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Conjugate and directional chaos control methods for reliability analysis of CNT–reinforced nanocomposite beams under buckling forces; a comparative study

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Abstract

The efficiency and robustness of reliability methods are two important factors in first order reliability method (FORM). The conjugate choice control (CCC) and directional chaos control method (DCC) were developed to improve the robustness and efficiency of FORM formula using the stability transformation method. In this paper, the CCC and DCC methods are applied for reliability analysis of a complex engineering problem as a nanocomposite beam, which is reinforced by carbon nanotubes (CNTs) under buckling force. The probabilistic model for nanocomposite beam is developed using buckling failure mode which is computed using the Euler-Bernoulli beam model. The robustness and efficiency CCC and DCC are compared using stable solution and number of call limit state functions. The results demonstrate that the CCC method is more robust than DCC for this example, while the DCC method is simpler than the CCC.

Keywords: Reliability analysis; Nanocomposite beam; Conjugate chaos control; Directional chaos control.

1. Introduction

The first order reliability method (FORM) is widely used to approximate the failure probabilities of the complex structural problems due to its simplicity and efficiency compared to the sampling methods i.e. Monte Carlo simulation [1, 2]. In FORM, the failure probabilities can be estimated by linearizing the limit state function (LSF) in the standard normal space as follows [3]:

$$P_f = \int_{g(\boldsymbol{X})\leq 0} \cdots \int f_{\boldsymbol{X}}(x_1,\ldots,x_n)dx_1\ldots dx_n \approx \Phi(-\beta) \tag{1}$$

where, $g(\boldsymbol{X})$ is the LSF, $g(\boldsymbol{X})\leq 0$ defines the failure region, $f_{\boldsymbol{X}}(x_1,\ldots,x_n)$ is the joint probability density function for the basic random variables $\boldsymbol{X}$ and Φ is the cumulative distribution function. β is the reliability index, which corresponds to minimum distance to the origin on the limit state function [4]. Generally, the main effort of FORM methods is to search the most probable point (MPP) i.e. $\boldsymbol{U}^*$ (the point of minimum distance to the origin on the limit state surface in the standard normal space). Several iterative formulas were developed to enhance the robustness and efficiency of the FORM based on the steepest descent search direction [3, 5-8] and the conjugate search direction [1, 4, 9- 11]. Yang [5] proposed the stability transformation method (STM) of chaos control (CC) for reliability analysis-based FORM. After that, Keshtegar and Miri [3] proposed a relaxed method-based chaos control with a dynamic step size named as relaxed HL-RF (RHL-RF). Recently, the FORM formula of STM-based CC was modified using the radial direction in terms of steepest descent search direction in the directional CC (DCC) method proposed by Meng et al. [8]. The CC approach was improved using the conjugate search direction with a dynamic line search in the conjugate chaos control (CCC), which was proposed by Keshtegar [9]. Keshtegar [9] showed the CCC method is more robust than the FORM formula- based steepest descent search direction such as CC [5], RHL-RF [3] and more efficient than the finite-step length (FSL) [6] and conjugate HL-RF (CHL-RF) [11] methods. Consequently, the performance convergences including both robustness and efficiency of the CCC and DCC can be investigated in a complex engineering problem.

The nanoparticles can be improved the mechanical, electrical and thermal properties of beams. The reliability analysis using a robust and efficient FORM method for the nanocomposite beams under axial compressive load can be provided a safety levels to successful apply of these complex engineering problems. The probabilistic model using the buckling force is important in the reliability analysis of nanocomposite beams. A nanocomposite beam reinforced by a single-walled carbon nanotube was analyzed using the airy stress-function method to determine local buckling [12, 13]. The buckling and bifurcation buckling behavior of composite plates which are reinforced by carbon nanotubes, armchair double walled boron nitride nanotubes were investigated by Ghorbanpour Arani et al. [14] and Rafiee et al. [15], Wattanasakulpong and Ungbhakorn [16], Kolahchi et al. [17], and Mosharrafian and Kolahchi [18]. The buckling failure mode is important issue to evaluate the nonlinear mechanical properties of the nanocomposite. Thus, the accurate analysis of the buckling force and reliability procedure can be provided a suitable reliability results to evaluate the failure probabilities. The CCC and DCC can be provided the stable and efficient results for highly nonlinear problems compared to other reliability method such as CHL-RF [11], CC [5], FSL [6] and RHL-RF [3].

In this paper, the CCC and DCC methods of FORM are applied for reliability analysis a nanocomposite beam-reinforced by the carbon nanotubes (CNTs). The probabilistic model of the buckling failure mode of the nanocomposite beams is developed using Euler-Bernoulli beam model, which involves various uncertainties including the length and thickness of beam, spring constant and shear constant of foundation, applied voltage, and volume fraction of CNTs. The efficiency and robustness of the CCC and DCC methods are investigated in a real complex nonlinear engineering problem under bucking performance function. The reliability index are extracted using the DCC and CCC methods for two random variables of the applied voltage in the range from -200 to 200 volte and volume fraction of nanoparticles in the range between 0 and 0.5. The results indicate that the CCC method provided stable results for nanocomposite beams under buckling forces compared to DCC method and the applied voltage is performed better to improve the confidence levels of beam than the volume fraction of nanoparticles.

2. Reliability methods

2.1. Directional chaos control (DCC) method

The efficiency of the stability transformation method (STM) is improved based on the directional sensitivity vector of random variables using the chaos control approach, named as the directional chaos control (DCC). The DCC iterative formula of FORM is proposed based on the STM as follows [8]:

$$\boldsymbol{U}_{k+1} = \beta_k \frac{\boldsymbol{n}_k}{\|\boldsymbol{n}_k\|} \tag{2}$$

where, $\boldsymbol{n}_k$ is radial direction which is along the reliability index β_k. The reliability index can be obtained as follows [1, 10]:

$$\beta_{k+1} = \frac{g(\boldsymbol{U}_k) - \nabla^T g(\boldsymbol{U}_k)\boldsymbol{U}_k}{\|\nabla g(\boldsymbol{U}_k)\|} \tag{3}$$

Where, $\boldsymbol{U}$ is the standard normal variable with the mean and standard deviation equal to zero and one, respectively. $\nabla g(\boldsymbol{U}_k)$ is the gradient vector of the LSF in the standard normal space at the point $\boldsymbol{U}_k$ as $\nabla g(\boldsymbol{U}) = [\partial g/\partial u_1, \partial g/\partial u_2, ..., \partial g/\partial u_n]^T$. The radial directional vector $\boldsymbol{n}_k$ is computed based on the STM of chaos control by the following discrete map [8]:

$$\boldsymbol{n}_k = \boldsymbol{U}_k + \xi \boldsymbol{C}(f(\boldsymbol{U}_k) - \boldsymbol{U}_k) \tag{4}$$

in which, $\boldsymbol{C}$ is the involutory matrix (namely, only one element in each row and each column in this matrix is 1 or -1 and the others are 0, usually, $\boldsymbol{C}$ is orthogonal matrix i.e. $\boldsymbol{C} = \boldsymbol{I}$). ξ is chaos control factor which is given as $0 < \xi < 1$. The factor ξ should be selected a small constant value (i.e. 0.1–0.001 [8, 5]) to achieve the stabilization for highly nonlinear LSFs. The $f(\boldsymbol{U}_k)$ is the discrete nonlinear map which is computed as

$$f(\boldsymbol{U}_k) = \frac{\nabla^T g(\boldsymbol{U}_k)\,\boldsymbol{U}_k - g(\boldsymbol{U}_k)}{\nabla^T g(\boldsymbol{U}_k)\,\nabla g(\boldsymbol{U}_k)} \nabla g(\boldsymbol{U}_k) \tag{5}$$

The control factor of radial direction can be controlled the instability of FORM formula and this radial directional vector in Eq. (4) –based chaos control is improved the computational cost of STM for highly nonlinear performance functions, remarkably.

2.2 Conjugate chaos control (CCC) method

The chaos control iterative FORM formula-based STM is improved based on the conjugate search by Keshtegar as follows [9]:

$$\boldsymbol{U}_{k+1}^{CCC} = \boldsymbol{U}_{k}^{CCC} + \xi[f^{C}(\boldsymbol{U}_{k}^{CCC}) - \boldsymbol{U}_{k}^{CCC}] \tag{6}$$

Where, $\boldsymbol{U}^{CCC}$ is the point which is determined based on the iterative formula of conjugate FORM with chaos feedback control. λ is control factor which can be generated as $0<\xi<1$. $f^{C}(\boldsymbol{U}_{k}^{CCC})$ is the solution of the kth iteration based on conjugate search direction at the point $\boldsymbol{U}_{k}^{CCC}$. $f^{C}(\boldsymbol{U}_{k}^{CCC})$ is determined based on a discrete nonlinear map as follows:

$$f^{C}(\boldsymbol{U}_{k}^{CCC}) = \frac{\nabla^{T} g(\boldsymbol{U}_{k}^{CCC})\boldsymbol{U}_{k}^{CCC} - g(\boldsymbol{U}_{k}^{CCC})}{\nabla^{T} g(\boldsymbol{U}_{k}^{CCC})\boldsymbol{\alpha}_{k}^{C}}\boldsymbol{\alpha}_{k}^{C} \tag{7}$$

where, $\boldsymbol{\alpha}_{k}^{C}$ can be defined as the normalized conjugate search direction vector or conjugate unit vector which is computed as follows [1, 11]:

$$\boldsymbol{\alpha}_{k}^{C} = \frac{\boldsymbol{U}_{k+1}^{C\lambda}}{\left\|\boldsymbol{U}_{k+1}^{C\lambda}\right\|} \tag{8}$$

The point $\boldsymbol{U}_{k+1}^{C\lambda}$ is along the conjugate direction at design point $\boldsymbol{U}_{k}^{CCC}$, which is computed as

$$\boldsymbol{U}_{k+1}^{C\lambda} = \boldsymbol{U}_{k}^{C} + \lambda_{k}\boldsymbol{d}_{k} \tag{9}$$

where, $\boldsymbol{d}_{k}$ is the conjugate search direction vector, which is defined as follows [10, 11]:

$$\boldsymbol{d}_{k} = -\nabla g(\boldsymbol{U}_{k}^{CCC}) - \frac{\left\|\nabla g(\boldsymbol{U}_{k}^{CCC})\right\|^{2}}{\left\|\nabla g(\boldsymbol{U}_{k-1}^{CCC})\right\|^{2}}\boldsymbol{d}_{k-1} \tag{10}$$

If λ is well-defined, then the convergence of conjugate FORM can be obtained in reliability analysis, accurately. The finite- step size λ_{k} in Eq. (9) is determined as follows:

$$\lambda_{k} = \begin{cases} \lambda_{k-1} & if \;\; \left\|\boldsymbol{U}_{k+1}^{CCC} - \boldsymbol{U}_{k}^{CCC}\right\| < \left\|\boldsymbol{U}_{k}^{CCC} - \boldsymbol{U}_{k-1}^{CCC}\right\| \\ 0.9\lambda_{k-1} & o.w \end{cases} \tag{11}$$

The initial finite-step size in Eq. (11) is defined based on the Armijo-line search rule as below

$$\lambda_{0} = \frac{25}{\left\|\nabla g(\boldsymbol{U}^{CCC}|_{\boldsymbol{U}=\mu})\right\|} \tag{12}$$

The above step size implies the sufficient descent condition as $\left\|\boldsymbol{U}_{k+1}^{CCC} - \boldsymbol{U}_{k}^{CCC}\right\| < \left\|\boldsymbol{U}_{k}^{CCC} - \boldsymbol{U}_{k-1}^{CCC}\right\|$, thus it can be concluded that $\left\|\boldsymbol{U}_{k+1}^{CCC} - \boldsymbol{U}_{k}^{CCC}\right\| \approx 0$ when $k \to \infty$. This means that $\boldsymbol{U}_{k+1}^{CCC} \approx \boldsymbol{U}_{k}^{CCC}$ and a fixed point is obtained based on conjugate iterative formula.

3. Probabilistic model of nanocomposite beam

The reliability analysis of nanocomposite beam can be done based on a probabilistic buckling model tha t this probabilistic model is included the various uncertainties of the beams in geometrical of beams, nanop articles and mechanical properties of foundation. Therefore, a limit state function which is separated the des ign domain into the failure and safe regions based on the buckling mode of the nanocomposite beams can be defined as follows:

$$g(L,h,kw,kg,V0,rho) = P_{cr}(L,h,kw,kg,V0,rho) - P \tag{13}$$

It can be supposed that the nanocomposite beams may be failed when the buckling forces (P_{cr}) is obtai ned less than the external compressive load (*P*) of the beams (P is given as 50GPa in this study). $P_{cr}(L,h,kw,kg,V0,rho)$ is the buckling force, which is depended on the length of beam (*L*), thickness of b eam (*h*), spring constant of foundation (*kw*), shear constant of foundation (kw), applied voltage (*V0*), and v olume fraction of CNTs in beam (*rho*). A polymeric-reinforced CNTs beam under external applied voltage, which is shown in Fig. 1, is considered for comparing the CCC and DCC methods in the reliability analys is based on the limit state function in Eq. (13).

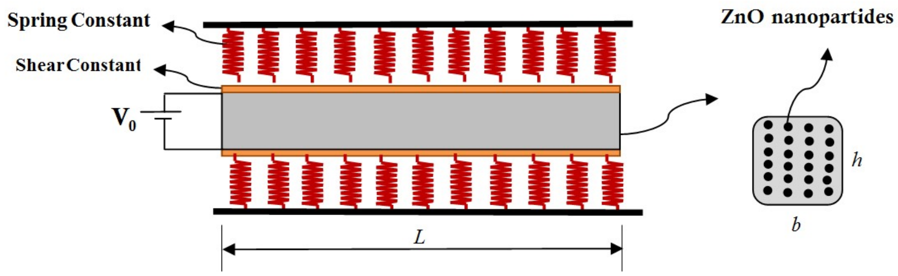

Fig. 1. The schematic view of the nanocomposite beam

The structure is modeled with Euler-Bernoulli beam theory where the displacement field can be given as follows [13]:

$$u_1(x,z)=U(x)-z\frac{\partial W(x)}{\partial x},\quad u_2(x,z)=0,\quad u_3(x,z)=W(x), \tag{14}$$

where $U(x)$ and $W(x)$ are displacement components in the mid-plane. The von Karman type nonlinear strain–displacement relations are given by

$$\varepsilon_x=\left(\frac{\partial U}{\partial x}\right)+\frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2-z\left(\frac{\partial^2 W}{\partial x^2}\right). \tag{15}$$

The stress-strain relations can be written as

$$\sigma_{xx}=C_{11}\varepsilon_x \tag{16}$$

where C_{11} is elastic constant which can be calculated by Mori-Tanaka model. The strain energy of the structure can be expressed as

$$U=\frac{1}{2}\int_0^L\int_A(\sigma_{xx}\varepsilon_{xx})\,dAdx. \tag{17}$$

Submitting Eqs. (14) and (15) into (17) gives

$$U=\frac{1}{2}\int_0^L\left\{N_x\frac{\partial U}{\partial x}+\frac{1}{2}N_x\left(\frac{\partial W}{\partial x}\right)^2-M_x\frac{\partial^2 W}{\partial x^2}\right\}dx, \tag{18}$$

where N_x is the resultant force and M_x is bending moment. The external work due to the foundation can be written as [14]

$$\Omega=\int_0^L\left(\underbrace{-K_wW+G_p\nabla^2W}_{q}\right)Wdx. \tag{19}$$

The governing equations of structure can be derived from the Hamilton's principle as

$$\delta U:\frac{\partial N_x}{\partial x}=0, \tag{20}$$

$$\delta W:\frac{\partial^2 M_x}{\partial x^2}-\frac{\partial}{\partial x}\left(N_x^M\frac{\partial W}{\partial x}\right)+q=0, \tag{21}$$

where N_x^M is the axial load applied to the concrete column. Introducing the following dimensionless quantities

$$\xi=\frac{x}{L},\ (\overline{W},\overline{U})=\frac{(W,U)}{h},\ \eta=\frac{h}{L},\psi=\overline{\psi},\ \overline{K}_w=\frac{K_whL}{C_{11}A},\quad \overline{G}_p=\frac{G_p}{C_{11}A},\overline{N}_x^M=\frac{N_x^M}{C_{11}A},\overline{I}=\frac{I}{AL^2},\ C_5=\frac{C_{55}}{C_{11}}. \tag{22}$$

and substituting Eqs. (21)-(22) into the governing equations yields

$$\frac{\partial^2\overline{U}}{\partial\xi^2}+\eta\frac{\partial^2\overline{W}}{\partial\xi^2}\frac{\partial\overline{W}}{\partial\xi}=0, \tag{23}$$

$$-\overline{I}\frac{\partial^4\overline{W}}{\partial\xi^4}-\overline{N}_x^M\eta\frac{\partial^2\overline{W}}{\partial\xi^2}-\overline{K}_w\overline{W}+\overline{G}_p\eta\left(\frac{\partial^2\overline{W}}{\partial\xi^2}\right)=0. \tag{24}$$

Finally, using DQM, we have

$$\left(\left[\underbrace{K_L + K_{NL}}_{K}\right] + P\left[K_g\right]\right)\begin{Bmatrix}\{d_b\}\\ \{d_d\}\end{Bmatrix} = 0, \tag{25}$$

where K_L is the linear stiffness matrix; K_{NL} is the nonlinear stiffness matrix and K_g is geometric stiffness matrix. Also, d_b and d_d represent boundary and domain points. Finally, based on an iterative method and eigenvalue prob lem, the buckling load of structure may be obtained. However, the critical buckling load (*Pcr*) is theoretically o btained based on the Eq. (25). It is supposed that a nanocomposite beam is under external comparative for ce i.e. P=50 GPa and the capacity of beam is obtained based on the buckling force in Eq. (25). Six basic random variables are given for this nanocomposite beam in Fig. 1 that these basic random variables are ext racted from Table 1.

Table 1: the basic random variables of the nanocomposite beam

Random variable	*h* (m)	*L* (m)	*kw* (N/m^2)	*kg* (N)	*rho*	*V0* (volte)
Mean	0.4	0.6	3.5×10^{12}	5	0–0.5	-200 – 200
Coefficient of variation	0.1	0.1	0.12	0.12	0.15	0.2
Distribution	Lognormal	Normal	Gumbel	Gumbel	Lognormal	Normal

4. Reliability results of nanocomposite beam

A nanocomposite beam under buckling failure mode is selected to illustrate the performance convergences both robustness and efficiency of the FORM formula-based CCC and DCC methods. The stable convergence and number of call functions to evaluate the buckling forces based on the Eq. (25) are used to compare the convergence performances of the nanocomposite beam in Fig. 1. The MATLAB programs are codded for reliability analysis of this complex example that the MATLAB codes can be evaluated the sensitivity vectors and simulated the normal and non-normal random variables. The gradient vector of limit sate function in Eq. (25) is determined based on the finite difference method.

The mean values for applied voltage and volume fraction of CNTs are respectively given as *V0*=100volte and *rho*=0.3 for reliability analysis. Based on the statistical properties of random variables in Table 1, the CCC and DCC methods are applied to approximate the reliability index. The results of reliability analysis including reliability index (β) the numbers of evaluating the gradient vector of limit state function (Iter), numbers of calling the limit state function (Call) for CCC and DCC methods with respect to different control factors λ=0.05, 0.1, 0.5, and 1 are tabulated in Table 2. The reliability index histories of CCC and DCC methods for λ=0.1 and 0.5 are shown in Fig. 2. The results from the Table 2 and Fig. 2 show that the convergence performances including both efficiency and robustness of CCC and DCC methods are strongly depended on the chaos control factor. The CCC method is yielded unstable results as chaotic solutions when the chaos control factor is given less than 0.1 for this nanocomposite beam. However, the CCC method is provided stable results because the initial finite-step size in Eq. (12) is adjusted based on the Eq. (11) for each iterations when the chaotic results are occurred from the iterative formula of CCC method. By selecting a large number for chaos control factor in CCC method, the numbers of evaluating the gradient vector (Iterations) for CCC method are slightly deceased to achieve the stabilization. The CCC method is converged, more efficiently when the λ is given equal to 1. The DCC is more efficient than the CCC method based on the λ=0.5. According to λ=1, the CCC is converged with the call functions about three times less than the DCC method. In addition, the CCC formula is more robust than the DCC method and the nonlinear dynamic map to search MPP based on the CCC method can be provided stable solutions for chaos control factor between 0.05 and 1. Howevere, the stability of the DCC is depended on the appropriate selection of control factor. The obtained MPP using the DCC and CCC based on λ=1 is ***X**** = (*L** = 0.395245m, *h** = 0.3959143m, *kw* = 3112.0935GPa, *kg* =19.5625N, $\rho = 0.29562479$, *V0* =100.7514volte).

Table 2: The converged results of CCC and DCC for different chaos control factor

Method	λ=0.05		λ=0.1		λ=0. 5		λ=1	
	β	Iter\Call	β	Iter\Call	β	Iter\Call	β	Iter\Call
CCC	3.555806	4559\523664	3.555796	2677\307211	3.555796	107\11983	3.555796	8\857
DCC	Chaos	Not converged	Chaos	Not converged	3.555798	75\17153	3.555796	13\3085

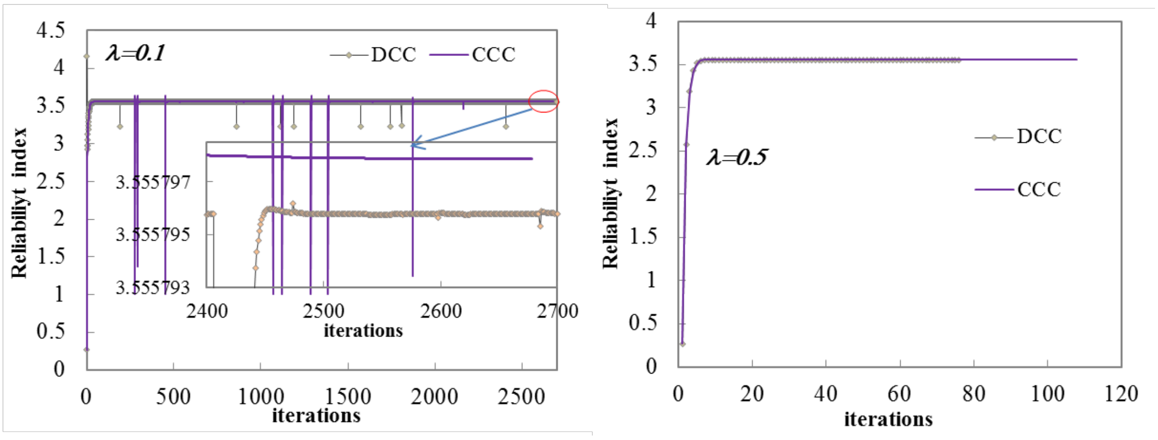

Fig. 2. Reliability index histories of CCC and DCC methods for chaos control factor 0.1 and 0.5

The reliability indexes of nanocomposite beam with respect to various applied voltages (V0) and volume fractions of CNTs (rho) are computed that the reliability indexes using the CCC method with chaos control factor of 1 are computed. Figures 3 and 4 illustrate the reliability index with respect to the various applied voltages and volume fractions of nanoparticles, respectively. The results from Figs. 3 and 4 showed that the reliability indexes is decreased by increasing the applied voltage while the volume fractions of nanoparticles can be improved the reliability index of nanocomposite beams. The reliability index is reduced from 3.7 to 3.5 when the applied voltages is changed from -200 to 250 volte in rho=0.3. However, the reliability index can be improved from 3.5 to 3.6 based on the volume fractions of nanoparticles 0.05-0.5 in applied voltage of 100volte. It can be concluded from the results of Figs. 3 and 4 that the decreasing applied voltage and increasing the volume fractions of nanoparticles can be improved the confidence level of the nanocomposite beams. The applied voltage is performed better than the volume fractions of nanoparticles on increasing the reliability index.

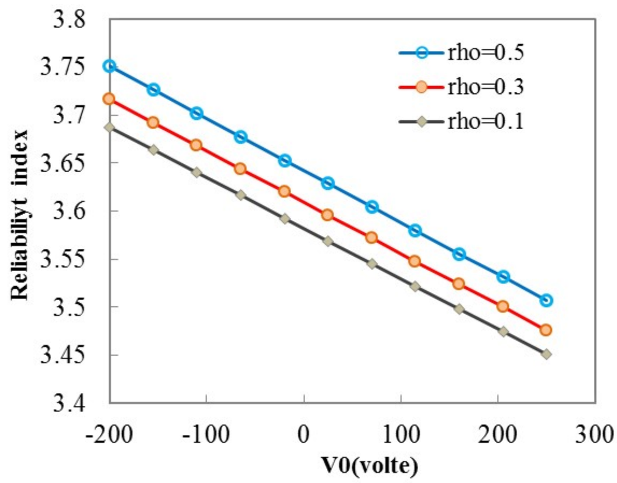

Fig. 3. Reliability index corresponding to various applied voltages based on the CCC for different rho

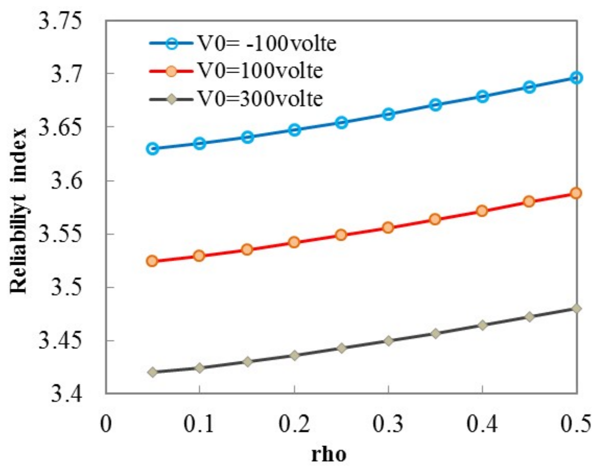

Fig. 4. Reliability index corresponding to various volume fractions of nanoparticles based on the CCC for different V0

5. Conclusion

The conjugate chaos control (CCC) and the directional chaos control (DCC) using stability transformation method were recently developed to improve the efficiency and robustness of iterative formula of first order reliability method (FORM). The capability and efficiency of these reliability method are performed better compared to the other reliability methods such as HL-RF, relaxed HL-RF, stability transformation method (STM). The CCC and DCC methods are implemented for reliability analysis –based FORM of a nanocomposite beam which is reinforced by the CNTs. The CCC and DCC methods are compared based on the nanocomposite beam to illustrate their robustness and efficiency for reliability analysis of the complex engineering problems in future. The stable r esults and number of evaluating the performance function of the buckling failure mode, which is determine d based on an implicit limit state function using Euler-Bernoulli beam model used to compere the CCC and DCC methods. A probabilistic model for buckling force is theoretically developed and then the reliability in dexes of the beam are computed with respect to different value of applied voltages and volume fractions of CNTs. The results of the reliability analysis for nanocomposite beam showed that the CCC method is a robust and efficient FORM formula in comparison with the DCC method when a large chaos control factor is selected. The CCC is converged to stable reliability index while the DCC is yielded unstable results as chaotic solutions for chaos control factor less than 0.1. Decreasing the applied voltage and increasing the CNTs in nanocomposite beam can be improved the reliability index for this complex reliability problem.

Acknowledgments

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