paper, the CHL-RF method is implemented to search MPP of a nanocomposite beam under the buckling failure mode because of its simplicity and robustness. The robustness of the CHL-RF method is guaranteed using the Armijo's rule based on the finite step length. A nanocomposite beam under the buckling forces is used to determine the probabilistic model that the implicit buckling performance function of a nanocomposite beam is theoretically estimated using the Timoshenko beam model and the Navier method. The random variables such as the length-to-thickness ratio of the beam, volume fraction of CNT, the shear and spring constant of the foundation are applied to reliability analysis of the nanocomposite beam, which are simulated with normal and non-normal distributions. The results illustrated that the CHL-RF method provided stable results for the iterative formula of FORM. Based on the reliability index, the length-to-thickness ratio of the beam (L/h) and the shear constant of the foundation are sensitive and insensitive basic variables among other basic variables.

2. The Conjugate HL-RF

In FORM, structural failure probabilities of the nanocomposite beams are approximated based on the reliability index as follows [13,18]:

$$P_f = \int_{g(X)\le 0} \cdots \int f_X(x_1,\ldots,x_n)dx_1..dx_n \approx \Phi(-\beta) \tag{1}$$

where β is the reliability index, which corresponds to minimum distance to the origin on the limit state function in the standard normal space [17]. Generally, the main goal of FORM is to search the most probable point (MPP i.e., $\boldsymbol{U}^*$) for computing the reliability index as $\beta = \left\|\boldsymbol{U}^*\right\|$. $g(\boldsymbol{X})$ is the limit state function or performance function that can be defined based on the bucking performance for the nanocomposite beam. In this paper, $f_X(x_1,\ldots,x_n)$ is the joint probability density function for the basic random variables $\boldsymbol{X}$, and Φ is the cumulative distribution function (CDF) of the standard normal variables. $g(\boldsymbol{X}) < 0$ defines the failure region as follows: $P_{cr} - P < 0$ in which P is the external compressive force which is given based on the applied loads and P_{cr} is the theoretical buckling force which is determined based on the capacity of the nanocomposite beam.

Typically, the FORM-based CHL-RF involves three steps for estimating the probability of failure as follows [13, 17]:

Step 1. Transform random variables in X-space into U-space by the following relation:

$$u = \frac{x - \mu_x^e}{\sigma_x^e} \tag{2}$$

where u is the standard normal variable with the mean and standard deviation equal to zero and one, respectively. μ_x^e and σ_x^e are the equivalent mean and standard deviation of the random variable *x*, respectively. It can be obtained for the normal random variable as $\mu_x^e = \mu_x$ and $\sigma_x^e = \sigma_x$. The equivalent mean and standard deviation of the non-normal random variables can be determined by the following equations [19]:

$$\sigma_x^e = \frac{1}{f_X(x)} \varphi[\Phi^{-1}\{F_X(x)\}] \tag{3}$$

$$\mu_x^e = x - \sigma_x^e \Phi^{-1}[F_X(x)] \tag{4}$$

where $F_X(x)$ is the cumulative distribution, $f_X(x)$ is the probability distribution, Φ^{-1} is the inverse standard normal cumulative distribution and φ is the standard normal probability distribution function.

Step 2. Find the most probable point (MPP) based on the iterative formula of CHL-RF.

The MPP $\boldsymbol{U}^* = (u_1^*, u_2^*, ..u_n^*)^T$ can be searched based on the conjugate search direction as follows [13]:

$$\boldsymbol{U}_{k+1} = \beta_{k+1} \alpha_{k+1}^{\lambda} \tag{5}$$

$$\beta_{k+1} = \frac{g(\boldsymbol{U}_k) - \nabla^T g(\boldsymbol{U}_k)\boldsymbol{U}_k}{\nabla^T g(\boldsymbol{U}_k)\alpha_{k+1}^{\lambda}} \tag{6}$$

where $\nabla g(\boldsymbol{U}) = [\partial g / \partial u_1, \partial g / \partial u_2, ..., \partial g / \partial u_n]^T$ is gradient vector of the limit state function at the design point $\boldsymbol{U}$. α_{k+1}^{λ} is the direction cosine vector or the sensitivity vector of the random variables, which can be computed as follows:

$$\alpha_{k+1}^{\lambda} = \frac{\boldsymbol{U}_{k+1}^{C\lambda}}{\left\| \boldsymbol{U}_{k+1}^{C\lambda} \right\|} \tag{7}$$

in which, point $\boldsymbol{U}_{k+1}^{C\lambda}$ is along the direction of the negative conjugate gradient vector at design point $\boldsymbol{U}_k$. The point $\boldsymbol{U}_{k+1}^{C\lambda}$ is determined by the following relation:

$$\boldsymbol{U}_{k+1}^{C\lambda} = \boldsymbol{U}_k + \lambda \boldsymbol{d}_k \tag{8}$$

where $\lambda > 0$ is the step length, and $\boldsymbol{d}_k$ is the conjugate search direction, which are computed by the following relations:

$$\boldsymbol{d}_k = -\nabla g(\boldsymbol{U}_k) - \frac{\left\| \nabla g(\boldsymbol{U}_k) \right\|^2}{\left\| \nabla g(\boldsymbol{U}_{k-1}) \right\|^2} \boldsymbol{d}_{k-1}$$

$$(9)\ \lambda_k = \begin{cases} \lambda_{k-1} & if \quad \left\| \boldsymbol{U}_{k+1} - \boldsymbol{U}_k \right\| > \left\| \boldsymbol{U}_k - \boldsymbol{U}_{k-1} \right\| \\ c\,\lambda_{k-1} & \text{otherwise} \end{cases} \tag{10}$$

where, $0 < c < 1$ is the adjusted factor which is given as 0.9 in this paper. The step length can provide a global convergence for the CHL-RF algorithm in the complex engineering problems with highly nonlinear performance function. The initial step size is determined based on the Armijo-type line search as follows [13, 17]:

$$\lambda_0 = \frac{50}{\left\| \nabla g(\boldsymbol{U}_\mu) \right\|^2} \tag{11}$$

Based on the Eq. (10), $\boldsymbol{U}_{k+1}^{C\lambda} = \boldsymbol{U}_k$ when the step length is $\lambda = 0$. This means that the $\boldsymbol{U}_{k+1}^{C\lambda}$ point is in a fixed position. According to the above equations, a cycle of the iterative CHL-RF formula is given for the reliability analysis as follows:

1- Compute the gradient vector of the performance function at point $\boldsymbol{U}_k$.

2- Compute the conjugate search direction $\boldsymbol{d}_k$ using Eq. (9).

3- Determine the point $\boldsymbol{U}_{k+1}^{C\lambda}$ in terms of Eq. (8).

4- Compute the sensitivity vector of the random variables based on Eq. (7).

5- Compute the reliability index and the new point using Eqs. (6) and (5), respectively.

6- Update the new step length based on the Armijo's rule and Eq. (10).

Step 3. Calculate the failure probability as $P_f \approx \Phi(-\beta)$.

It can be found out from the reliability analysis that the iterative approach of the FORM and the performance function are two important factors to evaluate the failure probability of the nanocomposite beams. The iterative formula has led to stable results; consequently, the CHL-RF approach can provide stable solutions for highly nonlinear performance functions [14]. The performance function is an essential key for accurate reliability analysis of the nanocomposite beams; thus, it can be defined as performance function of the nanocomposite beam.

The limit state function has separated the design domain into failure and safe regions based on the buckling mode of the nanocomposite beams. It is supposed that the nanocomposite beams may fail when the external compressive load (P) is more than the buckling forces (P_{cr}) of the beams. Consequently, the limit state function of the nanocomposite beams under the buckling failure mode can be given as follows:

$$g(L,h,kw,kg,rho) = P_{cr}(L,h,kw,kg,rho) - P < 0 \tag{12}$$

where $P_{cr}(L,h,kw,kg,rho)$ is the buckling force which can be obtained based on the Timoshenko beam theory. The displacements of an arbitrary point in the Timoshenko beam are [18] as follows:

$$\begin{aligned} u_1(x,z) &= U(x) + z\,\psi(x), \\ u_2(x,z) &= 0, \\ u_3(x,z) &= W(x) \end{aligned} \tag{13}$$

where ψ is the rotation of the beam cross-section.

$$\varepsilon_{xx} = \frac{\partial U}{\partial x} + z\,\frac{\partial \psi}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2, \tag{14}$$

$$\gamma_{xz} = \frac{\partial W}{\partial x} + \psi. \tag{15}$$

The stress-strain relations can be written as:

$$\sigma_{xx} = C_{11}\varepsilon_x, \tag{16}$$

$$\sigma_{xz} = C_{55}\left[\frac{\partial W}{\partial x} + \psi\right], \tag{17}$$

where C_{11} and C_{55} are the elastic constants which can be calculated by the Mori-Tanaka model as follows:

$$C_{11} = k + m, \tag{18}$$

$$C_{55} = m, \tag{19}$$

where

$$k = \frac{E_m\{E_m c_m + 2k_r(1+\nu_m)[1 + c_r(1-2\nu_m)]\}}{2(1+\nu_m)[E_m(1+c_r-2\nu_m) + 2c_m k_r(1-\nu_m-2\nu_m^2)]}, \tag{20}$$

$$m = \frac{E_m[E_m c_m + 2m_r(1+\nu_m)(3+c_r-4\nu_m)]}{2(1+\nu_m)\{E_m[c_m + 4c_r(1-\nu_m)] + 2c_m m_r(3-\nu_m-4\nu_m^2)\}}, \tag{21}$$

where c_m and c_r are the volume fractions of the polymer and the nanofibers, respectively; and kr, l_r, n_r, p_r, and m_r are the Hills' elastic modulus for the nanofibers. In addition, E_m and υ_m are the Young's modulus and the Poisson's ratio of matrix.

The strain energy of the structure can be expressed as:

$$U = \frac{1}{2}\int_0^L \int_A (\sigma_{xx}\,\varepsilon_{xx} + 2\sigma_{xz}\,\varepsilon_{xz})\,dA\,dx \tag{22}$$

Submitting Eqs. (13) to (15) into (22) yields Eq. (23) as follows:

$$U = \frac{1}{2}\int_0^L \left\{N_x\,\frac{\partial U}{\partial x} + M_x\,\frac{\partial \psi}{\partial x} + \frac{1}{2}N_x\left(\frac{\partial W}{\partial x}\right)^2 + Q_x\,\frac{\partial W}{\partial x} + Q_x\,\psi\right\}dx \tag{23}$$

The governing equations of the structure can be derived from the Hamilton's principle as:

$$\frac{\partial N_x}{\partial x} = 0, \tag{24}$$

$$\frac{\partial Q_x}{\partial x} - \frac{\partial}{\partial x}\left(N_x^M\,\frac{\partial W}{\partial x}\right) + q = 0, \tag{25}$$

$$\frac{\partial M_x}{\partial x} - Q_x = 0, \tag{26}$$

Based on the dimensionless parameters as follows:

$$\xi = \frac{x}{L},\ (\overline{W},\overline{U}) = \frac{(W,U)}{h},\ \eta = \frac{h}{L}, \psi = \overline{\psi},\ \overline{K}_w = \frac{K_w h L}{C_{11} A},$$
$$\overline{G}_p = \frac{G_p}{C_{11} A}, \overline{N}_x^M = \frac{N_x^M}{C_{11} A}, \overline{I} = \frac{I}{AL^2},\ C_5 = \frac{C_{55}}{C_{11}}. \tag{27}$$

the governing equations can be rewritten as:

$$\frac{\partial^2 \overline{U}}{\partial \xi^2} + \eta \frac{\partial^2 \overline{W}}{\partial \xi^2} \frac{\partial \overline{W}}{\partial \xi} = 0, \tag{28}$$

$$C_5 \left[\eta \frac{\partial^2 \overline{W}}{\partial \xi^2} + \frac{\partial \overline{\psi}}{\partial \xi} \right] - \overline{N}_x^M \eta \frac{\partial^2 \overline{W}}{\partial \xi^2} - \overline{K}_w \overline{W} + \overline{G}_p \frac{\partial^2 \overline{W}}{\partial \xi^2} = 0, \tag{29}$$

$$\overline{I} \frac{\partial^2 \overline{\psi}}{\partial \xi^2} + K_s C_5 \left[\eta \frac{\partial \overline{W}}{\partial \xi} + \overline{\psi} \right] = 0. \tag{30}$$

Based on DQM, the functions f and their k[th] derivatives with respect to x can be approximated as [21, 22] :

$$\frac{d^n f(x_i)}{dx^n} = \sum_{j=1}^{N} C_{ij}^{(n)} f(x_j) \qquad n = 1,\ldots,N-1, \tag{31}$$

where N is the total number of nodes distributed along the x-axis, and C_{ij} is the weighting coefficients, the recursive formula for which can be found in [21-24]. Using DQM, the governing equations can be expressed in the matrix form as:

$$\left(\left[\underbrace{K_L + K_{NL}}_{K} \right] + P\left[K_g \right] \right) \begin{Bmatrix} \{d_b\} \\ \{d_d\} \end{Bmatrix} = 0, \tag{32}$$

where K_L is the linear stiffness matrix, K_{NL} is the nonlinear stiffness matrix, and K_g is the geometric stiffness matrix. Also, d_b and d_d represent boundary and domain points. Finally, based on an iterative method and eigenvalue problem, the buckling load of the structure may be obtained.

3. Results and discussion

A nanocomposite beam with the limit state function in Eq. (12) under the external compressive load of 20 GPa i.e., P=20 GPa and the buckling capacity which is computed using Eq. (32) is applied to reliability analysis based on the CHL-RF method. The reliability index of the nanocomposite beam is obtained based on the five basic random variables with normal and non-normal distributions, whose statistical properties are tabulated in Table 1.

Table 1. Statistical properties of the basic random variables for the nanocomposite beam

Variable	Unite	Description	Mean	Coefficient of variation	Distribution
L	m	Length of beam	0.4,0.8,2,4	0.1	Normal
h	m	Thickness of beam	0.4	0.1	Normal
kw	GPa	Stiffness constant of foundation	100, 200, 500,1000	0.12	Gumbel
kg	N	Shear constant of foundation	0, 10, 20, 50, 100	0.12	Gumbel
rho	--	Volume fraction of CNT nanoparticles	0-0.5	0.15	Lognormal

The parameters of the CHL-RF for the reliability analysis are given as c=0.9 and the stopping criterion of $\left\| \boldsymbol{U}_k - \boldsymbol{U}_{k-1} \right\| < 10^{-5}$. Based on the Mean and COV from Table 1, the reliability indexes for different Means of the basic random variables are determined with respect to the various volume fractions of the CNT nanoparticles, which are changed in a range from 0 to 0.5 for nanocomposite beams. The reliability index is computed based on the applied compressive load of 20 GPa for this nanocomposite beam. The MPP and the reliability index are obtained using the CHL-RF method based on the statistical properties of the random variables in Table 1 with mean values of L=0.8m, h=0.5m, kw=500GPa, kg=50N, and rho=0.25 as $\boldsymbol{X}$*=(L*=0.6241391m, h*=0.3990155m, kw*=452.592126 GPa, kg*= 48.475307N, rho*=0.2467846) and β= 2.340344 after 31 iterations and 5203 numbers of call probabilistic function, respectively. Therefore, the CHL-RF method can provide stable results for this nanocomposite beam.

The reliability index of the probabilistic model in the Eq. (12) is estimated using the CHL-RF method for the nanocomposite beam with the mean values of the basic random variables as h=0.4 m, kw=500GPa, and kg=50N; the results of the reliability indexes for different length-to-thickness ratio of the beam are plotted in Fig. 1. The results obtained from Fig. 1 showed that the reliability index is increased by increasing the length-to-thickness ratio (L/h) of the beam. The increment rate of the reliability index with respect to the L/h is a nonlinear form so that the differences between the reliability indexes of L/h=1 and L/h= 2 are obtained more than the differences between the reliability indexes of L/h=2 and L/h= 5. The reliability index corresponding to the rho is not significantly changed compared with the variable L/h. It can be concluded from the results of Fig. 1 that very weak safety levels for the nanocomposite beam are obtained with respect to the L/h less than 1 (reliability index less than -3). A poor safety level is obtained for the nanocomposite beams when the variable L/h is selected as 2. Good safety levels are obtained with respect to the L/h=5, and the excellent safety levels can be provided when the L/h is selected more than 10 for the nanocomposite beam.

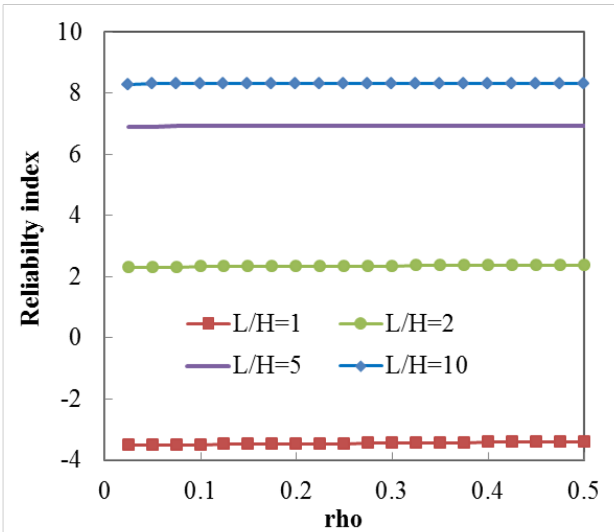

Fig. 1. The reliability indexes of beam for different length-to-thickness ratio

The effects of different spring constants of the foundation (kw) on the nanocomposite beam are evaluated based on the reliability indexes for the mean values of the random variables as h=0.4 m, L=1.2m, and kg=50N. The results of the reliability index with respect to various volume fractions of the CNT nanoparticles (rho) for different kws are shown in Fig. 2., which indicates that the variable kw significantly affects the safety levels of the nanocomposite beam compared with the rho. For L/h=3, the kw should be selected more than 200 GPa to obtain the reliability indexes larger than 2. The reliability indexes are increased by increasing the spring constants of the foundation as well as the length-to-thickness ratio of the nanocomposite beams. Good safety levels (reliability index more than 4) for the studied nanocomposite beam are obtained when the kw is given more than 500 GPa. The increment rates of the reliability index corresponding to the spring constants of the foundation are nonlinear. It can be seen that the reliability index is improved from 4.8 to 6.4 (relative increment is 25%) when the kw is increased from 500 to 1000 GPa, while the reliability indexes are increased from 1.9 to 4.8 (relative increment is about 60%) when kw is changed from 200 to 500 GPa. It can be concluded that an excellent safety level can be provided for the nanocomposite beam in the following conditions: L/h=3; h=0.4m, kg=50N, rho=0-0.5, and kw= 1000Gpa.

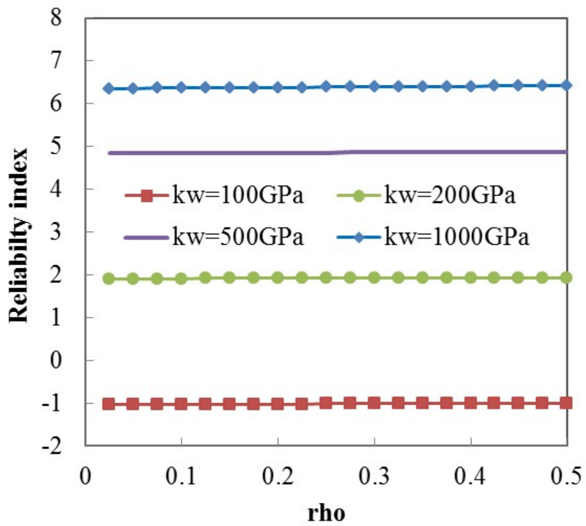

Fig. 2. The reliability indexes of the beam for different spring constant of foundation

Figure 3 illustrates the reliability index of the nanocomposite beam corresponding to various volume fractions of the CNT nanoparticles for different shear constants of the foundation (kg). The reliability indexes in Fig. 3 are computed using the CHL-RF method with the mean values of the basic random variables as h=0.4 m, L=1.2m, and kw=500GPa. It is obvious in Fig. 3 that the reliability indexes are increased by increasing the various volume fractions of the CNT nanoparticles (rho) as well as the shear constants of the foundations. Therefore, the effects of kg and rho on the reliability index are obtained. Based on the L/H=3, kw=500Gpa, good confidence levels (reliability index more than 4.5) are obtained for all kg and the reliability indexes are improved from 4.6 to 5.1 when the kg is increased from 0 to 100N. By comparing the results of Figs. 1-3, it is realized that the input basic variables length-to-thickness ratio of the nanocomposite beam (L/h) and the spring constants of the foundation (kw) can improve the performances of the beam under axial compressive loads, and significantly enhance the capacity of the nanocomposite beams in comparison with the volume fractions of the CNT nanoparticles (rho) and the shear constants of the foundation (kg). The reliability indexes are slightly increased by increasing the L/h and kw, while the variables' rho and kg are not affected, more reasonably.

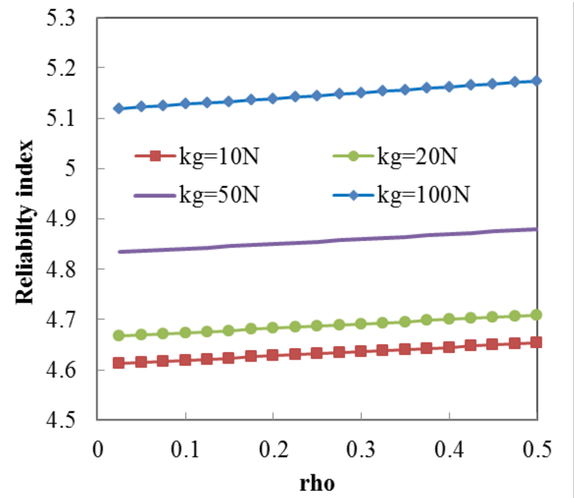

Fig. 3. The reliability indexes of the beam for different shear constant of foundation

As seen in Fig. 1 and 2, length-to-thickness ratio of the nanocomposite beam (L/h) and the spring constants of the foundation (kw) are effective input variables on the reliability index. The reliability indexes of the nanocomposite beam with respect to various L/h (i.e., 1-10) for different kw as 200, 300, 400, 500, 700, 1000Gpa are plotted in Fig. 4 with input variables kw=50N, h=0.4 m, and rho=0.25. It can be concluded that L/h can improve the safety levels of the nanocomposite beam more than the kw. The reliability indexes are obtained more than 4 by increasing the L/h more than 2 for kw ranging between 200 and 1000 GPa. The reliability indexes are significantly increased for the domain of the variables L/h less than 3 and kw less than 500GPa. The results of Fig. 4 show that an excellent confidence level (reliability index more than 6.5) can be obtained for the nanocomposite beams under an axial force of 20Gpa when the geometrical and mechanical conditions of the beams are given as L/h>7 in kw=200Gpa, L/h> 6 in kw=300Gpa, L/h> 5 in kw=400 and 500Gpa, L/h> 4 in kw=700 Gpa, and L/h> 3 in kw=1000Gpa. Consequently, the L/h more than 5 and kw more than 400 GPa can be suggested for an excellent robust design of the nanocomposite beams under an axial force of 20Gpa.

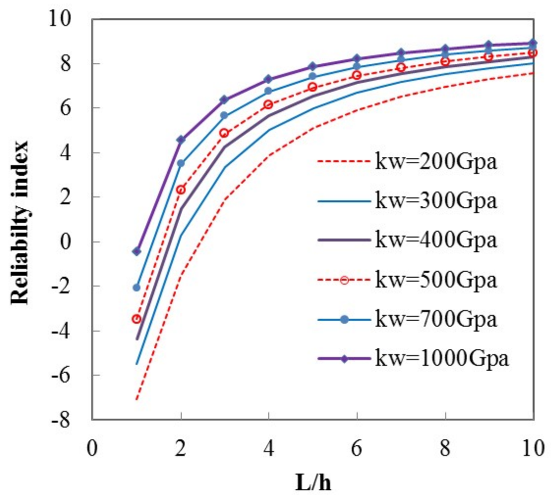

Fig. 4. Reliability indexes of the beam for different spring constants of foundation corresponding to various length to thickness ratios

4. Conclusions

The conjugate HL-RF (CHL-RF) with the Armijo line search is applied to evaluate the reliability index of the nanocomposite beams reinforced with the CNT nanoparticles. The conjugate dynamic discrete map is used to improve the robustness of the first order reliability formula in the reliability analysis of the nanocomposite beam under the buckling failure mode. A probabilistic model is developed based on the buckling forces on the nanocomposite beam using the Timoshenko beam theory and the Navier method. The input random variables such as the length of the beam, the thickness of the beam, the volume fractions of the CNT nanoparticles, the spring constant and the shear constant of the foundation are simulated using the normal and non-normal random variables for the reliability analysis of the nanocomposite beam. The results of the reliability analysis for the nanocomposite beam illustrated that the CHL-RF method can provide stable solutions and appropriate results for the reliability analysis of a complex real example. The shear constant of the foundation in the range from 0-100N and the volume fractions of the CNT nanoparticles in the range from 0-0.5 can improve the reliability indexes from 4.6 to 5.2. The spring constant of the foundation and the length of the beams are two effective variables to enhance the confidence levels of the nanocomposite beams compared with other random variables, i.e., kg and rho, more effectively. The nanocomposite beam reinforced with the CNT nanoparticles under an axial force of 20GPa can provide an excellent confidence level when the beam is designed with the robust conditions as length-to-thickness ratio of the beam more than 5 (i.e., $L/h \geq 5$), the spring constant of the foundation more than 400GPa (i.e., $k_w \geq 400$GPa), the thickness of the beam less than 0.4m (i.e., $h \leq 0.4$m), and the shear constant of the foundation more than 10 N (i.e., $k_w \geq 10$N) with volume fractions of the CNT nanoparticles in the range from 0.1-0.5.

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